Rail track resistance verification considering track-bridge-interaction

Verifizierung des Schienenwiderstands unter
Berücksichtigung der Gleis-Tragwerk-Interaktion

DISSERTATION

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Welcome to the world of rails.

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Rail track resistance verification considering track-bridge-interaction

Abstract: As rails are vital parts of a track system, it is essential to ensure their safe and reliable operation. The present verification approaches and limit values regarding the permissible additional stresses of the rail under compression and tension considering track-bridge-interaction (TBI) were developed in the 1980s. However, with the rapid development of the railway industry and the increasing of train speeds, rail infrastructures are subjected to ever more frequent, greater loads and more complicated loading conditions, especially in the area of bridges. Moreover, the manufacturing technologies of railway components have been further developed. Taking all the aforementioned variations into account, the current verification approaches and limit values do not apply properly today and shall be updated. For this purpose, new investigations are carried out in this dissertation.

As major parts of this cumulative dissertation, the published investigations are divided into three main blocks. The first block is the state of the art. In this block, a detailed background knowledge and a state of the art description of the permissible additional stresses in railway tracks due to TBI are given. Furthermore, the motivation for the studies within the scope of this dissertation is addressed. The second block deals with the rail resistance under compressive forces in ballastless track systems. Accordingly, numerical investigations on the behaviour of rails in ballastless track systems under compressive axial forces in the vicinity of bridge joints were performed. Experimental tests were also carried out on two 8.17 m long rails fixed with BSPFF-B-1 and SBS300-1 fasteners on the ÖBB-Porr slab track system. It was found that the rail resistance under longitudinal compressive loads can be largely increased. The third block focuses on the rail resistance under tension. First, extensive experiments were conducted on rail behaviour for up to five million cyclic loads in both vertical and transverse directions under different minimum stress levels. Subsequently, the sectioning method and the X-Ray diffraction method were applied to determine the residual stress distribution in the rail. Afterwards, the determined residual stress results and the fatigue test results are analysed together. As a result, a new comprehensive Smith-diagram, which took into account the actual rail residual stresses, up to five million load cycles in both vertical and transverse directions of the rail, was achieved.

In addition, two studies are supplemented. One deals with the fatigue behaviour of rails for up to 50 million load cycles and the other concerns the fatigue behaviour of rails from a different batch for up to five million load cycles. Based on all these aforementioned investigations, it is concluded that the current limit values and approaches regarding the rail resistance in ballastless track systems under compression and tension considering TBI are too conservative. In the end, new verification approaches and limit values are proposed.

Verifizierung des Schienenwiderstands unter Berücksichtigung der Gleis-Tragwerk-Interaktion

Kurzfassung: Da Schienen ein elementarer Bestandteil des Gleissystems sind, muss ihr sicherer und zuverlässiger Betrieb gewährleistet werden. Die gültigen Nachweisansätze und die Grenzwerte der zulässigen zusätzlichen Schienenspannung unter Druck und Zug mit Berücksichtigung der Gleis-Tragwerks-Interaktion wurden in den 1980er Jahren entwickelt. Mit der rasanten Entwicklung im Bereich des Eisenbahnverkehrs und den erhöhten Zuggeschwindigkeiten ist die Schieneninfrastruktur immer häufiger, höheren Belastungen und komplexen Belastungszuständen, insbesondere im Bereich der Brücken, ausgesetzt. Zudem haben sich die Fertigungstechnologien von Oberbaukomponenten weiterentwickelt. Unter Berücksichtigung aller vorgenannten Veränderungen sind die bisherigen Nachweisansätze und Grenzwerte für die heutigen Gegebenheiten nicht mehr geeignet und sollen aktualisiert werden. Die im Rahmen der Dissertation durchgeführten Untersuchungen befassen sich mit neuen Untersuchungen zu dieser Thematik.

Als wesentliche Teile dieser kumulativen Dissertation sind die veröffentlichten Untersuchungen in drei Abschnitte gliedern. Der erste Abschnitt zeigt den Stand der Technik. In diesem Block wird der Stand der Technik bezüglich der zulässigen Zusatzspannungen in Eisenbahnschienen infolge der Gleis-Tragwerks-Interaktion beschrieben und ein detailliertes Hintergrundwissen vermittelt. Weiterhin wird die Motivation für die im Rahmen dieser Dissertation durchgeführten Untersuchungen dargelegt.

Der zweite Abschnitt befasst sich mit dem Schienenwiderstand unter Druck in Feste Fahrbahn-Systemen. Dazu wurden numerische Untersuchungen zum Verhalten von Schienen in Feste Fahrbahn-Systemen unter axialen Druckkräften in der Nähe von Brückenfugen durchgeführt. Außerdem wurden experimentelle Versuche an zwei 8,17 m langen Schienen auf einem Festen Fahrbahn-System der ÖBB-Porr durchgeführt. Als Befestigung kamen die Typen BSPFF-B-1 und SBS300-1 zum Einsatz. Dabei wurde festgestellt, dass der Schienenspannungswiderstand unter Längsdruckbelastung deutlich erhöht werden kann.

Der dritte Abschnitt beschäftigt sich mit dem Schienenspannungswiderstand unter Zug. Zunächst wurden umfangreiche Versuche zum Schienenverhalten bei bis zu fünf Millionen Lastwechseln in Längs- und Querrichtung auf verschiedenen Unterspannungsniveaus durchgeführt. Anschließend wurden die Schnittmethode und die Röntgen-Diffraktometrie angewandt, um die Eigenspannungsverteilung in der Schiene zu bestimmen. Anschließend werden die ermittelten Eigenspannungsergebnisse und die Ergebnisse der Ermüdungsversuche gemeinsam analysiert. Als Ergebnis wurde ein neues umfassendes Smith-Diagramm erstellt, das die tatsächlichen Schieneneigenspannungen bis fünf Millionen Lastzyklen sowohl in vertikaler als auch in transversaler Richtung der Schiene berücksichtigt.

Darüber hinaus werden zwei Studien präsentiert. Die erste befasst sich mit dem Ermüdungsverhalten von Schienen für bis zu 50 Millionen Lastwechsel und die zweite mit dem Ermüdungsverhalten von Schienen aus einer anderen Charge für bis zu fünf Millionen Lastwechsel. Aus all diesen vorgenannten Untersuchungen wird gefolgert, dass die derzeitigen Grenzwerte und Nachweisansätze bezüglich des Schienenspannungswiderstandes in Feste Fahrbahn-Systemen unter Druck und Zug mit Berücksichtigung der

Gleis-Tragwerks-Interaktion zu konservativ sind. Abschließend werden neue Nachweisansätze und Grenzwerte vorgeschlagen.

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1 Introduction

1.1 Motivation

When the first segment of the Shinkansen train line (Japanese bullet, Tokyo–Osaka) with an operating speed of 210 km/h was opened in 1964 for the Olympic Games, high-speed railway (HSR) was born [1]. After the big success of the Shinkansen operation, several European countries, particularly France, Germany, Italy and the UK, developed new technologies and innovations aimed to establish the basis for the "passenger railway of the future" [2]. Finally, SNCF (the French national railway company) started the operation of the first HSR line between Paris to Lyon on 27th September 1981, at a maximum speed of 260 km/h. In contrast to the Shinkansen concept, the new European HSR was fully compatible with the existing railways, which facilitated further development of the system on the old continent [3]. Afterwards, this HSR construction fever spread across the world. Joining the group of countries offering HSR services were Italy in 1981, Germany in 1988 (ICE trains), Spain in 1992, Belgium in 1997, the UK and China in 2003, Switzerland and South Korea in 2004, the Netherlands and Turkey in 2009, Austria in 2012, Poland in 2015, and the USA in 2017 etc. So far, 21 countries/regions have developed HSR networks (with minimum operating speeds of 250 km/h) according to the newest statistic report of UIC in 2020 [4]. By this time, 52,484 km of HSR lines were in operation, 11,960 km were under construction and 23,343 km were under development. Fig. 1 shows the development of HSR lines since 1964. Nowadays, HSR transportation systems are preferred over other modes of transportation because of their high speed, better safety, and fewer environmental negative impacts [5–8]. The number of countries boosting HSR networks will likely continue to increase.

Fig. 1: Global HSR line development [3,4,9,10]

To cope with the rapid development of HSR, continuously welded rails (CWR) have been commonly constructed to gain better smoothness; longer bridges have been built to cross wider obstacles and to meet higher requirements of the lines. However, when CWR are laid on long bridges, additional rail stresses due to the coupling effects of tracks and deformed superstructures are superimposed on other existing stresses. As a consequence, this increases the risk of rail buckling in summer and rail fracture in winter. Hence, the additional stresses due to track-bridge-interaction (TBI) must be correctly considered and controlled to ensure safe operation [11,12]. For this purpose, the determination of structural parameters, related actions, the modelling and calculation methods of the combined TBI system as well as the design criteria have been specifically

regulated in both international and national codes [13–17] since the 1990s, and the regulations have remained almost unchanged in the last 30 years. These regulations are based on abundant investigations on ballast track systems in the 1970s to 1990s, mainly in Germany.

Due to the boost of HSR in the last years, railway technology has been greatly developed. The operating speed has reached 350 km/h in many countries. The next step to increase this speed is already on the edge; heavier rolling stocks are put into operation; longer bridges have been built to achieve better alignments and to overcome larger spans. The rail infrastructures are under ever more frequent, greater loads and more complicated loading conditions, especially in the area of bridges. Besides, new ballastless track systems have been commonly constructed for HSR lines in many countries such as Germany, China, and Japan instead of ballast track systems. Moreover, the manufacturing technologies of railway components have been further developed. Considering all these aforementioned variations, it can be presumed that the assumed conditions for the verifications of TBI originated in the last century do not apply properly nowadays. In some previous studies, several scholars have proved that the current limit values are too conservative, and new investigations in updating these regulations are necessary to be carried out [18,19].

1.2 Objective and scope

This work aims to investigate the rail track resistance in consideration of TBI in ballastless track systems and to contribute to the updating of the relating verification methods. The focus is to update the permissible additional compressive and tensile stresses due to TBI. To provide a better view of the TBI phenomenon and to address the boundary of this specific topic, an introduction of the TBI verifications in the current Eurocode [13], UIC 774-3 [14], and the German guideline provided by the railway authorities, DB Netz AG Richtlinie 804 [15] is given. The scope of the current verifications due to TBI has been related to four major parts: actions; the structural configuration; the modelling and calculation of the TBI and the resistance of the system due to TBI.

Part I – Related actions: For the verification of TBI, various actions including long-time actions, short-time actions and manufacture-related effects, are necessary to be considered. The long-time actions are the effects of the temperature on the rail and bridge as well as the creep and shrinkage of concrete. These actions induce deformations of the rail in the longitudinal direction. The short-time actions are mainly due to train traffic. For instance, the axle load in the vertical direction and the traction/braking forces in the horizontal direction. These loads introduce longitudinal displacements and rotations of the track-bridge system, respectively. In addition, the manufacturing process of the rail inevitably generates residual stresses in the rail cross-section. These residual stresses are superimposed on other stresses in the operating rail. All the above-mentioned actions have a major impact on the system's TBI behaviour.

Part II – Structural configuration: The track-bridge system is composed of the track system and the bridge system. The configuration of the track system depends on the rail alignment, the type of the track system (ballast/ballastless), the existence of special devices like rail expansion joints and turnouts as well as the properties of the applied components such as fasteners, rail pads etc. The configuration of the bridge relies on the static system of the bridge, geometries of both the superstructure and the sub-structure,

the used material, and other related components such as dampers and coupling devices if exist, etc. Correct analysis and consideration of the track-bridge system are essential for further calculations.

Part III – The calculation and modelling of TBI: This mainly concerns the modelling of the loads, the calculation and modelling strategy of the track-bridge system structurally, in particular the modelling of the load transmission or interaction behaviours between the track and the bridges. For instance, the coupling effects of the ballast track system in all six degrees of freedom, and the contact behaviour of the interfaces between different layers of ballastless track systems.

Part IV – The resistance of the track-bridge system: Some limit values, which correspond to either the resistance of the rail track or the track-bridge system, are regulated. These restrictions can be roughly divided into two parts. The first part is about the resistance of the railway track itself, more precisely, the additional stresses under compression and tension due to TBI must not exceed certain magnitudes. The second part deals with the limit deformation values of the track-bridge system. It concerns the relative longitudinal displacements between two neighbouring decks as well as between a deck and an adjacent abutment; the longitudinal displacement of the upper surface of the deck at the deck end; the vertical displacement of the upper surface of a deck relative to the adjacent construction. In addition, the uplift forces on rail supports and fastening systems of directly fastened rails are also limited.

To evaluate the TBI behaviour, all of these four parts are necessary to be considered. Many researchers dedicate themselves to certain topics in this area. This dissertation focuses on the resistance of the rail itself due to TBI, which falls within the scope of Part IV and aims to study and update the limit values of permissible additional stresses. To achieve this, literature, numerical and experimental studies have been carried out.

1.3 Outline

For the abovementioned purposes, comprehensive studies have been performed. These studies are categorized into three blocks. Fig. 2 shows an overview.

Fig. 2: Overview outline of this work

The first block is the state of the art. It contains one publication, additional information can be found in [20] (also see section 2.1). This block aims to provide detailed background knowledge and a state of the art description of the permissible additional stresses in railway tracks due to TBI. The current verification values of national and international codes (Germany, UIC, Eurocode, Spain, Italy and China) are firstly introduced. Subsequently, a detailed explanation of TBI including the decisive historical investigations, rail stresses and permissible additional stresses is given. Next, the background and recent investigations of the limit values of permissible additional rail compressive stress (72 MPa for ballast track systems and 92 MPa for ballastless track systems) and tensile stress (92 MPa) due to TBI in Eurocodes and German codes are specifically elaborated. Lastly, it is concluded that these permissible additional stress limit values are too conservative with the rapid development of HSR and rail manufacturing technologies. Consequently, new investigations are necessary to be carried out. This block explains the background and motivation of the further investigations in the following blocks.

The second block is the investigation of the rail resistance under compressive forces in ballastless track systems, which is directly related to the permissible additional compressive stress due to TBI. It has one publication, additional information can be found in [21] (also see section 2.2). According to this work, numerical simulations and experiments were performed to study the behaviour of rails in ballastless track systems under compressive axial forces in the vicinity of bridge joints. First, a numerical pre-study was carried out to find the proper simulation strategy for modelling the mechanical behaviour of the fasteners and the critical imperfections of the railway track system. Afterwards, experimental tests were carried out on two 8.17 m long rails fixed with BSPFF-B-1 and SBS300-1 fasteners on the ÖBB-Porr slab track system. Each rail was tested in three different situations: with all fasteners, with one pair of fasteners removed and with two pairs of fasteners removed. After each test, the related input parameters of the numerical models were adapted to the actual test conditions. The results of the numerical models had good consistency with the experimental results. In the end, it was concluded that the rail resistance under longitudinal compressive load can be largely increased.

The third block focuses on the rail fatigue resistance, which is directly related to the permissible additional tensile stress due to TBI. This block consists of three publications [22–24] (also see sections 2.3, 2.4 and 2.5). The three publications study the fatigue behaviour of rails in the vertical direction, in the transverse direction and the residual stress distribution of the rail cross-section, respectively. The first two publications provide a complete overview of the rail behaviour under cyclic loads in both vertical and transverse directions of the rail cross-section. To efficiently carry out multi-stress-level tests up to five million or even fifty million load cycles, a fatigue testing facility based on a resonant test concept by the Institute of Concrete Construction at Leibniz University Hannover was used. The tests were carried out according to the staircase method. In the vertical direction, 48 rail specimens were tested; it was planned to test three minimum stress levels, which were 50 MPa, 150 MPa and 250 MPa; for the minimum stress level of 150 MPa, both corroded and not-corroded rails were taken into account. In the transverse direction, 36 rail specimens were tested; it was planned to test two minimum stress levels, which were 150 MPa and 250 MPa; for the minimum stress level of 150 MPa, both corroded and not-corroded rails were taken into account. During the tests, the temperature, the supporting forces, the strains of the rail in the mid-span as well as the loading frequency were continuously measured; the load cycles were counted

consistently. The surface roughness condition of each rail was measured and evaluated after the test. As results, the rail failure modes, the rail material behaviour and the rail fatigue resistance were analysed and evaluated. At the end of each publication, a new Smith-diagram is recommended. The third publication in this block concentrates on the residual stress distribution of the 60E2 rail profile and its influence on rail fatigue behaviour. In this paper, two different methods, which are the sectioning method and the X-Ray diffraction method, were used to measure residual stresses. These two methods were compared and the results of the sectioning method were integrated into the Smith-diagram. Consequently, the residual stress distribution was found critical when determining the rail fatigue resistance. Finally, a new comprehensive Smith-diagram, which took into account the actual rail residual stresses, up to five million load cycles in both vertical and transverse directions of the rail, was achieved. On the one hand, these three publications are independent studies. On the other hand, they are deeply connected to as they all deal with the topic of rail resistance under cyclic loads.

In addition, two studies are supplemented. One deals with the fatigue behaviour of rails from the same batch for up to 50 million load cycles and the other with the fatigue behaviour of rails from a different batch for up to five million load cycles. Based on all the above-mentioned investigations, it is concluded that the current limit values and approaches regarding the rail resistance in ballastless track systems under compression and tension considering TBI are too conservative. In the end, new verification approaches are proposed and some outlooks are given.

2 Publications

This cumulative dissertation is composed of five papers. These papers have already been published in peer-reviewed journals. The five publications are listed below:

1) Publication No. 1

Kang, C., Wenner, M., Marx, S. (2021). Background investigation on the permissible additional rail stresses due to track/bridge interaction. *Engineering Structures*, 228: 111505.

DOI: 10.1016/j.engstruct.2020.111505

2) Publication No. 2

Kang, C., Bode, M., Wenner, M., Marx, S. (2019). Experimental and numerical investigations of rail behaviour under compressive force on ballastless track systems. *Engineering Structures*, 197: 109413.

DOI: 10.1016/j.engstruct.2019.109413

3) Publication No. 3

Kang, C., Schneider, S., Wenner, M., Marx, S. (2020). Experimental investigation on rail fatigue resistance of track/bridge interaction. *Engineering Structures*, 216: 110747.

DOI: 10.1016/j.engstruct.2020.110747

4) Publication No. 4

Kang, C., Schneider, S., Wenner, M., Marx, S. (2021). Experimental investigation on the fatigue behaviour of rails in the transverse direction. *Construction and Building Materials*, 272: 121666.

DOI: 10.1016/j.conbuildmat.2020.121666

5) Publication No. 5

Kang, C., Wenner, M., Marx, S. (2021). Experimental investigation on the rail residual stress distribution and its influence on the bending fatigue resistance of rails. *Construction and Building Materials*, 284: 122856.

DOI: doi.org/10.1016/j.conbuildmat.2021.122856

The information concerning the individual authorship of each paper is clarified respectively in the following sections.

2.1 Background investigation on the permissible additional rail stresses due to track/bridge interaction

First author:	Chongjie Kang
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Other co-authors	Marc Wenner, Steffen Marx
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This paper is based on [11,12], however, greatly further developed and completed with new sources and context. Chongjie Kang was responsible for the literature research, data curation, visualization and original draft writing. Marc Wenner was responsible for resources. The literature he provided found the initial basis of this work. Steffen Marx was responsible for the acquisition of the funding for this investigation. Many ideas were obtained in discussions and exchanges of the co-authors. The successful publication of this work was achieved due to the intensive cooperation of all co-authors. All co-authors reviewed this paper and provided meaningful comments during the process.

Background investigation on the permissible additional rail stresses due to track/bridge interaction

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ABSTRACT

The permissible additional rail stresses are essential when verifying the track/bridge interaction. Firstly, the current verification values of the German codes and other international codes (Eurocode, UIC, Spain, Italy and China) are introduced in this work. Subsequently, a detailed explanation of track/bridge interaction including the decisive historical investigations, rail stresses and permissible additional stresses is given. Afterwards, the background and related recent investigations on the limitation of additional permissible rail compressive stress (92 MPa/72 MPa) and tensile stress (92 MPa) due to track/bridge interaction in Eurocodes and German codes are comprehensively explained. It can be concluded that these permissible additional stress limitations are no longer dependable with the rapid development of high-speed railway and rail manufacturing technologies. New investigations are necessary to be carried out.

1. Introduction

The rail is the most significant track superstructure component of the railway infrastructure; it fulfils the functions of the driving surface, direction guidance as well as force transmission. With the introduction of continuous welded rail (CWR), an almost continuous driving surface was achieved, and therefore, higher speed and heavier axle loads could have been achieved. As an interface between train wheels and infrastructures, the rail track is under considerably high stresses. When the rails are constructed on bridges, the behaviour of bridges leads to additional stresses for CWR tracks. As a result, additional rail stresses due to the coupling effects of tracks and deformed superstructures are superimposed on other stresses. These additional stresses occur mainly because of the longitudinal elongation of the superstructure due to temperature effects, braking/traction forces, vertical bending effects as well as the creep and shrinkage [1–3]. This is no critical problem when the rails are not continuously welded since rail stresses can be released with the existing of rail joints, or when the bridge is short. However, due to the growing demand for a stable higher speed of railway traffic, especially high-speed railway (HSR) traffic, rails have been welded continuously to gain better smoothness; longer bridges have been constructed to cross wider obstacles and to meet higher requirements of the lines [4]. The track/bridge interaction (TBI) has to be, therefore,

precisely considered and calculated. The investigations on the verifications of track/bridge interaction can be classified into three major aspects [5]: actions like temperature [6], braking/traction loads [7,8], creep and shrinkage [9] etc. on track/bridge systems, resistance of the TBI system (limits and thresholds values) [10–12] as well as calculation, modelling and simulation of the structures and load transmissions in TBI system [13–17]. Many researchers carried out innovations studies with different focuses in these abovementioned fields. This work focuses on the background of the permissible additional stresses in rails due to TBI.

Aim to limit these additional stresses in rails, different investigations have been carried out at national and international levels to limit the permissible additional tensile and compressive stresses due to TBI. The resultant limit values from different codes and nations are listed in Table 1.

Table 1 shows that the limit values of Eurocode 1991, UIC 774-3, Richtlinie 804 (Germany) and IAPF (Spain) are the same except that the compressive stress limit of the German code is different for ballasted and ballast-less track systems. In comparison, the values of I/SC/PSOM/2298 Instruction (Italy) [20] and TB 10621–2014 (China) [21] are very different as they are based on different theoretical concepts. This work focuses on the background of the permissible additional stresses within the scope of German Richtlinie 804, Eurocode 1991 and UIC 774-3 (International Union of railways).

The limitations in Table 1 are based on the investigations on ballast

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Nomenclature			
a	Sleeper distance	σ_B	Additional tensile stress due to TBI
A_B	Bending work	σ_{BB}	Additional stresses due to bridge bending
A_E	Longitudinal expansion work	σ_{BL}	Additional stresses due to longitudinal displacements of bridges resulting from braking/traction
A_V	Transverse displacement work	σ_{BT}	Additional stresses due to bridge temperature variation
A_{SL}	Shear work in the longitudinal direction	σ_{limit}	Upper limit strength of the rail
A_R	Total area of both rail cross-sections of one railway track	σ_{min}	Minimum stress
A_{RT}	Rotation work of the rail	σ_{max}	Maximum stress
$A_{sleeper}$	Sleeper cross-section area	σ_{mean}	Mean stress
b	Width of assumed boundary	σ_Q	Bending stress due to the crossing of trains
C	Foundation modulus	σ_R	Residual stress
E_R	Young's Modulus	$\sigma_{S,M}$	Average yield stress value
f	Fictitious track direction error	σ_T	Temperature stress
$F_{R,crit}$	Critical normal force	$\sigma_{\Delta T_R}$	Temperature variation in comparison with the rail locking temperature
I	Inertia moment	τ	Proportionality factor
I_{Eq}	Equivalent moment of inertia of the two rail tracks	ΔQ	Additional force due to dynamic effects of trains
L	Elastic length	ΔT	Temperature variation
L_{span}	Span length of a bridge	$\Delta T_{R,crit}$	Critical temperature
n	Track condition parameter	$\Delta T_{R,max}$	Maximum allowed temperature
P	Longitudinal force in rail	$\Delta T_{R,S}$	Safety margin
Q	Vertical wheel force	$\Delta T_{R,B}$	Equivalent temperature of the longitudinal force due to TBI
$Q_{effective}$	Effective wheel force	$\Delta \sigma$	Stress range/allowable fatigue strength
R_t	Maximum peak-to-valley roughness	Abbreviations	
$\bar{s}$	Parameter calculated with n and φ	BMFT	German Federal Ministry of Research and Technology
v	Train speed	CWR	Continuous welded rail
w	Transverse displacement resistance	DB	German railway
$W_{R,foot}$	Section modulus of the rail foot	ERRI	European Rail Research Institute
Y	Transversal guiding force	FEM	Finite element model
α_R^{th}	Expansion coefficient	HSR	High-speed railway
μ	Influential parameter	ICE	Intercity Express
φ	Dynamic coefficient	TBI	Track/bridge interaction
$\sigma_{add,longitudinal}$	Additional longitudinal stress due to longitudinal displacement of the bridge	TUM	Technical University Munich
$\sigma_{add,bending}$	Additional bending stress due to offset and rotation of the bridge end	UIC	International Union of Railways
		ÖBB	Austrian Federal Railways

track systems in the 1980s or even previously developed theoretical models. Since then, these regulations have been merely changed. However, due to the rapid development of the railway industry, especially the HSR boom in the last 30 years, the train types, the rail properties, track systems and bridge constructions have been largely changed. First, due to higher speed and heavier trains, rails have to bear much larger and much more frequent wheel loads. Second, the manufacturing technologies for rails and relating components have been largely developed; the resistances of rails under compression and tension have been changed. Third, ballast-less track systems have become the most commonly constructed track systems instead of ballast track systems. Also, longer bridges have been commonly constructed [4]. Therefore, the conditions for railway tracks have been greatly changed, the coupling effects between bridges and rails (TBI) have become more significant.

Due to all these developments, these permissible additional stresses

need to be thus checked and updated at both national and international levels. However, the background of these present verification criteria and associated limit values remain un-clear or even unknown to the researchers in this field as it was poorly documented. To provide references and conditions for further investigations, the historical investigations and the development of these limit values are presented specifically in this work.

2. Brief history

Rails were produced in fixed lengths and joined end-to-end to form a continuous surface in the early 1920s [22]. To minimize the number of joints and thus the dynamic stresses due to collisions with train wheels, it was common to weld the rails to long sections from 1925 [23]. With the development of welding technology, continuous welded track (CWR) has become the regular type since 1954 in Germany as it has

Table 1
Permissible additional rail stresses for HSR due to TBI [MPa].

Stress	Track system	Eurocode [3]	UIC [1]	Germany [18]	Spain [19]	Italy [20]	China [21]
Compressive	Ballast	72	72	<u>72</u>	72	60	61
	Ballast-less	72	72	<u>92</u>	72	60	61
Tensile	Ballast	92	92	<u>92</u>	92	70	81
	Ballast-less	92	92	<u>92</u>	92	70	81

better lateral stability and provides continuous driving surfaces. This is in favour of avoiding collisions between wheels and tracks in contrast to jointed rails. However, at the transition zone from an embankment to a bridge, the track system's vertical stiffness changes significantly because of track-bed height reduction or even the elimination of the track-bed, thus, the deflections and settlements on both sides of the transition zone are different. These stiffness and deformation differences lead not only an increase in dynamic loads over time but also uneven rail geometry [24,25]. As a result, this causes large additional forces in rails and causes rail failures in the area of bridges. This issue becomes more significant for long bridges because of larger deformations and for ballast-less track systems due to greater stiffness variations. To deal with these additional forces in the area of movable bridge joints, two typical approaches have been regularly applied: one is to install rail expansion joints (for concrete bridges longer than 90 m, steel and composite bridges longer than 60 m [3]) to release rail stresses; the other is to construct sliding tracks on steel bridges to reduce the coupling effects [26]. However, the rail stresses for CWR in the bridge area have to be limited to avoid two major issues: the risk of buckling in summer and the risk of rail fracture in winter. The buckling risk can be avoided by optimizing the superstructure and applying safety factors. The rail fracture can be limited by controlling the forces of fasteners to a tolerable level. By applying special fasteners with reduced resistance in the area of bridge joints, the longitudinal sliding of the rail is spread in a larger length. Therefore, the stresses are more unevenly distributed and the maximum stresses in the rail can be reduced. As a result, the limit values are not exceeded. In general, the TBI can be avoided by installing rail joint expansion devices at the cost of introducing discontinuities in the track and additional maintenance.

One of the first theoretic investigations on TBI due to temperature effects dates back to the 1930s [27]. Meier developed a theoretical method to evaluate the stability of CWRs in the transverse direction based on an energy method [28]. Japanese researchers carried out specific investigations on TBI in 1962. This made it possible for the first time to evaluate TBI by using a computational approach [29]. Researchers in French [30] and the Netherlands [31] also did theoretical investigations in 1964 and 1966, respectively. According to these investigations, a non-linear behaviour, which describes the relationship of forces and displacements between tracks and bridges, must be considered. However, the application of analytical solutions reached its limits very soon due to the problem of non-linearity. On account of large discrepancies in the approach of railway authorities and the lack of a systematic investigation, UIC launched the issue ORE D 101 'Braking and Traction forces on bridges' at an international level. In 1974, the topic of TBI was included in the program of the UIC experts committee. As a result, complex analytical approaches to solve TBI were developed theoretically and metrologically. Even though a final determination of the permissible additional stresses was not achieved, the investigations had gathered good insights for further researches.

The planning of the high-speed railway (HSR) line Hannover – Würzburg and Mannheim – Stuttgart marked the beginning of the era of HSR in Germany. Aim to test and prove the technical and economic possibilities of HSR traffic, the German Federal Ministry of Research and Technology (BMFT) founded the Wheel/rail research program in 1968. The subproject 'R/S-Bücken', which was launched in 1975, carried out extensive studies on the requirements of bridges for HSR traffic. In addition to the study of vertical and horizontal behaviour as well as the formation of the track system on bridges, another focus was the verification of longitudinal forces in rails on high and long bridges. At that time, there were very few calculation tools (like computers) to calculate the complex force situations. Therefore, fundamental theoretical investigations and comprehensive experimental programs were established. The background of many current regulations as well as the development of ballast-less systems in Germany is based on the results of this research project [4]. However, there were still many unsolved problems left. With the development of computer technology and finite

element methods, it became possible to calculate more complex structures by considering non-linear coupling conditions between tracks and bridges from the middle 1980s [32,33].

After the opening of the high-speed railway line Hannover – Würzburg and Mannheim – Stuttgart in the early 1990s, extensive measurements were carried out on different bridges. The partially considerable deviations of measured and computationally predicted braking forces of experiments led to more detailed investigations on the longitudinal force transmission under braking loads [34].

As a part of ERRI D202, the project 'Improved knowledge of forces in CWR track' revised and broadened this topic to a relatively high international level in 1999. However, the work on TBI of this project was still limited to the development of the software CWERRI [35,36]. This program was developed by TU Delft and allows the longitudinal, vertical and lateral behaviour of CWR track to be modelled and calculated integrally. The description of track behaviour in the area of bridges along the longitudinal direction of this project is the background and the basis of the regulations in the UIC leaflet 774-3 (2001) as well as associated Eurocode regulations. In the last 15 years, long-term monitoring measurements of the rail stresses, rail temperature and superstructure temperature of five viaducts have been established and have provided very important information on the behaviour of tracks on (semi-) integral bridges in Germany [37]. Lou et al [38], Dai et al. [39] and Li et al [6] did many monitoring, experimental and numerical investigations on the temperature field of railway bridges in China.

3. Stresses in rails

Rails are under complex effects during operation. Fig. 1 demonstrates schematically the major stresses in rails.

In Fig. 1, $\sigma_{\Delta T_R}$ presents the temperature variation in comparison with the rail locking temperature ($23^\circ\text{C} \pm 3\text{ K}$ according to Deutsche Bahn); σ_R denotes the residual stress due to the rail manufacturing process (cooling and rolling process); σ_Q denotes the bending stress due to the crossing of trains (according to the theory of Zimmermann [12,40,41]); $\sigma_{\text{add, longitudinal}}$ denotes the additional longitudinal stress due to longitudinal displacement of the bridge; $\sigma_{\text{add, bending}}$ denotes the additional bending stress due to offset and rotation of the bridge end. These stresses can be classified into two categories: $\sigma_{\Delta T_R}$, σ_R and σ_Q are the standard stresses, which exist in the rail independent of the substructures; $\sigma_{\text{add, longitudinal}}$ and $\sigma_{\text{add, bending}}$ are the additional stresses in the bridge areas result from TBI.

To determine the permissible additional stresses, the fatigue resistance and buckling strength of rails need to be determined. However, these resistances have considerable scatters and are based on many assumptions. The evaluation of the permissible additional stress is therefore very sensitive to the change of some individual parameters. Hence, the limit values, which are written in the codes, are understood more as reference values or guide values during the design process rather than precise limits in the sense of strict verification. The primary goal of the longitudinal stress verification is therefore to guarantee the safe transmission and resistance of these additional stresses in the rail system. The background of these fixed values (see Table 1) and recent relating researches, will be explained in Sections 4 and 5.

4. Permissible additional compressive stress

4.1. Background of the 72 MPa and 92 MPa

On ballast track systems, large longitudinal compressive forces can cause the rail to buckle. In bridge areas, the vertical buckling tendency was locked out due to stiff superstructures and track systems [42]. The stability of the rail in the transverse direction depends on the superstructure form, the superstructure condition, the route curvature, the track system condition, and the longitudinal compressive force. A simplified theory based on an energy method to solve the rail stability

Fig. 1. Actions leading to rail stresses in the area of bridges [2].

problem was first introduced by Gruenewaldt/Meier [28,43] in 1931/34. With this deterministic approach, the critical compressive buckling force $F_{R,crit}$ of particular superstructures can be calculated. Historically, the $F_{R,crit}$ were equivalent to critical temperature warming $\Delta T_{R,crit}$ by applying the equation:

$$F_{R,crit} = E_R \cdot A_R \cdot \alpha_R^{th} \cdot \Delta T_{R,crit} \quad (1)$$

The critical temperature warming $\Delta T_{R,crit}$ for straight rails ($R > 800$ m) is calculated by the simplified equation of Meier; see Eq. (17) [43,44] and is named Meier Method in this work. Based on the energy method, the deformation work of rails depends on the deformation amplitude f under the temperature variation ΔT_0 and the compressive force P_0 . The compressive force remains constant when the lateral deflection develops. The given bending line for this calculation is based on actual in-situ occurred buckle wave (see Fig. 2) [45].

This wave can be expressed by the following equations:

$$y = \begin{cases} \frac{f}{2} \cdot (1 - \cos \frac{3\pi x}{l}), & 0 < x \leq \frac{1}{3}l \\ -f \cdot \cos \frac{3\pi x}{l}, & \frac{1}{3}l < x \leq \frac{2}{3}l \\ -\frac{f}{2} \cdot (1 + \cos \frac{3\pi x}{l}), & \frac{2}{3}l < x \leq l \end{cases} \quad (2)$$

The total work A of the deformation includes three parts, the work due to bending (A_B), the displacement work due to the displacement of the rail in the transverse direction (A_{DT}) and the expansion work of the rail in the longitudinal direction (A_E):

$$A = A_B + A_{DT} + A_E \quad (3)$$

Based on the forming of the rail, the individual work can be calculated.

1. The bending work A_B :

$$A_B = \frac{1}{2} \cdot \int EI \cdot y''^2 \cdot dx = \frac{81 \cdot EI \pi^4 \cdot f^2}{8 \cdot l^3} \quad (4)$$

2. The transverse displacement work A_{DT} :

$$A_{DT} = w \cdot \int y \cdot dx = w \cdot f \cdot l \cdot \frac{\pi + 2}{3\pi} \quad (5)$$

3. The longitudinal expansion work A_E :

$$A_E = -\lambda \cdot P \quad (6)$$

Here λ can be calculated:

$$\lambda = \frac{1}{2} \sum \int y'^2 dx = \frac{9\pi^2 f^2}{8 \cdot l} \quad (7)$$

And then

$$A_E = -\frac{9\pi^2 f^2}{8 \cdot l} \cdot P \quad (8)$$

The total work can be concluded as:

$$A = A_B + A_{DT} + A_E \quad (9)$$

$$\frac{dA}{df} = 0 = \frac{81 \cdot EI \pi^4 \cdot f}{4 \cdot l^3} + w \cdot l \cdot \frac{\pi + 2}{3\pi} - \frac{9\pi^2 f}{4 \cdot l} \cdot P \quad (10)$$

And f is considered as:

$$f = \frac{\frac{w \cdot l \cdot (\pi + 2)}{3\pi}}{\frac{81 \cdot EI \pi^4}{4 \cdot l^3} - \frac{9\pi^2 \cdot P}{4 \cdot l}} = -\frac{w \cdot 4l^2 (\pi + 2)}{27\pi^3 \left(\frac{9\pi^2 EI}{l^3} - P \right)} \quad (11)$$

$$\frac{df}{dl} = 0 = \frac{18\pi^2 EI}{l^2} - P_0 \quad (12)$$

The buckling length can be calculated:

$$l_0 = 3\pi \cdot \sqrt{\frac{2 \cdot EI}{P_0}} \quad (13)$$

So that the critical track direction error is:

$$f_{crit} = 8.7286 \cdot w \cdot \frac{EI}{P_0^2} \quad (14)$$

Fig. 2. Buckling shape for straight rails.

And after substituting (14) into (1), the critical temperature can be calculated as:

$$\Delta T_0 = \sqrt{\frac{8.7286 \cdot I_{Eq} \cdot w}{a_T^2 \cdot F^2 \cdot E \cdot f}} \approx 537.3 \cdot \sqrt{\frac{I_{Eq} \cdot w}{F^2 \cdot f}} \quad (15)$$

To avoid rail buckling, a design principle was established. The critical temperature change $\Delta T_{R,crit}$ must be larger than the maximum temperature warming of the rail $\Delta T_{R,max}$ plus a safety margin $\Delta T_{R,S}$. This margin was empirically determined and depends on the design speed of the lines, see Table 2 [46,47].

The critical temperature can be thus calculated as:

$$\Delta T_{R,crit} > \Delta T_{R,max} + \Delta T_{R,S} \quad (16)$$

$$\Delta T_{R,crit} = \sqrt{\frac{8.7286 \cdot I_{Eq} \cdot w}{\alpha_R^{th2} \cdot A_R^2 \cdot E_R \cdot f}} [K] \quad (17)$$

- I_{Eq} [mm⁴] is the equivalent moment of inertia of the two rail tracks
- w [N/mm] is the transverse displacement resistance, which is assumed to be constant
- α_R^{th} is the thermal expansion coefficient
- A_R is the total area of both rail cross-sections of one railway track
- E_R [mm²] is the E-modulus
- f [mm] is the assumed fictitious track direction error, $f = f_0 + 7$ mm, f_0 is the actual track direction error, which depends on the condition of the railway tracks [41].

It is noteworthy that I_{Eq} is an equivalent moment of inertia of the rail system. This parameter can be determined by experimental investigations [48]. In the bridge joint area, there exist additional longitudinal forces in continuous rails due to TBI. These forces (correspond to a temperature variation of $\Delta T_{R,B}$) must also be limited so that the safety capacity is kept and $\Delta T_{R,crit}$ is not reached. These effects are expressed as temperature variations. Eq. (16) can be, therefore, further developed as follows:

$$\Delta T_{R,crit} > \Delta T_{R,max} + \Delta T_{R,S} + \Delta T_{R,B} \quad (18)$$

The fixed limitation value 72 MPa in codes and standards today comes from the research project 'R/S-Brücken, Thema Nr.61'. The rail profile for the calculation was UIC60, the sleeper type was B70, the fastener type was W and the rail curvature radius was larger than 1500 m. Two different cases were calculated in the report [49], which are listed in Table 3. The maximum allowable temperature variation was $\Delta T_{R,B} = 30K$, it corresponded to an allowable compressive force or more specifically compressive stress of $E_R \cdot \alpha_R^{th} \cdot \Delta T_{R,B} = 72MPa$. It was clearly noted that this limitation was not to be understood as the stress certification but only as the limit value of additional compressive stress. The value 72 MPa has been fixed in present German standards as well as in international standards, see Table 1.

According to Meier's simplified theory, the stability of the rail tracks can be evaluated if the input parameters (see Table 3) are available. However, the deformation and stability of the rails are also influenced by other factors. In the 1980s, Chatkeo refined this theory. Besides the previous work, he took the shear work in the longitudinal direction (A_{SL}) and the rotation work of the rail (A_{RT}) into account. The equation (3) was developed as [50]:

$$A = A_B + A_{DT} + A_E + A_{SL} + A_{RT} \quad (19)$$

Table 2

The dependency of $\Delta T_{R,S}$ on train speed [46].

V [km/h]	≤80	≤100	≤120	≤140	≤160	≤230	≥230
$\Delta T_{R,S}$ [K]	10	20	25	30	40	50	60

Table 3

Calculation of the buckling stability for two conditions of ballast compaction [49].

	Parameter	Stabilized track	Un-stabilized track
Input	E_R	210 GPa	
	α_R^{th}	$1.15 \cdot 10^{-5}$	
	A_R	$A_R = 2.7648 \text{ mm}^2$	
	I_{Eq}	2200 cm ⁴ (according to the report no.583 of TUM)	
	w	$2/3 \cdot 14.1 = 9.4 \text{ N/mm}$	$0.8 \cdot 7.05 = 5.64 \text{ N/mm}$
	f	15 mm (based on HSR lines, good track maintenance)	
Results	$\Delta T_{R,crit}$	135 K	105 K
	$\Delta T_{R,max}$	50 K (neutral temperature: 23 K + 3 K according to DB)	
	$\Delta T_{R,S}$	50 K	25 K
	$\Delta T_{R,B}$	35 K	30 K

As the transverse displacement resistance and the rotational resistance between rails and sleepers usually showed non-linear mechanical behaviours, which were not able to be easily calculated with analytical equations, he developed a numerical approach [50]. To examine this improved method, three buckling tests (Table 4) were carried out on field tracks. The test results had a good agreement with the theoretic results. Besides, it was concluded that the results calculated with the Meier Method were in good agreement if the equivalent moment of inertia ($I_{Eq} = 1400 \text{ cm}^2$ for UIC 60 rails with W-fasteners on concrete sleepers) was correctly considered [51]. In the following text, this method is named as the improved Meier Method.

Despite the adaptation of the calculation, the limitation value remained unchanged at 72 MPa [18,49]. Afterwards, this value was adapted to 92 MPa for rails in ballast-less track systems within bridge areas. However, no solid experimental or theoretical evidence could be found in literature. This 92 MPa for rails in ballast-less track systems is too conservative and thus needs to be checked and updated in the opinion of some authors [10,12,22].

4.2. Recent related investigations

In 1998, Knoll [52] examined the phenomenon of rail buckling in unloaded and continuously welded narrow-gauge tracks in small-radius lines with different fasteners based on the improved Meier Method in his dissertation. He suggested some critical radius values for tracks with wooden and concrete sleepers under different circumstances. Iliev [48] carried out investigations on the concrete sleepers with or without elastic under-sleeper-pads regarding the transverse stability behaviour of rails. Based on the experimentally determined resistances of these different ballast track systems, he developed a simplified finite element model (FEM). In this model, the transverse displacement resistance, the longitudinal displacement resistance and rotating resistance were modelled as non-linear spring elements; rails and sleepers were modelled as 3D beam elements; the track direction error was inputted. The calculated results were in good agreement with the measured results in the field and were on the safer side in comparison with the results, which were calculated with the Meier Method. As a result, he did an extensive parameter study and provided multiple critical temperature

Table 4

Buckling tests in the 1980s [51].

Year	Track	Radius	Rail Profile	Sleepers		
				Material	Type	Distance
1984	Munich – Moosach	R = 246 m	S 49	Wooden	I	60 cm
1986	Munich – Daglfing	R = 360 m	S 54	Wooden	I	60 cm
1987	Wurzburg – Hannover	Straight track	UIC 60	Concrete	B 70	60 cm

limits for rails under various conditions in ballast track systems. Concentrated on the influence of fasteners, Liu [53] carried various experiments with different fasteners on the deformation behaviour and stress distribution of rails in ballast track systems in the transverse direction. She established a FEM, in which the rails and sleepers were modelled as solid elements; the contact relationship of the rail and under-pads was modelled with spring elements, which distributed uniformly in the contact area; the transverse and clamping behaviour of the fasteners were simulated with multiple spring elements. She concluded the influence of different fastener components on the deformation of rails in the transverse direction. With considering the rail breathing length, Pospichil [54] concluded the important factors, which could affect the stability of rail tracks in the transverse direction, and carried out field tests and long-term (two years) monitoring measurements on rails in ballast systems of ÖBB (Austrian Federal Railways) railway network. He evaluated the rail stability under different conditions and concluded different technical approaches to improve the rail stability in the transverse direction. In the aforementioned investigations, the transverse displacement resistance of the rail had been regarded and proved as the most important factor, which influenced rail stability. The influence of different factors on the transverse displacement resistance and further on rail stability in ballast track systems, and methods to improve this resistance, were important parts of these investigations.

With the introduction of eddy current brake technology by ICE 3 in the recent past years, the resulting additional warming effect to rail track stability had to be re-evaluated [44]. For this purpose, a semi-probabilistic study was carried out by Freystein [12] for different types of superstructures and various train speeds. The results of his work confirmed that the limit value of 72 MPa is sufficient in the area of bridges even with very heavy track structures according to Richtlinie 804.3401 [18]. This above-evaluated limit value relies on a good quality of the ballast, in which case the transverse displacement resistance of the ballast is not diminished by excessive deconsolidation. Therefore, in addition to the rail compressive force, the bridge and track displacement should be limited as well. The continuity of the lateral ballast boundary over the abutment should be guaranteed. In principle, higher additional rail compressive stress can be allowed if wide sleepers (e.g. BBS), twin sleepers (e.g. ZSX), Y-sleepers, or safety caps are applied [12]. However, the use of untypical track components in the sense of a standardized superstructure should be limited to individual cases. In contrast, the permissible additional rail compressive stress decreases when smaller rail profiles (54E4 or smaller) are applied [12]. Based on simulation results in correlation with field test parameters, 112 N/mm² permissible compressive stress was proposed for both ballast and ballast-less track system by Freystein [12]. Besides, Strauss et al. [55] carried out comprehensive investigations by means of monitoring-based nonlinear finite element modelling using advanced beam-spring interaction laws and specified thermomechanical considerations for capturing the real thermal expansion of bridge structures. They also concluded that the proofs of the rail stress with respect to predefined code based thresholds can be considered as very conservative.

4.3. Other aspects

Besides the energy method, another analytical approach based on the assumption of Winkler's continuously supported beam theory was developed by Kerr in 1974 [56–58]. The equation of the differential approach has a form of:

$$EI \frac{d^4 u}{dx^4} + (P - \tau) \frac{d^2 u}{dx^2} + w \cdot u = 0 \quad (20)$$

Where EI is the bending rigidity of the two rails, P is the longitudinal force in the rails, τ is a proportionality factor and w denotes the rail displacement resistance in the transverse direction. This approach was then further developed and validated by multiple researchers including Kerr himself in the following years. Kerr improved this approach by

presenting the axial resistance as a bi-linear response instead of a constant resistance in 1980. Kish and Samavedam further developed this approach by introducing the non-linearity of railway components properties such as the transversal displacement resistance and the torsional resistance. Moreover, they did many experimental and analytical investigations on the influence of dynamic vehicle induced loads on the lateral stability of CWR tracks [59–63]. Afterwards, they carried out parametric analyses on CWR track buckling and introduced probability functions to evaluate the rail buckling risks [64]. A summary of their work can be found in [65]. A characteristic of the energy method is that a bending line, which is geometrically compatible with the boundary conditions, can be assumed. The more the assumed bending line corresponds to the actual bending line, the more accurate the solution becomes. In comparison, only certain simplified bending curves can be applied with the differential approach [50]. In 2014, Iorio carried out multiple scenario field tests to investigate the transverse strength of railway tracks in Italy and obtained some load-displacement curves as results [66–68]. In addition, with the fast development of finite element modelling (FEM), many researchers also proposed simulation approaches, some of which can be referred to the work of Pucillo [69], Lim [70,71], and Yun [72], etc., to deal with this problem. It should be noted, the accuracy of numerical solutions depends on the accuracy of the assumed mechanical models of all parts, most of which need to be validated with experimental tests. However, all these aforementioned investigations are focused on the stability or transverse resistance of rails in ballast track systems. Even though the differences between the ballasted and ballast-less track system are obvious and large, there are still few investigations concerning the transverse resistance of ballast-less tracks.

In 2017, Kang et al. [10] carried out experimental investigations on an 8.17 m UIC 60 rail in the ÖBB-Porr ballast-less track system [73], which corresponded to the actual system in the new HSR line Berlin-Munich. They found out that the rail deformed abruptly first due to plasticity instead of buckling. This phenomenon was also verified by numerical simulations. In the FEM, the rail was modelled with beam elements; the fasteners were modelled with non-linear spring-elements, which were determined in advance by tests and adapted according to the actual experimental situation. Wenner et al. [74] did both experimental and numerical investigations on 31 m long UIC 60 rails in the ÖBB-Porr ballast-less system and found out that there was no sign of buckling even under twice the longitudinal force in comparison to historic investigations (see Table 3) on rails in ballast systems. They both suggested higher additional compressive stress than 92 MPa for rails in ballast-less track systems.

5. Permissible additional tensile stress

5.1. Background of the 72 MPa

Breaks due to cyclic loads are always the major cause of rail failures. 1083 occurrences of rail failure in the German railway network were recorded in the two years between 2012 and 2013 for a track length of about 63,000 km [11]. Very unfavourable cyclic stresses due to train loads and residual stresses from the manufacturing process occurred in the centre of the rail foot, which is normally regarded as the initial area of fatigue fracture. The bending tensile stresses in the rail foot and railhead were statistically proved by theoretical and experimental investigations at the Technical University Munich (TUM) in the 1970s [75,76]. The verification is related to two aspects, which are the fatigue strength of the rail profile and the fatigue-related stress range due to train traffic.

To obtain the fatigue strength, TUM carried out four-point-bending tests on UIC 60 rail specimens up to two million load cycles under a minimum stress level of 50 MPa in 1973. The rails were newly manufactured and almost without any corrosion. The maximum roughness depth R_t of the rails ranged from 10 μ m to 30 μ m, and the ultimate

strength of the rail steel was 900 MPa. The four-point-bending test setup is shown schematically in Fig. 3. Each specimen was 2.3 m long, the tested span was 1.9 m and the distance between the two loading points was 150 mm.

In this investigation, 27 specimens broke within two million load cycles; six specimens ran-out the tests. The lower boundary of 335 MPa, which was smaller than the stress range of all broken rails, was determined as the fatigue strength. The upper limit was calculated by considering a factor of 0.7 with the ultimate strength of the rail steel material, therefore, $\sigma_{\text{limit}} = 0.7 \times 900 \text{ MPa} = 630 \text{ MPa}$. Based on these results, a Smith-diagram was constructed (Fig. 4).

In the case of mean-stress-dependent fatigue strength analysis, the fatigue strength is expressed by the stress range $\Delta\sigma$ due to cyclic loads. This fatigue strength can be determined by cyclic rail bending tests. The influences of the rail profile, surface properties, and the manufacturing process (especially for residual stress) are thus directly included. With the purpose of determining the fatigue strength of rails under different loads, the Smith-diagram was applied. Fig. 4 shows how to derive a Smith-diagram using the minimum stress σ_{min} , the maximum stress σ_{max} and the mean stress σ_{mean} , which are obtained from bending fatigue test results. The horizontal axis and vertical axis present the σ_{mean} and σ_{max} (also σ_{min}) level, respectively. When the σ_{max} , σ_{min} and $\Delta\sigma$ of one load level have been determined, the σ_{max} and σ_{min} can be connected with the ultimate strength of the investigated rail material by two straight lines. Besides, an allowable maximum stress is set as the upper limit (σ_{limit}). The lower solid line presents σ_{min} and the upper solid line presents σ_{max} ; $\Delta\sigma$ is the corresponding allowable fatigue strength. The detailed principles of constructing a Smith-diagram can be found in DIN 50100 [78].

With this Smith-diagram and other stress information, the fatigue resistance of the rail under different stress levels can be thus determined. When the sum of the residual stress $\sigma_R = 80 \text{ MPa}$ and $\sigma_T = 100 \text{ MPa}$ due to temperature effects, which were research results of that time [77], were taken as the minimum stress level, the corresponding fatigue resistance was determined to be 282 MPa. Based on these discoveries, the directive document ‘Oberbauberechnung’ for track system calculations was issued in 1977 [80]. However, the permissible additional stresses of rails in bridge areas were not independently identified.

From 1977 to 1980, as a result of the research project ‘R/S-Brücke, Thema Nr.61’ for the new railway lines in Germany, the previous empirical approach to limit the effects by TBI was updated [49]. As an inevitable part of this project, fatigue strength tests on rails of open tracks have been carried out to supplement and extend the investigations on additional tensile rail stresses in bridge areas. Fig. 5 shows the resultant Smith-diagram. The values in Fig. 5 to determine the permissible additional tensile stress were derived from experimental results of heavily corroded S54/90 rails (the maximum roughness depth ranges from 502 μm to 533 μm). The different stresses were considered separately for the first time. The related parts are explained in the following text.

Fig. 4. Smith-diagram for a new rail UIC60 according to Oberweiler in 1973 [77,79].

Fig. 5. Smith-diagram for the rail S54/90 according to DB 061 078 in 1980 [81].

Fig. 3. Test setup of TUM in 1973 [77].

(a) Fatigue strength

Bending fatigue tests were carried out on 11 rail specimens, which included both operated and new rails (not operated), up to two million load cycles. As a result, nine rails failed within two million load cycles and the other two ran-out. The resultant S-N curve is shown in Fig. 6. The minimum stress range, which led to rail failure, was 210 MPa. The value of 205 MPa was regarded as the lower boundary stress and taken as the final fatigue strength (stress range $\Delta\sigma$).

However, all the assumptions and statements in Smith-diagrams only apply for the stresses within the elastic range. An upper limit value needs to be specified. The numerous tension tests in the 1970 s resulted in an average yield stress value of $\sigma_{S,M} = 525$ MPa and a lower 5-percentile value. This led to a final limit value of 470 MPa. Based on these results, a Smith-diagram was constructed; see Fig. 5.

(b) Residual stress

In comparison with Fig. 4, the residual stress is directly integrated as part of the minimum stress level to construct the Smith-diagram. Residual stresses are already in the rail even before the rail is loaded and cannot be avoided. They are introduced by heat treatment, cooling, and roller straightening during the manufacturing process [82]. The residual stress field is characterized by axial tensile stresses in the head and foot of the rail counterbalanced by compressive stresses in the rail web. The superposition of residual stress with other stresses results in a very high mean stress level during operation. However, the way they were taken into account in the Smith-diagram was very different and discussed controversially in literature [22]. Usually, to identify the individual stress parts in the Smith-diagram, rail specimens with actual residual stresses were adopted, so that the scattering of residual stresses of the specimens were directly included [49,83]. Sometimes the influence of residual stresses was already taken into consideration in the experiments to determine the permissible additional tensile stress and was thus not individually explained in smith-diagrams [75]. Sometimes it was only considered as minimum evaluation stress of experiments (Fig. 4). Sometimes it was considered as minimum evaluation stress of experiments and was integrated into the Smith-diagram (Fig. 5). In all these cases, the residual stress was assumed to be 80 MPa. This value fitted very well with the values that were given for S49 rails in literature [84]. Investigations on newly manufactured UIC60 from the 1980s suggested not only significantly larger values but also with bigger scattering [85].

(c) Temperature effects

When the rail temperature changes compared to the neutral temperature, the temperature stress of a continuous rail can be calculated with $\sigma_{\Delta T R} = -E_R \cdot \alpha_R^{\text{th}} \cdot \Delta T_R$. As part of the fatigue strength verification, a minimum tensile stress in the rail was assumed. The assumed minimum temperature in Germany was -25°C and the locking temperature was $23^\circ\text{C} \pm 3\text{ K}$, it led to a maximum cooling temperature gradient of $23 + 3$

Fig. 6. Summary of fatigue tests on corroded S54/900 rails [77].

- $(-25) = 51 \approx 50\text{ K}$. Based on the assumptions of $E = 210\text{ GPa}$ and $\alpha_R^{\text{th}} = 1.15 \cdot 10^{-5}\text{ K}^{-1}$, the corresponding evenly distributed tensile stress in the cross section was $\sigma_{\Delta T R} = 120\text{ MPa}$.

(d) Train loads

The tensile stress due to different trains is the decisive fatigue effect for rails, and must thus be considered in the bridge area. Fig. 5 shows the comparison of the maximum stress at rail foot under decisive wheel load and tolerable fatigue strength. As a structural component under train wheels, a rail is subjected to two major forces, which are the vertical wheel force Q and the transversal guiding force Y . For the stress calculation in the centre of rail foot, it was assumed in a simplified manner that the effects of the guiding force Y and the eccentricity of the wheel loading point on the railhead, were of no importance and were thus neglected [75]. This simplified calculation was based on the elastic beam model (Fig. 7) of Zimmermann [22]. According to this model, the transverse sleepers were transferred to a longitudinal beam with a width of b laying under the rail track; the bending stiffness of the rail is defined as $E_R I_R$; C is the supporting stiffness of the foundation.

The stress under the wheel load ($x/L = 0$ and $\mu = 1$) was thus calculated as:

$$\sigma_{\text{train}} = \frac{M}{W_{\text{UIC60}}} = \frac{Q_{\text{effective}} \cdot L}{4 \cdot W_{\text{UIC60}}}; \quad L = \sqrt[4]{\frac{4 \cdot E_R \cdot I_R}{b \cdot C}} \quad (21)$$

The determination of the bending stress was based on the then-effective 'Oberbauberechnung' (1977). For the track system UIC60-1667-B70W-60 on good a foundation, the following parameters were adopted: Young's modulus $E_R = 210,000\text{ N/mm}^2$; inertia moment $I_R = 3055\text{ cm}^4$ and section modulus of the rail foot $W_{R,\text{Foot}} = 377\text{ cm}^3$ for new UIC 60 rails; half of the sleeper cross-section area $A_{\text{sleeper}} = 5700\text{ cm}^2$ for sleeper B70; the sleeper distance $a = 600\text{ mm}$, and foundation modulus $C = 0.10\text{ N/mm}^3$. The elastic length of $L = 857\text{ mm}$ was set as the basic considered length for this situation. The calculation was made for both light and fast trains, as well as for heavy and slow trains. The combination of 21.0 t axle weight at 200 km/h speed was decisive. To determine the effective wheel force, the influence of the wheel-force displacement (e.g. curved alignment or evenly loaded wagons) was

Fig. 7. Calculation model according to Zimmermann [22].

evaluated by adding a ΔQ ; the influences of the dynamic effects of trains were considered with a coefficient $\bar{s}$, in which the factors n depends on the condition of the track system and φ took into account the influence of the train speed. Detailed information on these values can be found in [75,80].

$$Q_{\text{effective}} = (Q + \Delta Q) \cdot (1 + 3 \cdot \bar{s}) \quad \text{with } \bar{s} = n \cdot \varphi \quad (22)$$

To evaluate the rail tensile stress on high-speed railway lines in the project 'R/S-Brücken', it was decided that $\Delta Q = 0.2 \cdot Q$, $n = 0.2$ (good condition), and the considered train speed was $v = 200$ km/h. It was thus calculated that $\bar{s} = 0.2 \cdot 2 = 0.4$ based on Fig. 8. With these assumptions, the effective wheel force was $Q_{\text{effective}} = 277.2$ kN and the resultant maximum rail tensile stress was $\sigma_Q = 158$ N/mm². In the 1980s, in-situ measurements were carried out, the knowledge gathered in return led to significant changes in the calculation of $Q_{\text{effective}}$ (see Fig. 8). However, it must also be stated that the stress, calculated with Eq. (21), presents the maximum bending tensile stress, but not the stress range to be applied, which also takes into account the liftoff effects (see Fig. 7) [12]. The fatigue strength verification was revised in [12] based on this background.

(e) TBI related stresses

Based on the Smith-diagram and aforementioned values, the permissible additional stresses σ_B due to TBI was concluded to be 112 MPa (see Fig. 5). In sum, $\sigma_B = \sigma_{BT} + \sigma_{BB} + \sigma_{BL}$, where σ_{BT} denotes the additional stresses due to bridge temperature variation; σ_{BB} denotes the additional stresses due to bridge bending; σ_{BL} denotes the additional stresses due to longitudinal displacements of bridges resulting from braking/traction. Originally, a distinction was made in the project 'R/S-Brücken – Thema Nr. 61', whether the braking force should be handled as a fixed control threshold value (σ_{BL} as the minimum stress) or as an operational braking condition (σ_{BL} as fatigue-relevant stress variation). Since most of the structures were on open tracks, a braking force frequency $< 10^4$ was uniformly set, the sum of the three stress parts σ_{BT} , σ_{BB} and σ_{BL} were thus regarded as the minimum stress threshold in the Smith-diagram. Even for more frequent braking forces, the stress variation could only be superimposed with the static parts due to wheel loads of the trains, only the dynamic effect of the wheel loads in the vertical direction was considered as the fatigue related stress variation. As Fig. 5 shows, the reserved stress was 112 MPa for ballasted track rail systems. This value was then extended to ballast-less track systems.

To calculate the rail stresses on bridges, the non-linearity of the

Fig. 8. Comparison of the coefficient for the consideration of the driving dynamic according the track calculation guidelines from 1977 and 1992 [75,80].

displacement resistance must be taken into account. In the early 1980s, the effect of longitudinal displacement was calculated by a one-dimensional model. However, tools to calculate the effects of structure deflections were rare at that time. To avoid complex situations for individual cases, σ_{BB} was considered as 20 MPa and was then subtracted from the reserved stress σ_B [86]. This value was specified according to the calculation of rails on simply supported bridges with spans of $L_{\text{span}} = 44$ m, which was not only a proposed superstructure type by 'Rahmenplanung Talbrücken' [75] of the Deutsche Bahn AG but also the most commonly constructed superstructure type in the HSR line Hannover – Würzburg and Mannheim – Stuttgart [4]. The reserved stress was calculated as $112 - 20 = 92$ MPa for the remained two parts σ_{BT} and σ_{BL} .

The 'Oberbauberechnung' was updated to a new edition in 1992 [75]. Compared to the previous Smith-diagram (Fig. 5), the fatigue strength of the rail profile UIC60/900 [87] after corrosion was also determined. For the fatigue strength verification, the additional stress (Fig. 9) due to temperature variation was changed to 100 MPa ($\Delta T = 42$ K). The maximum fatigue-related stress range was determined to be 199 MPa. However, the allowable additional rail stress was not calculated from the construction of the diagram but kept to 92 MPa (see Fig. 9), with which σ_{BT} and σ_{BL} were covered [75]. In the current German code (Ril 804), Eurocode [3], and the international guideline of UIC [1], the permissible additional tensile stress is specified as 92 MPa.

5.2. Recent related investigations

However, compared to the 1980s, the current material properties have been greatly improved, changes in the manufacturing process have led to different residual stresses in the rail cross-section, the loads and occurring stress spectrum during operation can be more accurately estimated. Two obvious distinctions have been consistently discussed in the last years: the upper limit of the Smith-diagram and the residual stresses in rails. First, regarding the upper limit stress of the Smith-diagram, the 0.2% expansion limit of the rail steel R260 identifies a higher value of $\sigma_{S,M} = 600$ MPa nowadays. As this value is based on static tests, whether the yield strength can be set as the upper edge in the Smith-diagram is controversial and worth investigating. Second, different manufacturing processes lead to very different residual stresses. In general, residual stresses tend to decrease with the rapid development of manufacturing processes. For example, by optimizing the heat treatment method and omitting the rolling procedure, the residual stresses can be reduced to 50 MPa in the centre of the rail foot

Fig. 9. Smith-diagram for the rail UIC60/900 according the 'Oberbauberechnung' 1992 [75].

[88]. The considered residual stress values used in Figs. 4 and 5 are correct only if 80 MPa is the upper quantile value of the mean residual stress. Besides, it conflicts with the maximum allowable residual stress of 250 MPa in the rail foot according to EN 13674-1 as well. Experimental investigations on UIC 60 rails showed significant scattering concerning the stresses in rail foot [89–91]. One possibility to consider the fluctuating of residual stresses is to take the scattering degree into account. When constructing a Smith-diagram, the scatter degree of residual stresses should be determined by tests.

Besides, there have been very few investigations on the flexural fatigue behaviour of rail tracks. In the first decade of the new millennium, TU Dresden in Germany carried out detailed investigations on the non-linear superposition of different load cases, which include the temperature, structural bending, braking and other longitudinal forces due to train crossing [92–97]. Freystein did further investigations based on the component tests from the 1980s to determine the permissible additional stress for different track systems by evaluating the different structural properties and real operational loads instead of carrying out fatigue strength tests. Upon finishing his work, the permissible additional tensile stress of 72 MPa was proposed for most ballasted track systems. In addition, 92 MPa permissible additional tensile stress was proposed for ballast-less track systems and two special ballasted systems [12]. In recent years, the Institute of Concrete Construction at Leibniz University Hannover has carried out substantial monitoring work on the German HSR network and proposed higher permissible additional stresses for ballast-less track systems [2,9,22,98,99]. From 2014 to 2019, Kang et al. [11] did plentiful experimental investigations on the flexural fatigue behaviour of the typical high-speed rail profile UIC 60 up to five million load cycles under different load levels. In comparison with historic investigations of rails up to two million load cycles, the new rails showed better fatigue behaviour even for 2.5 times more load cycles. They concluded that the rail fatigue resistance in the transverse direction should be considered as well. In order to fully determine the additional permissible tensile stress due to TBI, further investigations concerning the residual stress distribution in rails and surface conditions were necessary to be carried out. Nowadays, The value 92 MPa is already planned to be updated to 112 MPa in the upcoming new Eurocode [100].

5.3. Other aspects

Despite substantial improvements in design, materials and non-invasive inspection, fatigue propagation and the failure of railway components remain an important issue for safety engineering [101]. Rail fatigue has become one of the major research topics. Different approaches have been carried out with the purpose of predicting rail fatigue life. On the basis of fracture mechanics, abundant investigations have been carried out. In 1997, the UIC/WEC began an international research project entitled 'Rail Defect Management' regarding defects in railhead. As a part of this huge project, laboratory and field tests were carried out to correlate rail defect data and engineering analyses. In this project, engineering fracture mechanic analyses were used to estimate the growth rate of railhead defects such as detailed fractures, tache ovale defects, and squat defects. Mensinger et al. [90,102,103] carried out experimental tests on different rail profiles by means of fracture mechanics and provided predictions on the crack growth rate on cracks starting from rail foot. Iorio compared different methods to verify the crack growth rate of rail steel [104,105]. Nejad et al. [106,107] did some experimental investigations on compact specimens with pre-fabricated cracks, which were extracted from rails on Iranian railways, according to ASTM 647 [108] under different ratios. The crack growth rates and stress intensity factors were evaluated and validated by numerical simulations based on Paris-law. Similar tests were also carried out by other scholars, such as Heshmat [109], Kim [110], Peixoto [111], etc. However, most of these fracture mechanical based investigations were usually restricted to either specific testing specimens or pre-determined defects. As this field is extensive and not related to the

background of the anchored limit value (92 MPa), it is not further discussed in this work.

6. Discussion

Throughout Europe, especially in Germany, the investigation results of the project 'R/S-Brücken' in the 1975, which is the basis of the present calculation models, verification concepts, limit values and calculation assumptions, are still anchored in the current codes. The present acceptance of permissible stresses in Eurocode and German code of ballast and ballast-less track system are results of a long-term development but still based on the abovementioned experimental investigations on ballast track systems more than 40 years ago. They have provided essential reference values for TBI analysis. However, the previous empirical practice-based approach to determine the need for rail expansion joints has reached its limit in large-scale structures and is not proper for ballast-less track systems. Meanwhile, the manufacturing technology of rails and relating devices has been further developed as well. For example, the resistance of the fastener devices has changed; the residual stresses in rails has largely changed with the rail manufacturing process. These 72 MPa/92 MPa limit values are outdated, and have become obstacles for the design of longer railway bridges. In the last years, new corresponding investigations on the resistance of rails under compressive and tensile stresses on ballast-less track systems have been carried out. Higher limits values have been often recommended.

Although the current verification concepts in the norms lead to a safe evaluation of the rail in the bridge areas, the optimization of these concepts could in many cases exploit existing reserve. For this purpose, further researches not only on the resistance of rails but also on the residual stresses of rails, mechanical behaviour of the connecting components should be carried out. To finally improve the design concept of railway and railway bridges with considering TBI, it is essential to take the realistic rail resistance, actions and the corresponding transmission mechanism between actions and resistance (nonlinear modelling of TBI) into account. In addition, other verification methods that correspond to the present regulations of fatigue damage evaluation, and safety philosophy should not be neglected.

CRediT authorship contribution statement

Chongjie Kang: Investigation, Writing - original draft, Visualization, Data curation. **Marc Wenner:** Review, Resources. **Steffen Marx:** Supervision, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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2.2 Experimental and numerical investigations of rail behaviour under compressive force on ballastless track systems

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Experimental and numerical investigations of rail behaviour under compressive force on ballastless track systems

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ABSTRACT

Aiming to verify the behaviour of ballastless track systems under high compressive rail forces occurring in the vicinity of bridge joints due to track bridge interaction, numerical and experimental studies are described in this paper. At first, a short summary of background information on the allowable rail stresses under compressive force on ballasted track systems is presented. Subsequently, a numerical pre-study for ballastless systems is performed. The unfavourable imperfection cases and loading procedures in transverse direction for experimental tests are determined. Based on these results, two rails are experimentally tested to achieve practical results. To ensure safety and reliability of the system, a specific testing program and a measurement concept are developed. Main testing results including the rail geometry, deformation and stress development are compared and discussed. Additionally, numerical models to evaluate the rail behaviour are improved and verified. A good consistency between the finite element model and the testing results could be achieved. The performed investigations give a scientific basis in order to increase the allowable rail compressive stresses in comparison with the existing values in the codes. A higher allowable rail stress considerably improves the design conditions for long railway bridges.

1. Introduction

The rail is an essential part of the railway system. Its main functions are to transmit wheel forces to the track supporting structures and, together with the tread and flange of the wheel to guide vehicles [1]. At the beginning of railway history, jointed track was used. The rails were connected by means of joints to allow length changes caused by temperature variation of the rail itself. Using joints prevented the development of axial forces and the consequent risk of track buckling at high temperatures [2]. In the early 1950s, the jointed track gave way to continuously welded rail (CWR). CWR offered considerable advantages in terms of time and maintenance costs, noise reduction, passenger comfort and performance of railway vehicles. However, CWR still suffered some drawbacks, among which the most important is the tendency to buckle, mainly in the horizontal plane, when the rail temperature and the corresponding constraining compressive forces rises beyond a certain limit [3–8].

The first investigations of rail buckling effect can be dated back to the end of the 1920s. These studies were made on the basis of energy approach with virtual displacement by Gruenewaldt [9], who derived the basic relationships for both straight and curved rails that are still

used today. Subsequently, Meier concluded this as a computational feasible method [10,11]. With this deterministic approach, the critical compressive buckle force $F_{R,crit}$ can be calculated for particular superstructures. Previously, the compressive force $F_{R,crit}$ was equivalent to critical temperature rise $\Delta T_{R,crit}$ by applying the equation [12]. In 1968, a wheel/rail research program called 'R/S-Brücken' was launched by the German Federal Ministry of Research and Technology [13]. As part of this program, the rail stability was specifically investigated based on the aforementioned theory, a critical temperature variation $\Delta T = 105$ K (1.94 MN) was calculated as the critical buckling load for UIC60 rail profiles on ballast track systems. On the basis of this research, the permissible compressive stress of 72 N/mm^2 due to track/bridge interaction [12,14,15] was concluded and introduced in German and European standard [16,17]. In the 1980s, the theory of Meier was calibrated by Chatkeo based on experiments results. Chatkeo corrected the equivalent moment of inertia and introduced the assumed fictitious track direction error, which depends on the train speeds [18–20]. Furthermore, the applied safety distances have been more precisely defined [21]. Despite the adaptation of the calculation, the limit value remained unchanged at 72 N/mm^2 . All investigations are only valid for ballast tracks. This value was extended to 92 N/mm^2 for rails on

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E [MPa]	A [cm ²]	G [kg/m]	I _x [cm ⁴]	I _y [cm ⁴]
210,000	76.48	60.03	3021.5	510.5

technology with ICE 3 in the last few years, the resulting additional warming effect to rail track stability needed to be evaluated [22]. For this purpose, a semi-probabilistic study was carried out by Freystein [20] for different types of superstructure and train speed with a classical, deterministic and semi-probabilistic verification. These new findings confirmed that the limit value of 72 N/mm^2 for ballast track systems is sufficient even in the area of bridges. Compared to ballasted track structures, ballastless structures have apparently higher equivalent inertia strength because of their geometry and their increased self-weight and larger lateral resistance due to their construction type. Additionally, field experience shows almost no rail track buckling failure on ballastless systems. Instead, driving safety and passenger comfort has become the criteria and concentration for the design and verification of railway bridges. Therefore, the deformation behaviour of railway track on ballastless system should be the focus in this aspect. In the current German standards [15,23] and international standards (Eurocode, UIC code) [17,24], the permissible compressive stress due to track/bridge interaction remains 92 N/mm^2 for ballastless track systems. This value is assumed to be very conservative in the opinion of Wenner [19]. Because maximum rail stress is a crucial influence parameter in railway bridge design, new investigations need to be carried out.

2. Background

After the German reunification, a large traffic program called 'VDE' (German unification transport project) was launched to connect East and West Germany [25]. As part of this program, the number 8 sub-project (VDE 8) focused on the new high-speed railway line between Berlin and Munich. Within this project, high compressive forces of rails on specific bridges were calculated and exceeded the permissible additional compressive stress of 92 N/mm^2 according to EN 1991-1 [17] and Richtlinie 804 [15] in Summer. For the purpose of validating the safety of the already constructed line, investigations regarding the behavior and stability of the rail under high compressive forces were done

Fig. 3. Fasteners (a) regular fastener SBS300-1; (b) fastener BSPFF-B-1 in transition area.

Fig. 4. FE-model details.

Fig. 5. Mechanical model of fasteners.

Fig. 6. Simplified continuous supported rail (left) and simplified two points supported rail (right).

at the Institute of Concrete Constructions, Leibniz University Hannover. Fig. 1 shows the ÖBB-PÖRR [26] slab track system, which has been constructed on the investigated bridges. The prefabricated slab, which constitutes the upper part of the ballastless track system was chosen in this paper to be studied. Since the deformations of the concrete slab itself are negligible compared to the rail displacements, the investigated system can be reduced on the fixed slab track. Numerical and experimental studies to explore the rail behaviour under compressive load on this system in the area of bridge joints are presented. The influence of bridge joints is modelled by boundary conditions in the model as well as

in the test.

3. Numerical pre-study

In order to prepare the experiments, a numerical study was carried out. The investigated system, consisting of the slab, the fastening elements and the rail, has been modelled in an FE model. The system behaviour under increasing compressive axial forces and various imperfect situations was investigated. These preliminary results made it possible to define the critical imperfections to be introduced into the rail and to determine the measurement layout for the monitoring. Moreover, this preliminary analysis provided reference values for the design of the experiments and for the comparison of the monitored results while running the test.

3.1. FE model properties

In the model, the rail steel was defined in simplified form as elastic material with an E-modulus of $E = 210.000$ MPa. Fig. 2 shows the 60E2 cross section. In addition, other structural details of the rail are listed in Table 1.

The total length of the rail is 8.17 m. The longitudinal distance between adjacent fasteners is 650 mm. The distance from the end of the rail to the first fastener on both sides is 510 mm. There are 12 pairs of fasteners in this system. The six pairs in the middle are BSPFF-B-1 fasteners (Fig. 3b), the other six pairs on both sides together are SBS300-1 fasteners (Fig. 3a).

Fig. 7. Validation of the boundary conditions for rail rotation.

To approach the problem in a realistic way, a 3D numerical model of the rail track was developed in ABAQUS. The rail track is simulated with beam elements. The fasteners are modelled with non-linear springs, the movement of the track slabs on bridges leading to geometric imperfections of the rail are modelled by boundary conditions. Details of the model are shown in Fig. 4.

The mechanical models of the fasteners, which are based on experimental investigation, are shown in Fig. 5. For a specific explanation of these models please refer to [28].

In reality, the rail is continuously bedded on the surface of the rail pads, which belong to the fastening devices. However, the realisation of a continuous boundary simulation will expend a lot of computation time and may cause convergence problems due to the high level of nonlinearity. Hence, a simplification analysis was carried out to improve computational efficiency. The goal was to find a configuration of only two springs that creates an equivalent rotational and vertical stiffness compared to the combination of a continuous bedding below the rail and the tension clamps above the rail foot (Fig. 6).

For the simplified continuous boundary condition, as long as the pretension of the supporting force is not overcome ($F < 27$ kN for BSPFF-B-1 refer to Fig. 5), practically only the continuous springs on the lower side of the rail will work in vertical direction. When tension occurs ($F > 27$ kN), the springs on the upper side of the rail will be fully activated. For the simplified boundary condition, the compression effect as well as the tension effect of the fasteners are combined into two springs on the lower side of the rail. When the distance between the two springs is selected to be 95 mm, these two models correspond to

each other very well. An obvious difference is observed when the rail rotation is larger than 35 mrad as Fig. 7 shows. Under this circumstance, the simplified model tends to have conservative results. Taking the real situation into consideration, the rail rotation will be less than 35 mrad. Within this range, the rotation stiffness of the fasteners could be simulated with the two springs. The simplification of the constraint condition is approved to be effective and will be applied to the models in the following chapters.

3.2. Simulated imperfection conditions

Due to manufacture processes, structural effects on bridges and some other influences, rails are subjected to imperfections. Because of these imperfections, rails will deform or even buckle under longitudinal pressure. After implementing imperfections in the model, a longitudinal compressive load was introduced. Table 2 lists all the considered imperfection cases for the simulation, differentiated between transverse and vertical direction. The aim of this investigation was to find the most critical imperfection leading to the lowest acceptable compressive force.

The cases H1, and V1 are due to relative displacements in the joint between neighboured superstructures. The values of the deformations are determined by a model of the bridge for the load cases temperature variation, pier temperature variation and traffic load. Detailed calculation can refer to [29]. The limitation of H2, H5 and V2 is defined in the German code Richtlinie 804 [15]. The shape of the rail imperfection H3 and V3 are limited in the German rail code DIN EN 13674-1 [30]. Finally, H6 and H7 are the first and second Eigenmodes calculated by numerical simulation. The amplitude of the imperfection considers the maximum allowable geometrical imperfection, which is 1.0 mm according to [27]. For all the aforementioned cases, the rail works with all fasteners functioning. Since the bedding conditions given by the fastening devices majorly influence the system behaviour, the accidental case of failed fastenings was also investigated. When a fastener fails, it provides no lateral or vertical support to the rail. Fig. 8 shows the two additional investigated situations. In one case, fastener No. 6 fails, in another case, both fasteners No. 6 and No. 7 fail. These system configurations were slightly adjusted for the lab tests (refer to Section 4.3).

3.3. Results

In the numerical simulations for the preparation and analysis of the test, the simplified assumption has been taken, that the rail material behaves elastic until it reaches a yield strength (in this case, the design

Table 2
Imperfection cases.

Transverse			Vertical		
H1		$U_y = 2$ mm	V1		$U_z = 5.4$ mm
H2		$\phi = 2$ mrad	V2		$\phi = 5$ mrad
H3		$H < 0.6$ mm (see H6 & H7)	V3		$V < 0.4$ mm to 3 m or < 0.3 mm to 1 m (neglected)
H4		$F_h = 50$ kN	V4		$F_v = 110$ kN
H5	Rail line radius	$R = 2200$ m	–	–	–
H6		1 mm	–	–	–
H7		1 mm	–	–	–

Fig. 8. Rail system with failure of fasteners.

Table 3
Simulation results of **transverse** load cases.

LC	Deformed shape	$\Delta_{\max}$ [mm]	LC	Deformed shape	$\Delta_{\max}$ [mm]
Legend H1 0MN 5MN Δ					
H1		3.25	H5		1.47
H2		3.42	H6		3.66
H4		3.19	H7		3.36

Fig. 9. Maximum displacement development (legend refers to Table 3).

Fig. 12. Maximum displacement development.

Fig. 10. Load application.

Fig. 13. Rail shape change.

Fig. 11. Target and calculated rail shape.

yield stress was 600 MPa). This stress corresponds to a maximum axial load of $F = 600 \text{ MPa} \times A_{\text{rail}} = 4.59 \text{ MN}$ if the influence of moments is not considered. Therefore, the maximum load considered for the design

of the test was $F_{N,\max} = 5 \text{ MN}$.

For the transverse imperfection cases (Table 2), the rail deforms obviously under axial compressive load. Table 3 lists the simulation results of the transverse cases. All the load cases, except for H4, show more than 3 mm deformation in lateral direction under $F_{N,\max} = 5 \text{ MN}$. The deformation shape (line Δ) of H1 is similar to H7. The deformation shape of H2, H4 and H5 are similar to that of H6. This indicates that regardless of the initial shape, the rail tends to deform similarly to its eigenform shape.

For the vertical imperfection cases, load cases V1, V2 and V4 were calculated. The maximum deformations under $F_{N,\max} = 5 \text{ MN}$ are 0.23 mm, 0.33 mm and 1.42 mm, respectively. These deformations are much smaller in comparison with those of the transverse cases. It can be

Fig. 14. Schematic diagram of the test platform.

Fig. 15. Installation in the laboratory.

concluded that the considered system reacts much stiffer in vertical direction than in transverse direction. In other words, the deformation of the rail is more critical in transverse direction under axial compressive loads. Hence, the following analyses focus on rail behaviours in transverse direction. The maximum deformation positions on the rail under 5 MN of each case can be determined after the simulation. In Fig. 9 it can be observed that the displacements of the rail in those positions under case H1, H2, H6 and H7 are very similar to each other. The displacements develop very slowly and are less than 1.5 mm within 3 MN. After that, the displacements increase much faster as the load

risks; the maximum displacement can be larger than 3.5 mm under 5 MN compressive load. In comparison, the rail behaves much stiffer for case H5; the maximum deformation is only 1.47 mm under 5 MN load. As for case H4, the displacement develops much more uniformly when the load increases; it can also reach 3 mm under 5 MN.

Based on the simulation results, H1, H2, H4, H6 and H7 should be considered for the lab test. However, due to the limited space and equipment in the laboratory, H4 was not considered here, but will be considered in future field tests. Although H6 is more unfavourable than H7 when considered alone, H7 tends to be more unfavourable in combination with H1 and H2. In conclusion, case H1, H2 and H7 will be considered for carrying out the experimental tests.

In order to achieve the worst case, different combinations of the three aforementioned cases were also simulated. The combination of all three cases would lead to a larger deformation, but only the combination of H1 and H7 would have similar results and can be more realistically reached in practice. The target shape was designed to be achieved by applying four transverse forces on the rail in different positions, as Fig. 10 shows. The target shape and the shape calculated by applying four loads are shown in Fig. 11. Figs. 12 and 13 show the maximum displacement development and rail shape change under this circumstance. The maximum deformation can be as large as 3.4 mm under 5 MN load.

The rail behaviour of a rail with one or two pairs of fasteners (Fig. 8) removed is similar, but with bigger magnitudes according to the calculation. As the rail geometry and system configuration would be adjusted during the test, a detailed pre-study was not carried out. Verification calculations will be introduced in Section 5.2.

4. Experimental tests

Based on the results of the numerical pre-study, an experimental investigation was performed. In order to perform the tests, a hydraulic loading jack with a capacity of 5 MN was installed, and a test platform and a rail track slab system were constructed.

Fig. 16. Longitudinal load introduction.

Fig. 17. Transverse load introduction.

Table 4
Major measurements concept.

Name	Sensor	Sensor numbers	Measurement output
G1	CCR	–	Initial position of the rail, the fasteners and the track base plates
G2	Leica T-Probe	34	Initial position of the rail after pre-test
G3	CCR	9	Position of the rail during the test
D1	Strain gauges	8	Compressive forces of the rail
D2	Strain gauges	6	Stress due to longitudinal force and bending moment
W1	Laser sensors	2 × 9	Transverse deformation of the rail

Fig. 18. Measurement arrangements along the rail longitudinal direction, distances in [mm].

Fig. 19. Measurement layout across the sectional direction.

4.1. Experimental platform

To carry out the experiment investigation, a 10.8 m × 2.3 m test platform (Fig. 14) next to the track slab system was designed and constructed. The frame is composed of two concrete beams, two steel cross beams, a hydraulic jack and two units of guidance devices. Each concrete beam includes six uniformly distributed steel bars in the cross section, which can be pre-stressed. These pre-stressing bars lead to a compressive force of 2.5 MN for one beam. Therefore, the whole frame

Fig. 20. Loading program of TV1c.

was able to withstand elastically a maximum load of 5 MN introduced by a hydraulic jack. The concrete beams are connected to each other with two HEA 600-steel beams at each end. A 5 MN hydraulic jack was placed at the left end of the rail. The other end of the rail was fixed to the frame. The roller guidance devices (Fig. 16) were constructed to allow longitudinal movement by preventing transverse movement in order to achieve controlled bearing conditions.

Up to four additional hydraulic jacks were installed in transverse direction. Thus, the calculated rail imperfection according to Fig. 10 could be implemented (Fig. 17). Fig. 15 shows a full view of the completed test platform. The movement of the track slab was controlled with a clamp device on both sides of the slab.

Table 5
Investigated cases.

Case	Test Section		$F_{N,max}$ [MN]	Starting Rail geometry	Comment
–	–	TV2a	2,7	Starting geometry (Plate movement + plate rotation)	Test trial
Case 1	TV1a	TV2b	2,7	Starting geometry	Target curve implementation
Case 2	TV1b	TV2c	3,0	End geometry of Case 1	–
Case 3	TV1c	TV2d	2,5	End geometry of Case 2	Fastener No. 6 removed
Case 4	TV1d	TV2e	2,5	End geometry of Case 3	TV1d: Fastener No. 6 and No. 7 removed TV2e: Fastener No. 5 and No. 6 removed
Case 5	TV1e	TV2f	3,7	End geometry of last section	–

Fig. 21. Rail geometry before and after the transverse loading procedure.

Fig. 22. Additional deformation of the rail after case 1.

4.2. Measuring concept

In order to obtain the initial rail geometry information, and also to monitor and record the behaviour of the tested rail during the experiment, a measurement concept was developed. The major measurements are listed in Table 4.

The G1, G2 and G3 measurements were carried out by the Geodetic Institute of Leibniz University Hannover. Leica's AT960LR laser tracker monitored Corner Cube Reflectors (CCRs) on the track plates, rails and abutments. The individual measurements were carried out with an accuracy of < 0.1 mm (maximum permissible error, MPE) in the entire experimental area. To define the initial position of the track slab, the

position of the fastenings characterized by 24 angle guide plates per rail was detected (G1). In addition, a reduced number of CCRs was measured at the beginning of each test section (see Table 4). To determine the rail axis, 156 rail profiles (G2a) with 40 measuring points on the rail head were measured by Leica T-Probe across both rails (TV1 and TV2) at a specified distance. For each of the recorded profiles, a CAD model of the rail profile was adapted to the measured data. The adaptation was carried out by means of least squares adjustment. To define the rail geometry at the beginning of each test section, the CCRs on the rails (see Fig. 15) and all points on the track base plates (G2b) were measured. Subsequently, the changes in x, y and z directions to the initial state of the rail axis can be calculated from G2a. During the tests (G3), measurements of CCRs at defined load levels were carried out to monitor the positions and relative changes of the rails.

Figs. 18 and 19 present the detailed layout of the D1, D2 and W1 sensors in longitudinal direction.

As Fig. 19 shows, the strain gauges D1 are located on both sides of the rail web in order to monitor the longitudinal force in the rail and its development over the rail length. The strain gauges D2 are applied to the rail foot. Therefore, the stress due to longitudinal load and transverse bending of the rail at specific locations can be investigated. The laser sensors W1 are oriented towards the head and foot of the rail to measure the transverse displacement of the rail.

Additionally, the longitudinal displacement of the rail, oil pressure of the hydraulic jack, force of the hydraulic jack, rail temperature, ambient temperature, longitudinal and transverse displacement of the track slab, steel beam and concrete beam are also monitored and recorded during the experiment to ensure the safety and reliability of the tests.

4.3. Test program

Two rails were tested so that the reproducibility of the test can be checked. The two test parts were defined as TV1 and TV2, which consisted of several cases. The corresponding cases, which are experimentally investigated and numerically validated, are listed in Table 5. For each rail, the initial imperfection geometry introduced in Section 3.3 was applied to the rail by plastic deformations under combined effects of longitudinal and bending stresses in the first case. In case 2, the intact system with all fasteners was tested within a compressive load of 3.0 MN. Subsequently, defective systems with one or two pairs

Fig. 23. Rail deformation curve for different cases.

Fig. 24. Rail deformation development for different cases.

Fig. 25. Rail stress development for different cases.

of fasteners removed were tested within 2.5 MN load. In the last case, the compact system was tested again with a larger load until the rail reached large plastic deformations. After each case, the fasteners were removed to ensure that no lateral forces remained in the fasteners.

4.4. Loading and imperfection implementation procedure

In order to obtain stable and reliable results, the loading procedure was carefully controlled. At first, the longitudinal force was increased up to the first load level and then kept constant for 3 min. Subsequently, the load was reduced to the basic load level (300 kN) and also kept constant for 3 min. This was repeated three times until the next load

level was implemented. As an example, Fig. 20 shows a typical loading program of section TV1c. The G3 measurements took place every time when the load was kept constant. The laser sensors and the strain gauges measured the deformation continuously with a frequency of 20 Hz.

The calculated target shape for both tested rails (TV1 and TV2) was the same in order to achieve the same target geometry, different transverse load arrangements based on a FEM simulation were designed to be implemented. Afterwards, these transverse loads were removed. As a result, some plastic deformations due to the introduced imperfection remained in the rails. The rail geometry of both rails before and after the implementation are shown in Fig. 21. The remaining

Fig. 26. The gap between rail and angle guide plate of the fastener device.

Fig. 27. Revision of the fastener models.

plastic deformations from the implantation procedure are shown in Fig. 22.

As Fig. 21 shows, the initial displacement difference between the first and last G2 measurement positions of TV1 and TV2 is 10.3 mm and 10.1 mm respectively, which are very close to one another. This initial geometry was achieved by the movement and rotation of the slab plates. However, the remaining deformation of TV1 was much larger than that of TV2 after the transverse loading procedure. This indicates that larger plasticity, which had not been considered by the elastic pre-study simulation, has been implemented in TV1 rail rather than in TV2 rail. The magnitudes of the measured deformation curvatures were larger than the calculated geometry due to the assumption of the elastic material, but the shapes were already very similar.

5. Results of the experiments

5.1. Main test results

5.1.1. Rail deformation curve

After the implementation of the initial rail geometry, the experiments proceeded according to the sequence as listed in Table 5. The rail deformation curves, which are based on the test results, are presented in Fig. 23.

The laser sensors were set to zero before each test ($F = 0$ MN). The maximum deformations were measured by W1.3.1 or W1.4.1, the minimum deformation was measured by W1.7.1 or W1.8.1. However, the absolute value of maximum deformations was much bigger than the

minimum deformation due to the asymmetrical initial rail geometry. It can be observed from Fig. 23(a) that the deformation shapes of the two rails under 1.0 MN load are very different. Subsequently, the difference becomes smaller as the load increases. The rail geometry of both rails is very similar when the load reaches 3 MN. Under this load, the maximum deformation difference is smaller than 0.15 mm. The same phenomenon can also be observed in Fig. 23(b) within 2.5 MN and Fig. 23(d) within 3 MN. It can be concluded that the rail behaviour is first of all influenced by the initial geometry of the rail. This influence decreases as the load becomes greater.

Fig. 23(b) indicates that the removal of one fastener in case 3 has no obvious influence on the deformation shape of the two rails. In contrast, Fig. 23(c) shows that the removal of two fasteners affects the rail behaviour significantly. The differences become even larger as the load increases. It can be concluded that the development of the rail deformation was measurably influenced by the fact which fasteners were removed from the rail curve. Moreover, Fig. 23 shows that the removal of rail fasteners will reduce the stiffness of the rail system. With a load of 2.5 MN, the maximum deformation of the rail system with one fastener removed was 2.46 mm which is similar to that of an intact system. In contrast, the rail system, where two pairs of fasteners were removed, is 5.46 mm, this is considerably bigger than the value of an intact system with all fastening devices.

5.1.2. Deformation and stress development of the rails

The maximum deformation development curves of W1.3.1 and W1.4.1 for different cases are given in Fig. 24. The development of the

Table 6
Deformation results of the tests and FEMs.

Test part	Load [MN]	W1.3.1			W1.4.1		
		Test [mm]	FEM [mm]	Deviation[%]	Test [mm]	FEM [mm]	Deviation [%]
TV1b	2.0	1.643	1.514	-7.85%	1.737	1.553	-10.60%
	3.0	2.322	3.020	30.06%	2.541	2.957	16.37%
TV2c	2.2	1.027	1.470	43.12%	0.930	1.333	43.34%
	3.0	2.463	2.412	-2.05%	2.290	2.202	-3.84%
TV1c	2.1	2.061	2.489	20.76%	2.280	2.672	17.19%
	2.5	2.471	3.425	38.60%	2.757	3.654	32.54%
TV2d	2.1	1.745	1.884	7.97%	1.783	1.831	2.69%
	2.5	2.320	2.552	10.00%	2.382	2.541	6.68%
TV1d	2.1	2.022	3.302	63.30%	2.630	3.907	48.55%
	2.5	2.640	3.891	47.39%	3.574	4.761	33.21%
TV2e	2.1	4.106	4.563	11.13%	3.872	4.353	12.42%
	2.5	5.465	6.953	27.23%	5.173	6.620	27.97%
TV1e	2.1	2.084	1.792	-14.01%	1.553	1.765	13.65%
	3.0	2.139	3.203	49.74%	2.380	3.053	28.27%
TV2f	2.2	1.415	1.501	6.08%	1.344	1.362	1.34%
	3.0	2.366	2.472	4.48%	2.278	2.226	-2.28%

Fig. 28. TV2f rail deformation curve under different loads of FEM and test.

calculated stresses ($\sigma = \epsilon \cdot E$, $E = 210$ GPa) is shown in Fig. 25. The deformation and stresses of TV1 rail in case 2, case 3 and case 5 increases almost linearly within 3 MN, while the deformation of TV2 rail develops non-linearly from 0.0 MN for the same cases. The main reason for this different behaviour is that the TV1 rail has produced a larger plastic deformation during the imperfection implementation process (refer to Section 4.4). Aside from that, the TV1 rail also deforms faster than the TV2 rail due to its larger initial rail geometry (Fig. 22). This results in a maximum difference of 1 mm when comparing the two rails in the same situation. However, the difference decreases as the load increases. Furthermore, both rails begin to show a large non-linear deformation when the load exceeds 3 MN. This indicated that the rail began to yield further and reached much more plasticity.

As for case 4, both rails deform faster and nonlinearly from the beginning as the load increases. The deinstallation of two fasteners reduced the rail lateral stiffness. Therefore, the rails deform further in lateral direction and the bending effects contribute considerably to the rail stress. The non-linear development of strains due to the geometrical non-linearity or to the material non-linearity cannot be considered separately from one another.

Fig. 29. TV2f deformation results of FEM and test.

5.2. Recalibration of the numerical models

In order to not only validate the experiment results but also to provide reliable numerical models to analyse rail behaviour under various situations, the numerical models from Section 3, which were also based on an elastic material E-Modulus = 210 GPa, were calibrated and improved to verify the test results. In this case, the assumption of a constant E-Modulus is valid for forces up to 3 MN because of the loading history of the investigated rail. For future analyses, in order to determine the allowable compressive rail force, the non-linearity of the material has to be taken into account from the beginning.

Two major improvements were made to revise the models. First, the actual rail curvature was applied to the rail. During the test, the rail curvature was always changing, the rail shape before each test section was imported to the numerical model. Second, the relative positions between the rail and fastener device also changed after each test. Ideally, the fasteners should function at the neutral point, which is the position $S = 0$ in Fig. 27. However, the actual starting position of the rail might not be at the exact centre of each fastener pair. Before each test section, the gap $L1$ and $L2$ between the rail foot and fastener was measured, as Fig. 26 shows. The rail was supposed to be at the centre position of a pair fastener if $L1 = L2$. Otherwise, the fastener model should have to be calibrated. A movement $\Delta = (L1 - L2)/2$ was applied to revise the fastener models as Fig. 27 shows.

After the calibration, the numerical simulation results and experimental results for forces < 3.0 MN generally have a good consistency. Table 6 lists the main deformation comparison results of the tests and FEMs.

It can be observed from Table 6, within the range of 3 MN, the FEM results are on the safe side in general. These deviations vary from 1.34% to 63.33%. There are also very few cases, where the FEM provides values which are smaller than the measured values. In addition, under critical situations, such as an intact system under a load of 3 MN and a system without having one fastener or two fasteners under 2.5 MN, the simulation results are either larger than the test results or smaller than the test results because of negligible deviation. However, when the load exceeds 3 MN (3.5 MN for TV1e and TV2f, 3.7 MN for TV2f), the FEM results are much smaller than the tests results. In these cases, the rail begins to show plastic deformation, therefore, the FEM with elastic material assumption can be no longer applied.

In order to take a closer look at the differences between tests and FEMs, TV2f is taken as an example to be further investigated. The rail deformation curve under each testing level is shown in Fig. 28.

It can be observed from Fig. 28, as the load increases, that the deformation curve of test results and FEM results show a good consistency within 3.0 MN. W1.3.1 and W1.4.1 measured the maximum deformation which also corresponds to the FEM results. However, when the load is larger than 3.0 MN, the difference becomes bigger. The deformation curve under loads of 3.5 MN and 3.7 MN are very different. The deformation and stress development of the unfavourable measurement positions are shown in Figs. 30 and 29.

Fig. 30. TV2f calculated stress results of FEM and test.

Fig. 29 shows that the deformation develops almost linearly for FEM and the test measurement within 3 MN in both cases. When the load is larger than 3 MN, the FEM results still develop almost linearly, while the test results begin to develop non-linearly. Under 3.5 MN and 3.7 MN, the test results are much larger than the FEM results. A similar phenomenon can also be observed from Fig. 30 for the calculated stress results. The calculated stresses of the tests at each side of a rail section (e.g. D2.1.1 and D 2.1.2) develop non-linearly in different directions depending on the deformation direction.

In conclusion, the FEMs and the test results show a good consistency when the load is within 3 MN. This indicates that the elastic material assumption can be applied within certain loads (3 MN). For larger loads, the non-linear material behaviour of the rail steel must be considered in FEMs.

6. Conclusion and outlook

A series of tests and numerical simulations have been undertaken to investigate the behaviour of ballastless track system under high longitudinal compressive rail stresses. Based on the results of these investigations, the following conclusions can be drawn: the rail behaves much stiffer in vertical direction than in transverse direction. Under axial compressive loads, the rail transverse behaviour (displacement) is decisive; the vertical deflections of the rail are relatively small. The maximum transverse deformation is less than 2.5 mm under a load of 3 MN for the intact system with all fastening devices; when the load is larger than 3 MN, the rails deform non-linearly rapidly; the deinstallation of one pair of fasteners has only a small effect on the rail behaviour (displacement and stress); the deinstallation of two pairs of fasteners greatly reduced the rail structure stiffness, the position of the deinstalled fasteners on the rail geometry wave also has a great influence.

The corresponding theoretical buckling load in the rail of the track on ballasted track system is 1.94 MN according to previous investigations. No safety factors have been taken into considered for this value. Since the ballastless track system reacts stiffer than ballasted track, a higher possible rail force is expected. On the basis of the aforementioned investigation, it could be shown that for the tested boundary conditions and for axial forces up to 3.0 MN, rail deformation stays smaller than 2.5 mm and no indication for system instability occurs. According to this research, the rail resistance under longitudinal compressive load can be largely increased based on the considered condition. There remains still the problem that no consistent safety concept for allowable rail stresses under track/bridge interaction in existing codes. This paper provides a good basis and direction for further research to finally determine and update the permissible compressive rail stress under bridge and track interaction. For this purpose, the development of large-scale tests, more detailed numerical analysis and safety considerations have been under way [31]. Further results will be

published in the near future.

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2.3 Experimental investigation on rail fatigue resistance of track/bridge interaction

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Chongjie Kang was responsible for the investigation, data curation, visualization and original draft writing. Sebastian Schneider was responsible for the realization of the test setup as a part of the investigation. Marc Wenner was responsible for the management of the research project and the initial test program. Steffen Marx was responsible for the acquisition of the funding for this investigation. The successful completion of this investigation was achieved due to the intensive cooperation and exchange of all co-authors. All co-authors reviewed this paper and provided meaningful comments during the process.

Experimental investigation on rail fatigue resistance of track/bridge interaction

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ABSTRACT

This paper presents an experimental study on the flexural fatigue behavior of the typical high-speed rail profile 60E2 (UIC60) for five million load cycles. A resonant machine was applied to achieve a test program of different load levels with high loading frequency. This investigation focuses on the rail fatigue resistance under multiple minimum stress levels and different corrosion conditions. In comparison with historic investigations of rails for two million load cycles, this new rail profile shows better fatigue behavior even for 2.5 times more load cycles. In the end, a new Smith – diagram for five million load cycles is proposed.

1. Introduction

Railway transportation system is usually preferred over other modes of transportation because of its higher speed, better safety, and less environmental negative impacts [1,2]. The steel rails are at the heart of the world's railway system. Their main functions are to transmit wheel forces to the track bed and, together with the tread and flange of the wheel to guide vehicles [3]. The avoidance of rail failure is a primary goal of track safety [4]. According to unpublished statistical data from the Deutsche Bahn AG (German Railway Company), 1,083 occurrences of rail failure in the German railway network were recorded in the two years between 2012 and 2013 for a track length of about 63,000 km. Despite substantial improvements in design, materials used and non-invasive inspection, fatigue propagation and the failure of railway components remain an important issue for safety engineering [5]. Various damage and failure types of different rail profiles have been investigated [3–10] and classified in both German standard [11] and international codes [12,13]. The fracture of a rail track as a result of fatigue cracking is considered as a serious incident in the railway industry, as it is likely to result in vehicle derailment with a high probability of loss of life [14].

Railway infrastructure managers are facing different challenges: first, to improve railway safety by reducing track failures and associated risks, which implies having high-quality rails and a good maintenance policy; second, continuously increasing train frequency, running speed

(high-speed railway) and axle load [15]. In recent years, wear and rolling-contact fatigue (RCF) were identified as the two main failure mechanisms for the rails. A lot of investigations have been conducted in these two fields [16]. However, the flexural fatigue strength, which is also an essential property of rail tracks especially regarding the additional stresses in the rail in the area of bridges, has been neglected in recent years. Due to the rapid development of the railway industry, the rail material, profile and manufacturing process have been largely improved. As a consequence, new knowledge of these rails is now necessary.

2. Research background

In principle, bridges are the critical area of rail failures. Bridge/track interaction [17] introduces significant additional stresses in the rail as well as in the bridge. This is no critical problem when the rails are not continuously welded (rail stresses can be released with the existing rail joints) or when the bridge is short. However, due to the rapid development of railway traffic especially high-speed railway traffic, rails have been welded continuously to gain better smoothness; longer bridges have been constructed to cross wider obstacles. The bridge/track interaction has to be, therefore, precisely considered and calculated. In order to limit these additional stresses in the rail, different investigations have been carried out at national and international levels to determine the permissible additional tensile and compressive

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Table 1
Permissible additional stresses at different levels for HSR [MPa]

Stress	Track system	Eurocode[17]	UIC[18]	Germany[19]	Italy[20]	Spain[21]	China[22]
Compressive	Ballast	72	72	72	60	72	61
	Ballastless			92			
Tensile	Ballast and ballastless	92	92	92	70	92	81

stresses due to bridge/track interaction. The resultant limit values from different codes and nations are listed in Table 1.

Table 1 shows that the limit values of Eurocode, UIC, Germany and Spain are the same except that the compressive limit values of Germany are different for ballast and ballastless systems. In comparison, the values of Italy [20] and China [22] vary very widely as they are based on different theoretical concepts. The investigations of this work focus on the permissible additional tensile stress within the scope of Eurocode, UIC (International Union of railways) and German concept. The historic development of the limit value 92 MPa will be thus at first explained in this section.

Rails are under complex loading conditions during operation. The major actions and stresses are:

- (1) σ_{ATR} : Temperature variation in respect to the neutral temperature ($23^\circ\text{C} \pm 3\text{ K}$ according to German Railway).
- (2) σ_R : Residual stress due to rail manufacturing process (cooling and rolling process).
- (3) σ_Q : Bending stress due to the crossing of train traffic (according to the theory of Zimmermann [26–28]).
- (4) $\sigma_{add, longitudinal}$: Additional longitudinal stress due to track/bridge interaction (longitudinal displacement of the bridge).
- (5) $\sigma_{add, bending}$: Additional bending stress due to track/bridge interaction (longitudinal displacement and rotation at bridge joints).
- (6) Other effects (snow etc.).

The major stress parts are schematically shown in Fig. 1.

Very unfavorable cyclic train loads and residual stresses due to manufacturing process occurred in the middle of rail foot, which is normally regarded as the initial area of fatigue break. The bending tensile stress of the rail foot and rail head was statistically proved by theoretical and experimental investigations at the Technical University Munich (TUM) in the 1960 s [30,31]. In the case of mean-stress-dependent fatigue strength analysis, the fatigue strength will be expressed by the variation of stress (stress range $\Delta\sigma$) due to cyclic loads. This fatigue strength can be determined by cyclic rail bending tests. The influence of rail profile, surface properties and manufacturing process (especially for residual stress) can be thus directly included. The tests of TUM in the 1973 were carried out under a minimum stress level of 50 MPa. With the purpose of determining the fatigue strength of the rail, the Smith-diagram (see Fig. 2) was applied. To construct a Smith-diagram, the minimum stress, maximum stress and mean stress, which can be obtained by the fatigue bending tests, are needed. The horizontal axis and vertical axis present the mean stress level and maximum/

Fig. 2. Smith-diagram for a new rail UIC60, the value of 282 MPa is the stress limit according to Oberbauberechnung 1977 [33]

minimum stress (σ_{max} , σ_{min}) level, respectively. After the σ_{max} , σ_{min} and stress range ($\Delta\sigma$) of one load level are decided, the σ_{max} and σ_{min} can be connected with the ultimate strength, which is 900 MPa of the rail steel for both axis, respectively. Additionally, an allowable maximum stress should be set as the upper limit (σ_{limit}). The lower solid line presents the minimum stresses (σ_{min}) and the upper solid line presents the maximum stresses (σ_{max}); the stress range ($\Delta\sigma$) was the corresponding allowable fatigue strength. As a result, the fatigue strength of the rail under different stress levels can be predicted. The detailed principles of constructing a Smith-diagram can refer to DIN 50100 [32].

It can be observed from Fig. 2 that when the minimum stress level $\sigma_{min} = \sigma_T + \sigma_R = 180$ MPa was determined, the allowable stress range (fatigue strength) $\Delta\sigma = 282$ MPa due to other effects can be thus concluded. However, the additional rail stresses in bridge area were not clarified until 1977. Based on the aforementioned discoveries, the ‘Oberbauberechnung’ was issued in 1977 [33]. Later on, as a result of the research project ‘R/S-Brücke, Thema Nr.61’ for the new railway lines in Germany, the previous empirical approach to limit the effects by bridge/track interaction was updated. The fatigue strength tests on rails of open tracks have been carried out to supply and further the

Fig. 1. Distribution of stresses over rail height [29]

Fig. 3. Smith-diagram for the determination of the allowable additional tensile stress in the rail foot due to track-bridge interaction [30]

investigations on additional tensile rail stresses in bridge area from 1977 to 1980. Fig. 3 shows the resultant Smith-diagram.

The values in Fig. 3 to determine the permissible additional tensile stress are derived from experiment results of heavily corroded rails with S54/90 profile (maximum roughness depth approximately 500 μm). In these tests, both operated and not operated rails were tested. According to the results, the structural fatigue strength of the rail was identified as $\Delta\sigma = 205$ MPa under a minimum stress level of 50 MPa for two million load cycles. To obtain this value, associated bending fatigue tests were carried out on 11 rail specimens for two million load cycles. As a result, nine rails failed within two million load cycles and the other two ran out. The resultant S-N curve is shown in Fig. 4. The minimum stress range, which led to rail failure, was 210 MPa. The value 205 MPa was concluded as the lower boundary and regarded as the final fatigue strength (stress range). In the 1980s, studies on UIC60/900 rail were carried out [34]. It concluded that higher values than 205 MPa could be set for UIC60/900 rail compared to S54 rail.

However, all the assumptions and statements in Smith-diagrams only apply for the stresses within elastic range. An upper limitation in Fig. 3 should be prescribed. As results of numerous tension tests in the 1970s, an average yield stress value of $\sigma_{S,M} = 525$ MPa and a lower 5-percentile value, which led to a final limit value of 470 MPa, were identified in the diagram. The 0.2% expansion limit of rail steel R260 identifies a higher value of $\sigma_{S,M} = 600$ MPa nowadays. As this value is based on static tests, whether the yield strength can be set as the upper edge in the Smith - diagram is also worth investigating.

The additional stresses due to bridge/track interaction can be

Fig. 4. Summary of fatigue tests on corroded rail S54/900 from [18], determination of fatigue strength as a lower boundary curve.

calculated as $\sigma_B = \sigma_{BT} + \sigma_{BD} + \sigma_{BV}$, in which σ_{BT} denotes the additional stresses due to bridge temperature variation; σ_{BD} denotes the additional stresses due to bridge bending; σ_{BV} denotes the additional stresses due to longitudinal displacements of bridges result from braking/traction. The limitation of σ_B is obtained from the constructing of the Smith-diagram. Originally a distinction was made in the project ‘R/S-Brücken – Thema Nr. 61’, whether the braking force should be handled as fixed control threshold value (σ_{BV} as the minimum stress) or as an operational braking condition (σ_{BV} as fatigue-relevant stress variation). Since most of the structures are on the open tracks, a braking force frequency $< 10^4$ was uniformly set, so that the three stress parts σ_{BT} , σ_{BD} and σ_{BV} were regarded as the minimum stress threshold in the Smith-diagram. Even for more frequent braking forces, the stress variation could only be superimposed with the static parts due to wheel loads of the coming trains, only the dynamic effect of the wheel loads were considered as the fatigue stress variation. As Fig. 3 shows, the reserved stress is 112 MPa for ballast track rail system. This value was also extended to be applied for track slab systems.

To calculate the rail stresses on bridges, the nonlinearity of the displacement resistance must be taken into account. In the early 1980 s, the effect of longitudinal displacement were calculated by one dimension models. However, tools to calculate the effects of structure deflections were rare at that time. To avoid complex situations for individual cases, σ_{BD} was considered as 20 MPa and was then subtracted from the reserved stress σ_B [35]. This value was also applied to rails on simply supported bridges of $L = 44$ m span in the Deutsche Bahn AG ‘Rahmenplanung Talbrücken’ [30]. The reserved stress could be then calculated as $112 - 20 = 92$ MPa for the remained two parts σ_{BT} and σ_{BY} .

The ‘Oberbauberechnung’ was updated by a new edition in 1992 [30]. Compared to the previous Smith - diagram (Fig. 3), the fatigue strength of rail profile UIC60/900 [34] after corrosion was also defined. For the fatigue strength verification, the additional stress (Fig. 5) due to temperature variation was changed to 100 MPa ($\Delta T_R = 42$ K). However, the allowable additional rail stress was not calculated from the construction of the diagram but kept to 92 MPa, with which σ_{BT} and σ_{BV} were covered [30].

In the current German code (Ril 804), Eurocode [17] and international guideline of UIC [18], the permissible additional tensile stress is fixed to 92 MPa. However, compared to the 1980 s, the current material properties have been greatly improved, changes in manufacturing process has led to different residual stresses in rail cross section, the loads and occurring stress spectrum during operation can be more

Fig. 5. Smith-diagram for the rail UIC60/900 according the ‘Oberbauberechnung’ 1992 [30]

accurately estimated. Today, there are other verification methods which correspond to the present regulations of fatigue damage evaluation and safety philosophy. Freystein concluded that for most superstructures, the limit value of rail 60E2 on ballast track system and track slab track system can be increased to 112 MPa provided that the proportion of σ_{BD} is calculated explicitly. This presents an important step on updating the limits [28]. Moreover, this adjustment has also been proposed for the next update of the Eurocode. Extensive experiment investigations on the limit values concerning the rail fatigue strength have been carried out in this work.

3. Experimental investigation

3.1. Specimen description

The rail specimens are from the new high-speed railway line Erfurt-Leipzig/Halle project in Germany. The rails have E602 profile (former UIC 60) with a steel grade of R260, which are the most widely used rail profile and material on HSR lines. Related basic properties of this rail profile are: elastic modulus $E = 210$ GPa, cross-sectional area of entire rail $A = 76.48$ cm², line weight 60.03 kg/m, vertical bending inertia for the entire rail $I_{yy} = 3,021.5$ cm⁴. The rails specimens for the tests were produced in 2010. The chemical composition of R260 steel is listed in Table 2. According to the norm, “the microstructure shall be pearlite but grain boundary ferrite may occur in these grades. There shall be no martensite, bainite or grain boundary cementite. Detailed information of the profile can be found in EN 13674-1 [36].

These specimens were newly manufactured and had not been put into operation after production. For the whole research project, a total number of 206 specimens were produced from eight pieces of 60 m long rails. For the bending fatigue tests, the rails were cut into 2.2 m long specimens. Except for the rails during the corrosion test, the rails were in direct contact with the natural environment. The corrosion and hence the roughness developed during the time between production and test.

Additionally, standard tension tests according to DIN 50125 [37] were carried out on steel specimens with dimensions of $5 \times 16 \times 50$ mm to validate the rail material before the fatigue tests. The specimens were cut out from the foot center. The stress – strain curves of the specimens are shown in Fig. 6, it shows that the steel material has an almost linear stress – strain relationship until 550 MPa. The results are listed in Table 3.

3.2. Fatigue test principle and test program

Referring to the Eisenmann's experiments in the 1980 s at TU Munich [38,34,39], the supporting span and loading distance were set to 1.9 m and 150 mm respectively to carry out the four-point bending tests as Fig. 7 illustrates.

The total length of one specimen was 2.2 m. Each end of the rail was laid upon an individual half cylinder which had a 40 mm radius. All configurations met the EN 14730 standard [40]. The objective of the tests was to investigate the fatigue strength of the rails along the stronger axis of the section until $N = 5 \cdot 10^6$ load cycles.

Rail fatigue tests under three different minimum stress (σ_{min}) levels, 50 MPa, 130 MPa and 210 MPa, have been planned to be carried out. In order to evaluate the rail fatigue resistance with the staircase method, at least 10 specimens were required for each level. The influence of

Fig. 6. Stress-strain behavior of tested standard specimens.

Table 3

Results of the tension tests.

Specimen	0.2% yield strength [MPa]	Tensile strength [MPa]	E-modules [GPa]
No.1	614	999	208.5
No.2	589	997	199.6
No.3	594	1,002	214.9
Mean Value	599	999.3	207.7

Fig. 7. Principle design for the vertical tests.

Table 4

Test program.

Minimum stress σ_{min} [MPa]	Corrosion condition	Specimens	Starting stress range $\Delta\sigma$ [MPa]
50	not-corroded	12	310
130	not-corroded	12	280
130	corroded	12	250
210	not-corroded	12	250

surface roughness was also intended to be concluded in the experiments. Table 4 gives a full overview of the test program. For each load level, the minimum stress remained always constant. When performing one test series by applying the staircase method, the first specimen can be tested at a certain stress range. If the test was completed with a failure within $5 \cdot 10^6$ load cycles, then the stress range for the next specimen was decreased by 10 MPa. If the test was completed without a failure after $5 \cdot 10^6$ load cycles, then the stress range for the next test was increased by 10 MPa. The tests would proceed until all 12 specimens were completed.

Table 2

The chemical composition of the R260 rail specimens [36]

Element	C	Si	Mn	P max.	S max.	Cr	Al max.	V max.	N max.	O	H
Value	[%]	[%]	[%]	[%]	[%]	[%]	[%]	[%]	[%]	[ppm]	[ppm]
	0.60–0.82	0.13–0.60	0.65–1.25	0.030	0.030	≤ 0.15	0.004	0.030	0.010	0.002	0.00025

Table 5
Surface roughness parameters [44]

Value	Description
R_a	Arithmetic mean roughness $R_a = \frac{1}{l} \int_0^l Z(x) dx$
P_t	Maximum height of the assessed profile curve
R_z	Average of maximum roughness height per sampling length
R_q	Root mean square roughness $R_q = \sqrt{\frac{1}{l} \int_0^l Z^2(x) dx}$
R_t	Maximum peak-to-valley roughness

3.3. Corrosion experiments

The fatigue life of metal structures is known to depend heavily on the surface quality [41]. The mechanism was investigated and explicated on a micro-scale by means of many theoretical and experimental studies [42]. Corrosion tests were carried out on 12 additional rail specimens to obtain a more prominent difference in fatigue strength compared to the not-corroded rails.

During the fatigue tests, the roughness was always measured after the completion of each fatigue test. The surface roughness develops during the fatigue tests according to both numerical and experimental investigations [42,43]. However, in comparison with the absolute surface roughness values of all tested rails, the variation is very small. This can, therefore, be neglected. The surface roughness of both not-corroded and corroded rail specimens was measured and documented according to the ISO standards [44,45]. The measured values are listed in Table 5. In the cases of failed rails, the measurement took place from the broken edge to the other end of the rail. In the case of run-out rails, the measurements took place in the mid-span behind the polished surfaces of the strain gauges.

The rails were placed in a spray chamber for 10 h in the environment of 5% sodium chloride solution at 35 °C according to the NSS test from ISO 9227:2017 [46] (see Fig. 8 and Fig. 9).

Stylus-based equipment was the chief way of measuring surface roughness [47]. A contact roughness measuring device (Fig. 10) was used for the rail surface. The measurement paths are shown in Fig. 11. The test needle (sensor) had a radius of 2 μm, the measuring distance was 40 mm according to [45,48], and the measuring speed was 0.3 m/s.

In order to control the degree of rail corrosion, three rails were measured after each corrosion test. The target values were determined on the basis of a historical investigation of the rails, which had been operated for more than 10 years [34]. Fig. 12 demonstrates the roughness of the rails before and after the corrosion tests. Before the corrosion test, the roughness parameters are approximately half of the target values. In contrast, after corrosion these parameters were quite close to each other and about twice greater than the target results. Under these circumstances, the fatigue test results should be on the safe

Fig. 9. Rails after corrosion.

Fig. 10. Roughness measuring device.

Fig. 11. Roughness measuring paths.

Fig. 8. Rails before corrosion.

Fig. 12. Corrosion control results.

Fig. 13. Resonant test stand.

Fig. 14. Associated equivalent mechanical model of the resonant test stand.

side in comparison with historical tests.

3.4. Test facility

A newly developed fatigue testing facility based on a resonant test concept by the Institute of Concrete Construction of Leibniz University Hannover was used to carry out the fatigue tests. More detailed information can be found in the work of Schneider [49]. Figs. 13 and 14 show the major components and associated equivalent mechanical model of the test facility, respectively.

Based on the theory of Rayleigh method, the fundamental natural frequency of the mechanical model (see Fig. 14) can be calculated as follows:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{E \cdot I_y \cdot \pi^4 + 2 \cdot L^3 \cdot \sum k_{II}}{L^3 \cdot (\mu \cdot L + 2 \cdot m_{II})}} \quad (1)$$

The resonance machine is able to generate excitation frequencies up to 25 Hz. However, the first bending eigenfrequency of the tested rail system with a rail span of 1.90 m in the vertical direction, without any additional weight, is $f_0 \approx 140$ Hz. In order to reduce this frequency to the allowable frequency range of the machine, an additional 1.6 t mass (resulting in a resonating mass of 2.4 t including the springs) was introduced in the middle of the span. Consequently, the first eigenfrequency of the rail system was reduced to $f_0 \approx 21$ Hz. The associated equivalent mechanical model of the resonant test stand is shown in Fig. 14. The stiffness of the springs is $k = 667.4$ kN/m. More detailed interpretation about the resonant machine and control system of the test facility can be found in [49].

The experiments should only introduce tensile stresses at the rail foot. As the vibrators generate a sinusoidal load function, the rail must be preloaded, so that no alternating tensile and compressive stresses were produced, and the required maximum and minimum stresses

could therefore be generated. These vertical mean bending preload F_m was introduced by the tension springs. In comparison with the stiffness of the rail, the stiffness of the springs was very small. The rail deflection would produce only insignificant changes in the spring forces during the tests. The experiments were controlled by specific control parameters, which in this case are the measured strain values at the center of rail foot. By applying a control loop, the excitation frequency of the resonance vibrators was continuously adjusted so that the required strain range could always be produced and held.

3.5. Measurement and test control

Before each test, the rail was mounted on the test facility. After that, the additional mass and the transverse beam were attached to the rail. Subsequently, the springs were prestressed and therefore the vertical tensile load could be transferred to the rail by the transverse beam. By controlling the magnitude of vertical prestressing, the required mean stress σ_m was produced at the bottom of the rail. Then, the electric motors generated a cyclic load. The rotation frequency increased at a constant rate until the planned stress range $\Delta\sigma$ was reached, which occurred after 3,000 to 4,000 load cycles. Finally, the stress range was kept constant until the end of the test; see Fig. 15. The rotation frequency of the electric vibrators was about $f \approx 20$ Hz. A run-out test with $N = 5 \cdot 10^6$ load cycles lasted about three days. This is four times faster than traditional experiments that are carried out by hydraulic

Fig. 15. Increasing rotation frequency and stress range at the beginning of a fatigue test.

Measurement sensors layout on the longitudinal direction [mm]

Cross section A-A [mm]

Fig. 16. Arrangement of the strain gauges and temperature sensors.

(a)

(b)

(c)

(d)

Fig. 17. Additional sensors to monitor the test stand (a) strain gauges and temperature sensor; (b) load cell, (c) limit switch, (d) photoelectric sensor.

testing machines with 5 Hz load frequency.

The tests were carried out with a strain-controlled procedure. Three strain gauges were installed directly on the bottom surface of a rail at the mid-span right under the applied loads. One gauge was installed at the center of the rail foot. The other two were installed parallel in the same longitudinal position with a distance of 20 mm to the foot edge (see Figs. 16 and 17). The strain gauges were aligned along the longitudinal direction of the rail. Before the DMS was installed on the rail surface, the corroded surface was carefully removed with the help of a small belt sander. A template was used to limit the sanding area to adhere the strain gauges. Subsequently, the surface was carefully cleaned. After the installation of the DMS, additional protection glue was also used for protection. Therefore, the influence of sanding can be controlled as little as possible. Temperature sensors PT100 were installed to monitor the rail and air temperature during the fatigue tests.

The tests were controlled by the measurement values of strain gauge 1. Strain gauges 2 and 3 were installed to measure the stress distribution across the foot and to serve as backup gauges for strain gauge 1. Four load cells were installed under the bearings to record the supporting forces of rails during testing. One laser sensor was applied to monitor the displacement of the mass which corresponds to the rail deformation at mid-span. Further strain gauges were also set on the vertical steel bars to measure the forces introduced by the springs. The rotational frequencies of the vibrators were recorded with the help of Hall effect sensors [50]. In addition, four limit switches [51] were set so as to have contact with the rail specimens and one photoelectric sensor was installed, so that the test could be automatically terminated in case the specimen broke or behaved abnormally.

4. Results and discussion

4.1. Rail failure

The test specimens had to complete a total of five million load cycles without any fracture or failure to pass the test according to [40]. If no sign of failure occurred during the five million cycles, the test was stopped and the specimen was considered to have 'qualified'. At the end of each test completed without failure, the rail was visually checked for the incidence of any fatigue cracks. If the rail failed, then the test was stopped and the fractured surfaces were inspected to identify the cause of the failure.

Of all the 48 tested rails, 24 (50%) of them were fractured. A fatigue

Fig. 19. Fatigue fracture type 2.

crack could be clearly observed on the fractured surface of each broken rail specimen. Four stages, comprising crack initialization, crack growth, crack propagation and rupture, could be observed from the fracture surfaces [52]. Almost all the rails were broken in the central 150 mm field along the longitudinal direction. However, there were three different types of fracture over the rail cross section. Most of the rail cracks started from the bottom surface of the rail foot, which in this case is called type 1 cracking, as depicted in Fig. 18. Subsequently, the crack was propagated in a semi-elliptical profile until it broke. The glossy crack surface profiles revealed a long-term wear caused by friction during the test, while the chevron patterns and the radial marks indicated not only a brittle fracture of the rail but also that the brittle fracture radiated from the crack profile through the whole rail section to the edges. This corresponds to the observations made by other researchers [14,38,34,53–55]. The other two fracture types (2 and 3) are illustrated by Figs. 19 and 20. The phenomenon was similar to type 1, but the initiation points were different. As for type 2, the crack initiated from the foot edge and propagated from the edge to the center of the rail foot. The front of the crack was arc-shaped. Type 2 occurred in two

Fig. 18. Fatigue fracture type 1.

Fig. 20. Fatigue fracture type 3.

Fig. 21. Measured crack dimensions.

rail specimens. In the case of fracture type 3, the crack initiated above the rail foot edge at the bottom of rail web. This only happened to rail specimen 1–18. The possible cause for type 2 and type 3 fracture could be the presence of relatively large pre-existing cracks in the respective positions or irregularities in the rail surface before the tests.

In order to capture the characteristics of all broken rails, the position and dimension of all rail cracks were measured, as depicted in Fig. 21 and listed in Table 6. As for the failed rails, 20 of them were fractured within two million load cycles. Only two of the rails endure more than three million load cycles. The area of crack profiles showed an obvious tendency to decrease when the load level increased. This indicates that the crack profile area is related to the mean stress level of the rail. The greater the mean stress is, the smaller the area will be. Additionally, the mean crack area S_m of the corroded rails was smaller than that of not-corroded rails under the same load level (130 MPa).

4.2. Staircase method analysis

Compared with static load, cyclic load leads to substantially lower strength limits for rails. Frequently, S-N tests are performed to obtain information about the quality of the components in a running process [54]. Historical tests were carried out by Eisenmann to obtain the fatigue strength of rail track profiles S54 and UIC60 during the last

century [38,34,39]. The staircase method (or up-and-down method) is often used for the determination of fatigue resistance [56–58].

During the staircase tests, a broken rail was recorded as a failure. If it did not break, it was recorded as a ‘run-out’. Both failures and run-outs were obtained to carry out the staircase method analysis. The mean fatigue strength $\Delta\sigma_{D, m}$ was calculated by using the formula (see also [40]):

$$\Delta\sigma_{D, m} = \sigma_0 + d \cdot \left(\frac{A}{N} \pm 0.5 \right) \quad (2)$$

where σ_0 is the lowest stress range at which tests with the less frequent result were conducted, $d = 10$ MPa, $A = \sum i \cdot n_i$ (i ranges from 0 to z), $N = \sum n_i$ = total number of less frequent events, n_i = number of less frequent events at the i -th stress level above σ_0 , i = coded stress level ($i = 0$ for σ_0), z = number of stress levels above σ_0 at which testing has been undertaken.

And the standard deviation was calculated using the formula:

$$s = 1.62 \cdot d \cdot \left(\frac{B \cdot N - A^2}{N^2} + 0.029 \right) \quad (3)$$

where $B = \sum i^2 \cdot n_i$.

In order to apply the fatigue strength from the test results to the design values, the quantile value of fatigue strength with a given survival probability must be determined. According to DIN EN 1990, the lower characteristic value of building material strength should correspond to 5% quantile for limit state verification. For constructions, the confidence level of 75% has prevailed. Based on the aforementioned information of the 5% quantile value, a confidence level of $\gamma = 0.75$, a factor of 2.048 can be thus determined according to [59]. As a result, the 5% quantile fatigue strength is calculated as:

$$\Delta\sigma_{D, 0.05} = \Delta\sigma_{D, m} - 2.048 \cdot s. \quad (4)$$

The evaluated results are demonstrated in Figs. 22 and 23.

The 5% evaluated quantile results by staircase method as well as the lower boundary strength result, which is the minimum test result where failure occurs, are listed in Table 7. It can be concluded that the results evaluated by the reliability method with 5% quantile are less than the

Table 6
Rail specimen break status.

Load Level	Rail specimen No.	Load cycles [million]	Fracture type	D [mm]	R [mm]	L_0 [mm]	Crack profile area S [mm ²]	Mean values S_m [mm ²]
50 MPa	1–29	1.60	1	35.0	15.4	64.5	423.3	336.87
	3–7	0.90	1	34.0	14.0	127	373.8	
	3–8	1.90	1	33.0	13.0	42.5	336.9	
	3–18	1.10	1	33.5	15.1	64.0	397.3	
	3–20	1.17	1	29.0	12.0	64.5	277.9	
	3–22	0.67	1	24.1	11.2	98.2	212.0	
130 MPa	2–1	2.14	1	24.9	11.0	97.5	215.1	254.72
	2–4	1.88	1	20.0	9.2	91.0	152.4	
	2–6	3.64	1	26.1	12.2	81.0	250.3	
	2–17	1.27	1	30.0	13.9	85.0	327.5	
	2–18	3.52	1	35.0	15.0	65.5	412.3	
	2–22	1.25	1	20.5	10.6	78.9	170.7	
210 MPa	1–18	1.91	3	–	–	–	–	127.64
	5–1	1.08	1	17.0	8.0	89.8	106.8	
	5–3	1.19	2	12.0	11.5	12.0	108.4	
	5–5	2.35	1	17.5	8.0	70.0	110.0	
	5–6	1.04	1	25.5	10.5	31.5	206.2	
	5–8	0.98	1	17.0	8.0	81.5	106.8	
130 MPa(corroded)	5–16	0.95	2	11.4	14.9	0.0	133.4	178.73
	5–18	0.62	1	25.1	12.0	56.0	236.6	
	5–19	1.01	1	15.2	8.3	71.0	99.1	
	5–21	1.40	1	20.0	11.5	61.0	180.6	
	5–22	1.16	1	24.5	11.0	64.0	211.7	
	5–25	0.64	1	24.0	11.4	61.5	211.0	

Note. ‘–’ means the data is unavailable.

Fig. 22. Staircase method testing results for 5·10⁶ cycles.

results evaluated by the lower boundary method. The fatigue strength of the rail decreases as $\sigma_{\min}$ increases. However, the decrease of the fatigue strength is much slower than the increase of $\sigma_{\min}$. From $\sigma_{\min} = 50$ MPa to 210 MPa, the rise of stress is 80 MPa for each level, while the decrease of $\Delta\sigma$ is from 289 MPa to 250 MPa, which is only about 39 MPa.

4.3. Corrosion effect

The surface conditions of the tested rails have demonstrated an impact on the fatigue resistance. For the $\sigma_{\min} = 130$ MPa experimental series, both not corroded rails and corroded rails were tested. The fatigue strengths of the two specimen types (with and without corrosion) are presented in Fig. 23 and also in the S-N diagrams Fig. 24. The fatigue strengths of the corroded specimens were clearly lower than those

of the not-corroded rails. Due to the corrosion, the scattering was much higher than those of the not corroded rails. The mean stress of not-corroded and corroded rails were 277 MPa and 255 MPa respectively. The difference was only 22 MPa. However, by taking the standard deviation into consideration, the 5% quantile values of fatigue strength $\Delta\sigma_{D,0.05}$ were 271 MPa and 199 MPa respectively. The evaluated fatigue strength of not corroded rails was 72 MPa (36.2%) larger than that of the corroded rails.

4.4. Summary

In comparison with the historical investigation results of the directive 'Oberbauberechnung' [30], this paper has made the following progress:

Thirty-six not-corroded rail specimens covering three different

Fig. 23. Graphical demonstration of rail fatigue strength.

Table 7
Test results evaluated by lower boundary and reliability method.

Minimum stress	Not-corroded rails		Corroded rail	
Results type	Lower boundary	5% quantile	Lower boundary	5% quantile
$\sigma_{\min} = 50$ MPa	$\Delta\sigma = 300$ MPa	$\Delta\sigma = 289$ MPa	–	–
$\sigma_{\min} = 130$ MPa	$\Delta\sigma = 280$ MPa	$\Delta\sigma = 271$ MPa	$\Delta\sigma = 240$ MPa	$\Delta\sigma = 199$ MPa
$\sigma_{\min} = 210$ MPa	$\Delta\sigma = 260$ MPa	$\Delta\sigma = 250$ MPa	–	–

Fig. 24. S-N diagram of tested rails.

minimum stresses of 50 MPa, 130 MPa, 210 MPa and 12 corroded rail specimens under 130 MPa minimum stress were tested. The plausibility and reliability of the results were verified in the aforementioned sections. A more in-depth insight into rail fatigue behavior under different conditions was obtained. High stress levels also correspond to actual situations where heavier axle loads and higher train velocities meet high quasi permanent rail stresses in the area of bridges [29]. In contrast, historical tests were only carried out on corroded rails under 50 MPa minimum stress. The resistance under higher minimum stresses needs to be extrapolated.

New fatigue experiments were carried out until five million load cycles instead of two million. The surface roughness of the corroded rails of the tests in this paper was also worse than that of the historical investigated corroded rails and of corroded rails after 10 years of normal use.

Finally, the evaluation method has changed. In the 1970–80 s, the lower boundary, which is the minimum test result, was taken as the design value. For the new results, the reliability method was applied to evaluate the results. The results produced by the later method are more reliable than those delivered by the former method.

Despite the above-mentioned, more scientific and critical experimental procedure and evaluation method, 60E2 (of R260 steel grade) rails still do not show worse fatigue resistance than the S54/90 (1978) and UIC60/900 (1980) rails. The results are listed in Table 8 for comparison. The historical results of untested load levels are extrapolated

Fig. 25. Smith diagram of the new tests results for $N = 5 \cdot 10^6$.

using the Smith-diagram. Under 130 MPa minimum stress level, the extrapolated fatigue strength of S54 (1978) and UIC60 (1980) is 183 MPa and 202 MPa respectively, while the new experimental-based fatigue strength is 199 MPa. The new result of a 60E2/UIC60 profile keeps almost the same even for three million more load cycles and more critical evaluation method.

The whole rail strength also depends on the residual stresses in the rail. A residual stress scattering magnitude of 80 MPa is determined in historic literature [30,28,60]. With the rapid development of rail manufacturing technology, the residual stress distribution in rails has changed. New investigations to determine the modern rail track residual stresses need to be carried out. With an assumption of 80 MPa residual stress scattering also for new rails, Fig. 25 provides a comparison of the Smith - diagram based on both historic and new test results. The stress range under different loads can be directly observed. It can be concluded that the not-corroded rails have much greater

Table 8
Comparison with historic results.

	1978 S54/90, 2 mill. LC corroded rail	1980 UIC60/900, 2 mill. LC corroded rail	New results 60E2/R260, 5 mill.LC not-corroded rail	New results 60E2/R260, 5 mill LC corroded rail
R_t [μm]	70–180	70–180	60–100 ^{*1}	170–250 ^{*2}
$\sigma_{\min} = 50$ MPa	$\Delta\sigma = 205$ MPa lower boundary	$\Delta\sigma = 225$ MPa lower boundary	$\Delta\sigma = 289$ MPa 5% quantile	
$\sigma_{\min} = 130$ MPa	$\Delta\sigma = 183$ MPa extrapolated	$\Delta\sigma = 202$ MPa extrapolated	$\Delta\sigma = 271$ MPa 5% quantile	$\Delta\sigma = 199$ MPa 5% quantile
$\sigma_{\min} = 210$ MPa	$\Delta\sigma = 162$ MPa extrapolated	$\Delta\sigma = 178$ MPa extrapolated	$\Delta\sigma = 250$ MPa 5% quantile	

Note: In Table 8, the mean values of R_t are listed. However, it was later found that here the maximum values of R_t should be written. Therefore, a correction has to be made: ^{*1}: 92–190 and ^{*2}: 209–431.

fatigue resistance (larger stress range) under specific minimal load levels than corroded rails on the basis of this assumption. Moreover, the new results of corroded rails also show a better rail fatigue resistance than the historical results. The historical assumption of the permissible strength of $\sigma_{\text{limit}} = 470 \text{ MPa}$ [38,30] is also too conservative. The maximum stress reaches 540 MPa under the minimum stress level of 210 MPa with the same assumption of equivalent residual stress scattering, this indicates a much greater potential of steel fatigue strength and also better stress resistance due to track/bridge interaction. Further investigations concerning the ultimate limit strength of rail track under cyclic loads and up-to-date residual stress are needed to be carried out to determine the permissible stress due to track/bridge interaction.

5. Discussion

This project involved carrying out experimental studies including four-point bending fatigue tests under different minimum stress levels and corrosion tests of typical rail profile 60E2 (former UIC60) with R260 steel grade material in accordance with the relevant sections of EN and German standards. The tests were monitored and carefully controlled. Most of the broken rails developed cracks from the bottom surface of the rail foot. After a certain propagation of the crack area, the crack immediately expanded to the whole rail cross-section and led to brittle failures. Only a few rails developed cracks from the edge of the rail foot and rail web. The staircase method was applied to carry out the test program and calculation. Reliability analyses with 5% quantile of the rail fatigue strength were additionally evaluated to provide scientific design values. Surface roughness influenced the strength of rail. The results confirmed that current 60E2 R260 rails have an equivalent or even better fatigue strength for five million load cycles than the rails (UIC 60) from the historical experiments for two million load cycles.

The results of these experiments provide important reference values for railway design, bending fatigue research and especially for track/bridge interaction design. Further investigations are necessary to determine the residual stress in the rail, so that the absolute rail strength can be determined. Furthermore, investigations to quantify the influence of surface roughness, which could be valuable for the whole life evaluation of rail, also need to be carried out. Moreover, research on the fatigue resistance of rail under higher minimum stress levels, greater loads and variable load sequences is also required.

CRedit authorship contribution statement

Chongjie Kang: Investigation, Writing - original draft, Visualization, Data curation. **Sebastian Schneider:** Investigation, Validation, Writing - review & editing. **Marc Wenner:** Project administration, Writing - review & editing, Resources. **Steffen Marx:** Supervision, Funding acquisition, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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2.4 Experimental investigation on the fatigue behaviour of rails in the transverse direction

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Chongjie Kang was responsible for the investigation, data curation, visualization and original draft writing. Sebastian Schneider was responsible for the realization of the test setup as a part of the investigation. Marc Wenner was responsible for the management of the research project and the initial test program. Steffen Marx was responsible for the acquisition of the funding for this investigation. The successful completion of this investigation was achieved due to the cooperation and exchange of all co-authors. All co-authors reviewed this paper and provided meaningful comments during the process.

Experimental investigation on the fatigue behaviour of rails in the transverse direction

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HIGHLIGHTS

- Bending fatigue tests on UIC60 rails in the transverse direction have been carried out.
- 24 not-corroded and 12 corroded rails were tested up to five million load cycles.
- Fracture characteristics of rails are summarized and analyzed.
- A comprehensive Smith-diagram based on results in both transverse and vertical directions is proposed.

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ABSTRACT

This paper presents an experimental investigation on 60E2 (UIC 60) rails loaded in the transverse direction up to five million load cycles. A resonance testing machine was applied to achieve a test program with different load levels and a loading frequency of 15 Hz. First, the experimental approach, which includes the specimens, the test setup, the test program, the corrosion treatment as well as the additional slow bending tests, is introduced in detail. Subsequently, the rail steel material behaviour, the fracture characteristics of the rails, and the corrosion effects are analysed and summarized; the fatigue strength of the rails is evaluated with the staircase method and compared with the fatigue strength of rails loaded in the vertical direction. In general, the rails show good fatigue resistance in the transverse direction, which is larger than that in the vertical direction and close to the static yield strength. As a result, a new comprehensive Smith-diagram of the UIC 60 rail for five million load cycles is presented. In the end, necessary further investigations are discussed.

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1. Introduction

As vital parts of a railway system, steel rails transmit wheel forces to the track-bed and, together with the tread and flange of the wheel, they guide vehicles [1,2]. Although substantial improvements in design, material, and non-invasive inspection have been achieved in the last years, fatigue propagation and failure of railway components remain an important issue for rail transportation safety engineering. According to statistical data from the Deutsche Bahn (German Railway), 1,083 occurrences of rail failure in the German railway network were recorded in the two years 2012

and 2013 for a track length of about 63,000 km [3]. The fracture and failure of rail tracks are considered as serious incidents in railway transportation, as they are likely to induce vehicle derailment with a probability of loss of life [4].

In recent years, thanks to the rapid development of the high-speed railway industry [5], the rail material, profile and manufacturing process have been largely improved. Therefore, new knowledge about these rails is necessary. As a result, investigations have been carried out on flexural bending fatigue strength, which is an essential property of rail tracks especially regarding the additional rail stresses in the area of bridges. In the 1980 s, Eisenmann et al. [6,7] carried out four-point bending fatigue tests (up to two million load cycles) on S 54 and UIC 60 rail profiles of 900A steel (same grade with the new classification R260). The rails were loaded under a minimum stress level of 50 MPa in the vertical direction. They took the lowest test results (lower boundary), which were

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Fig. 1. Illustration of wheelset and track dimensions for tracks on curvature.

$\Delta\sigma = 205$ MPa and $\Delta\sigma = 225$ MPa, as the limit fatigue strength of the S 54 and UIC 60 rails, respectively. Subsequently, the directive ‘Oberbauberechnung’ [8], which aimed to analyse different effects of the rail on bridges, limited the permissible additional normal tensile stress to 92 MPa for rails due to bridge/track interaction [9,10] on ballast track systems. To determine this value, the main stresses including the residual stress due manufacturing process, the thermal stress due to temperature variation and the stresses due to train traffic should be previously decided, detailed calculation assumptions and method can be found in [3]. This limit value was then extended to rails on ballast-less track systems in the 1990 s and has been written in German guidelines [11], the Eurocode [12] as well as the UIC standard [13]. This limitation has provided essential reference value for the design and construction of railway bridges, especially for bridges with long expansion lengths. It remained unchanged, although the production technology of rails has been changed significantly. Since 2010, Freystein et al. [14], Wenner et al. [15] and Kang et al. [3] have suggested that this value should be increased based on simulations and multi-load level experimental investigations. However, these investigations have only focused on the flexural bending fatigue strength of the rails loaded in the vertical direction. There has been no experimental evidence to ensure the fatigue safety of rails loaded in the transverse direction. In reality, rail tracks are under both vertical and transverse forces (see Fig. 1). Besides, the transverse load becomes larger when the rail track is on curves or in the area of bridges, where additional stresses are introduced and superimposed in the rail foot [16].

Experimentally, in the 1970 s, Birmann et al. [17] and Eisenmann et al. [18] carried out investigations in the laboratory and the field on S 49 rails on curved tracks ($R = 190$ m). The ratio (Y/Q) of the vertical wheel load (Q) to the transverse load (Y) was $Y/Q = 5/10.65 = 0.47$. They concluded that the ratios (σ_F/σ_M) of the measured stress in the edge of rail foot (σ_F) to the stress in the centre of rail foot σ_M were 1.80 and 1.62 in the laboratory and the field, respectively.

As for theoretic analysis, Fig. 2 shows a simplified method for the superposition of stresses due to wheel loads according to Franz

[19]. The wheel load is divided into three parts, which include a vertical load through the shear centre, a torsion moment, and a transverse load through the shear centre. The vertical load introduces uniformly distributed stresses on the surface of the rail foot. In contrast, the torsion moment and the transverse load introduce linearly distributed stresses in the rail foot. The superimposed stress σ_F at the edge can be larger than σ_M . Franz took the UIC 60 rail as an example and calculated a ratio of $\sigma_F/\sigma_M = 2.36$ when $Y/Q = 60$ kN/100 kN = 0.6.

Moreover, bridges are critical areas of rail failures. Bridge/track interaction [12] introduces significant additional stresses in the rail. When a rail lays on a bridge joint, the translation of the bridge superstructure in the transverse direction results in large horizontal forces in the rail [20], therefore, causes additional stresses in the edge of the rail foot. This phenomenon is more obvious for rails on ballast-less track systems than for rails on ballast track systems due to the greater structural stiffness of ballast-less track systems [21–23]. Wenner et al. [24] carried out experimental investigations in the laboratory on 60E2 rails, which were installed on the ÖBB-Porr track system (ballast-less) [20,25]. Under the condition of ± 1 mm translation in the transverse direction, which meets the limitation of Richtlinie 804 [11], they measured 30 MPa more stress in the rail foot edge than in the rail foot centre.

In general, the rails have larger stresses at the edge of rail foot than in the centre of rail foot under cyclic wheel loads. Even though this phenomenon has been addressed by the aforementioned authors, there exist almost no investigations in dealing with this issue. With the help of a resonance test machine, the authors carried out comprehensive experimental investigations concerning this issue. The experiments and results are presented in the following sections.

2. Experimental investigation

2.1. Specimen description

The rail specimens were obtained from the new high-speed railway (HSR) line Erfurt – Leipzig/Halle project in Germany. The rails have an E602 profile (former UIC 60) with a steel grade of R260. This is the most commonly used rail type on HSR lines. Detailed information about this profile can be found in EN 13674-1 [26]. The related basic properties of this rail profile are: Young’s modulus $E = 210$ GPa, cross-sectional area of an entire rail $A = 76.48$ cm², line weight $G = 60.03$ kg/m, and the bending inertia of the rail in the transverse direction $I_y = 510.5$ cm⁴. The rail specimens for the tests were produced in 2010. The chemical composition of R260 steel is listed in Table 1. According to the norm EN 13674, “the microstructure shall be pearlite but grain boundary ferrite may occur in these grades; there shall be no martensite, bainite or grain boundary cementite”.

The tested specimens had not been put into operation after manufacture and are considered as “new” concerning the loading history. For the bending fatigue tests, the original 60 m long

Fig. 2. Superposition of stresses in a rail due to wheel loads [19]

Table 1

The chemical composition of the R260 rail specimens [26]

Element	C [%]	Si [%]	Mn [%]	P max. [%]	S max. [%]	Cr [%]	Al max. [%]	V max. [%]	N max. [%]	O [ppm]	H [ppm]
Value	0.60-0.82	0.13-0.60	0.65-1.25	0.030	0.030	≤0.15	0.004	0.030	0.010	0.002	0.00025

manufactured rails were cut into 2.2 m long specimens. Except for the time during the performed corrosion treatments (see section 2.4), the rails were in direct contact with the natural air environment. Hence, the surface roughness developed slowly between manufacture and the tests.

Additionally, standard tension tests according to EN ISO 6892-1 [27] and DIN 50125 [28] were carried out on steel specimens with dimensions of $5 \times 16 \times 50$ mm. These three specimens were cut out from the rail foot centre. The stress-strain curves of the specimens are shown in Fig. 3 and indicate that the steel material has an almost linear stress-strain behaviour until 550 MPa. The results are listed in Table 2.

2.2. Test setup

The objective of the tests was to investigate the fatigue strength of the rails along the y-axis (Fig. 4) for a maximum of $N = 5 \cdot 10^6$ load cycles. The configuration of the test setup was identical with the tests, which had been carried out in the vertical direction according to Kang et al. [3]. The supporting span and loading point distance were set to 1.9 m and 150 mm, respectively, to carry out the four-point bending tests; see Fig. 4. Each end of the rail was laid upon an individual specially designed 'rail shoe', which fit the rail profile and allowed longitudinal movement along the x-axis and rotation of the y-axis.

A newly developed fatigue testing facility based on a resonant concept by the Institute of Concrete Construction of Leibniz University Hannover was used to carry out the fatigue tests. Detailed information about the controlling program of this test facility can be found in the work of Schneider et al. [29]. Fig. 5 and Fig. 6 display the associated equivalent mechanical model and major components of the test facility, separately.

Fig. 3. Stress-strain behaviour of tested standard specimens.**Table 2**

Results of the tension tests.

Specimen	0.2% yield strength [MPa]	Tensile strength [MPa]	Young's modulus [GPa]
No.1	614	999	208.5
No.2	589	997	199.6
No.3	594	1,002	214.9
Mean Value	599	999.3	207.7

Fig. 4. Principle design for the vertical tests. **Fig. 5.** Associated equivalent mechanical model of the resonant test stand.

The resonance machine is able to generate excitation frequencies up to 25 Hz. Based on the theory of Rayleigh method, the fundamental natural frequency of the mechanical model (see Fig. 5) can be calculated with the following equation:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{E \cdot I_y \cdot \pi^4 + 2 \cdot L^3 \cdot \sum k_{II}}{L^3 (\mu \cdot L + 2 \cdot m_{II})}} \quad (1)$$

Where

$\sum k_{II} = 133.5$ kN/m, $\mu = 60.03$ kg/m, $m_{II} = 740$ kg, $L = 1.90$ m. The first bending eigenfrequency of the designed test set-up was calculated to be 15.57 Hz.

2.3. Test program

The rail fatigue tests were planned to be carried out under two different minimum stress (σ_{min}) levels, which were 150 MPa and 250 MPa based on the assumption of $E = 210$ GPa. To evaluate the rail fatigue resistance with the staircase method, at least 10 specimens were required for each level [30]. The influence of surface roughness was also intended to be concluded in the experiments (see section 2.4). Table 3 gives a full overview of the test program. For each load level, the minimum stress remained always constant. When performing one test series by applying the staircase method, the first specimen can be tested at a certain stress range. If the test was completed with a failure within $5 \cdot 10^6$ load cycles, then the stress range for the next specimen was decreased by 10 MPa. If the test was completed without failure up to $5 \cdot 10^6$ load cycles, then the stress range for the next test was increased by 10 MPa. The tests would proceed until all 12 specimens were completed.

Before each test, the rail was mounted on the test facility. Subsequently, the springs were pre-stressed and therefore the vertical load could be transferred to the rail by the transverse beam. By controlling the magnitude of the vertical pre-stressing, the required mean stress σ_m was generated at the lower edge of the

Fig. 6. Resonant test set-up.

Table 3
Test program.

Minimum stress $\sigma_{\min}$ [MPa]	Corrosion condition	Specimens	Starting stress range $\Delta\sigma$ [MPa]
150	not-corroded	12	410
150	corroded	12	330
250	not-corroded	12	330

rail foot. Then, the electric motors generated a cyclic load; the rotation frequency of the motors increased at a constant rate until the planned stress range $\Delta\sigma$ was reached, which occurred after 2000 to 4000 load cycles. Finally, the stress range was kept constant until the end of the test; see Fig. 7. The actual rotation frequency of the electric vibrators was about $f \approx 15$ Hz. One run-out test for $N = 5 \cdot 10^6$ load cycles lasted about four days, which was three times faster than traditional hydraulic test facilities with a maximum loading frequency of 5 Hz [29].

Fig. 7. Increasing rotation frequency and stress range at the beginning of a fatigue test.

The tests were carried out with a strain-controlled procedure. Two strain gauges were installed directly on the edges of the rail foot at the mid-span right under the applied loads (see Fig. 8). The strain gauges were aligned along the longitudinal direction of the rail. The corroded surface was carefully removed with the help of a small belt sander before the strain gauges were installed. A template was used to limit the sanding area to adhere the strain gauges. Hence, the influence of sanding can be kept to a minimum. Temperature sensors PT 100 were installed to monitor the rail and air temperature during the fatigue tests. Fig. 9.

The tests were controlled by the measurement values of strain gauge 1. Four load cells were installed under the bearings to record the supporting forces of rails during testing. Two laser sensors were applied to monitor the displacement of the rail close to the mid-span. Two strain gauges were set on the vertical steel bars to measure the forces introduced by the springs. The rotational frequencies of the vibrators were also recorded with the help of Hall effect sensors [31]. Besides, four limit switches [32] were set as to be in contact with the rail specimens, and one photoelectric sensor was installed, so that the test could be automatically terminated in case the specimen broke or behaved abnormally.

2.4. Specimen corrosion

To determine the fatigue life of rails under different surface conditions, comparable tests with corrosion treatment have been carried out. The surface roughness parameters, especially the arithmetic mean roughness R_a , the maximum peak-to-valley roughness R_t and the average of maximum roughness height per sampling length R_z , were taken as the control values of different surface conditions [33,34]. The target values were determined on the basis of an investigation 40 years ago on several rails, which had been operated for more than 10 years [7]. To achieve the target values, the rails were placed in a spray chamber for at least 10 h in the environment of 5% sodium chloride solution at 35 °C according to the NSS test from ISO 9227:2017 [35] (see Fig. 10 and Fig. 11).

Fig. 8. Arrangement of the strain gauges and temperature sensors.

Fig. 9. Additional sensors to monitor the test stand: (a) Lasers, (b) load cell, (c) limit switch, (d) photoelectric sensor.

Fig. 10. Rails before corrosion.

Fig. 11. Rails after corrosion.

The chief way of measuring this surface roughness was stylus-based equipment [36]. A contact roughness measuring device (Fig. 12) was used for the rail surface. The measurement paths are shown in Fig. 13. The test needle (sensor) had a radius of 2 μm , the measuring path was 40 mm according to [37,38], and the measuring speed was 0.3 m/s. Before measuring, the rust and other impurities on the surface were carefully removed with an air pressure gun without damaging the steel surface.

To control the development of rail surface roughness, the corrosion treatments were carried out in several partial phases. After

Fig. 13. Roughness measuring paths.

Fig. 14. Measured surface roughness values.

each partial test, two rails were measured. Fig. 14 demonstrates the roughness of the rails before and after the corrosion tests. The roughness values of the rail specimens after corrosion were quite close to each other and much greater than the target values. Under these circumstances, the fatigue test results are supposed to be on the safe side in comparison with historical test results.

2.5. Slow bending test

After the fatigue tests, several slow four-point bending tests with the same test geometry configuration were carried out. The aim of these experiments was to investigate rail behaviour after five million load cycles. An additional new rail (8-8), which had not been loaded before, was also tested for comparison. The test set-up and measurement plan are shown in Fig. 15 and Fig. 16. Fourteen lasers were arranged to measure the deflection and rotation of the rail. Strain gauges S1 and S2 at the rail's feet measured the strain.

The loading histories of the specimens are listed in Table 4. The maximum loaded stress was 560 MPa for the rail specimens 3-12 and 4-2.

To obtain stable and reliable results, the loading procedure (Fig. 17) was carefully controlled. At first, the longitudinal force was increased up to the first load level and then kept constant for two minutes. Subsequently, the load was reduced to a basic load level (5 kN) and also kept constant for two minutes. This was repeated three times until the next load level was implemented. The target result for analysing (section 3.5) each level was the maximum value during the third loading period. The maximum loaded force was 150 kN, which corresponds to a stress level of 964 MPa. This was calculated with equation (2) (see section 3.2). However, if abrupt deformation changes occurred before this maximum force was reached, the test was stopped for operational safety.

Fig. 12. Roughness measuring device.

Fig. 15. Static test stand.

Fig. 16. Sensor arrangement of the static experiment (Unit: mm).

Table 4

Loading history of the specimens for slow bending tests.

Rail specimen	Load cycles [million]	Minimum stress	Maximum stress
6–15	5.01	130	455
6–23	5.01	130	485
3–12	5.01	220	560
4–2	5.01	220	560
8–8	0	0	0

3. Test results

3.1. Rail fracture

A test specimen had to complete a total of five million cycles without any fracture or failure to pass the test according to [30]. If no sign of failure occurred up to five million load cycles, the test was stopped and the specimen was considered 'run-out'. At the end of each test completed without failure, the rail was visually checked for the incidence of any fatigue cracks. If the rail failed, then the test was stopped and the fractured surfaces were inspected to identify the cause of failure.

Of the 34 rails tested, 19 (55.9%) failed before the target number of load cycles. An oxidized surface associated with a pre-existing crack was observed on the fractured surface of each broken rail specimen. Four typical stages comprising crack initialization, crack growth, crack propagation and rupture [39], were observed on the fracture surfaces. The crack initiated at the edge of the rail foot. Subsequently, the crack propagated along the boundary of the rail foot profile when a certain crack surface was reached, the rail failed instantly. The glossy crack surface profiles revealed long-term wear caused by friction during the test, while the chevron patterns and the radial marks indicated not only a brittle fracture of the rail but also that the brittle fracture radiated from the crack profile through the whole rail section to the other edges. The front of the crack forms an arc curve, see Fig. 18.

Fig. 17. Loading procedure of the slow bending tests.

Fig. 18. Fatigue crack profile of rail 4-1 after failure.

Fig. 19. Definition of measured crack dimensions.

Aiming to capture the cross-sectional characteristics of all broken rails, the dimensions of all rail cracks were measured, as depicted in Fig. 19 and listed in Table 5. As for the failed rails, 17 out of 19 were fractured within one million load cycles. Only two of the rails endured more than one million load cycles. The area of crack profiles showed a tendency to decrease when the load level increased. This indicates that the crack profile area is related to the mean stress level of the rail. The greater the mean stress is, the smaller the area will be.

Table 5
Crack dimensions.

Stress level	Rail specimen No.	Load cycles[million]	D1[mm]	D2[mm]	H[mm]	Crack profile area S [mm ²]	Mean values S _m [mm ²]
140 MPa(not-corroded)	4-15	0.49	5.0	12.5	12.0	105.0	125.6
	4-16	0.52	10.0	13.7	12.8	151.7	
	4-17	0.50	12.5	10.0	12.0	135.0	
	4-18	0.41	5.0	11.5	12.0	99.0	
	4-21	0.31	13.1	11.2	12.2	148.2	
	4-25	0.64	10.0	11.5	13.0	139.8	
	4-26	0.53	13.5	8.5	12.0	132.0	
	3-11	0.65	9.0	10.6	11.8	115.6	
	3-13	0.66	8.0	11.0	11.0	104.5	
	4-1	0.66	10.0	3.0	11.5	74.8	
220 MPa(not-corroded)	4-3	0.79	9.5	10.0	12.0	117.0	111.5
	4-6	0.56	11.0	6.5	12.0	105.0	
	4-9	0.58	11.5	5.0	12.0	99.0	
	4-11	1.76	11.0	14.0	13.0	162.5	
	4-13	1.21	13.0	4.0	13.0	110.5	
130 MPa(corroded)	6-19	0.82	14.0	14.1	12.0	168.6	145.7
	6-22	0.47	12.0	11.0	12.0	138.0	
	6-25	0.55	10.0	14.0	11.0	132.0	
	6-26	0.37	10.0	14.0	12.0	144.0	

Test results showed the fracture sections of rails that were tested in the transverse direction were normally relatively upright. In contrast, the vertically tested rails showed a very different cracking path according to different literature [3,4,40,41] along with the height of the rail cross-section. Both of these different crack paths are schematically shown in Fig. 20.

3.2. Material behaviour

With about 15 Hz loading frequency, the maximum temperature increase of the rails during fatigue tests ranged from 4.7 K to 12.8 K. This meets the limitation ($\Delta T = 30$ K) for ferritic steel according to the standard DIN 50100 [42].

As aforementioned, all the tests have been carried out under a strain-controlled procedure. The stresses calculated from the strain gauges are based on the Hooke's law ($\sigma_{\text{strain gauge}} = \varepsilon_{\text{strain gauge}} \cdot E$) with Young's modulus of $E = 210$ GPa. This assumption is based on a linear material behaviour of rail steel. However, the rail material behaves in a slightly non-linear way before yielding. Fig. 3 shows an obvious non-linear behaviour of rail steel before it reaches the 0.2% yield strength of 600 MPa. In order to confirm this

assumption, the stress–strain behaviour of rail steel is analysed in this section.

The test results of the rail 4–24 are taken as examples and shown in Fig. 21. The rotation frequency (see Fig. 21a) of the motor remained very stable at 15 Hz during the whole test. This indicates that the geometry configuration, weight, damping and stiffness of the test system remained unchanged during the five million load cycles. Also, the deviation of the planned strain range remains smaller than 2.5%, which is within the limits of 3% stipulated in DIN 50100 [18] and acceptable for this application. However,

Fig. 20. Schematic illustration of crack paths of rails in longitudinal direction.

Fig. 21. Monitoring results of rail 4–24 during the test.

Fig. 21b shows that both the minimum strain and the maximum strain increase slightly due to the plastification of the material, even though the strain range remains constant during the whole test.

To validate the results by considering the aforementioned plastification, another alternative to calculate stresses was applied. The supporting forces of the rails during the fatigue tests were measured (Fig. 4) as well; based on the Bernoulli beam theory, the stress on the foot of a rail in the mid-span can also be evaluated as follows:

$$\sigma_{\text{Load cell}} = \frac{M}{W_y} = \frac{F_{\text{Total}} \cdot L}{2 \cdot W} = \frac{(F_{\text{Left}} + F_{\text{Right}}) \cdot L}{2 \cdot W_y} \quad (2)$$

Where F_{Left} and F_{Right} are measured forces at both sides of the rail; $L = 875 \text{ mm}$; $W_y = 68.1 \cdot 10^3 \text{ mm}^3$. The calculated results were compared with the previously planned stress levels and listed in Table 6.

According to Table 6, the minimum stress levels were reduced by 10 MPa, 20 MPa and 30 MPa, respectively, in comparison with the corresponding planned stress levels. Therefore, the stress range was changed by 10 MPa, -5 MPa and 10 MPa, accordingly. These differences showed that the assumption of a perfect linear elastic material behaviour was not fully accurate for these specimens. The behaviour of the calculated stresses and directly measured strains of rail 4–24 and rail 4–10, which were tested under different minimal stress levels, are taken as examples and shown in Fig. 22. It can be observed that the rail material is not perfectly elastic under these relatively high-stress levels. Some permanent strains occur and stay in the rails for the first load cycles. The plastic strains develop as the number of load cycles increases. However, after a certain number of load applications, the permanent strains settle into a closed cycle, which is known as a plastic shakedown behaviour [43,44]. As the maximum stress level of rail 4–24 is larger than that of rail 4–10, the remaining plastic strain of rail 4–24 is also larger.

Because of the non-elastic rail material and plastic shakedown behaviour, the assumption of $E = 210 \text{ GPa}$ is questioned, and therefore, needs to be validated. By applying the $\sigma_{\text{Load cell}}$ and the measured strain values, the secant modulus before and during the fatigue experiments is evaluated. Two different secants E -moduli are evaluated in this work. E_s is evaluated when the mean stress σ_{mean} is reached (before the starting of the cyclic loading). It can be calculated with $E_s = \sigma_{\text{mean}} / \varepsilon_{\text{mean}}$. E_d is the secant modulus evaluated during the fatigue tests, which is calculated with $E_d = (\sigma_{\text{max}} - \sigma_{\text{min}}) / (\varepsilon_{\text{max}} - \varepsilon_{\text{min}})$. The E_d is evaluated for different load cycles. Therefore, the development of E_d can be obtained.

It can be observed from Fig. 23 that all the evaluated results of E_s are smaller than the assumption of $E = 210 \text{ GPa}$. The E_s decreases as the mean stress level increases. The maximum difference is as large as 23 GPa under the minimum stress level of 220 MPa. This verifies the necessity of the correction of the stress levels. The developments of E_d of run-out rails and fractured rails are illustrated in Fig. 24, and Fig. 25, separately. It can be observed that the E_d results of both run-out and fractured rails remain relatively stable during the tests up to five million load cycles or before failure. However, the E_d of most fractured rails experiences a small

Table 6
Comparison of the planned test program and the modified test program.

Minimum stress σ_{min} [MPa]		Corrosion condition	Specimens	Starting stress range $\Delta\sigma$ [MPa]	
Planned	Calculated			Planned	Calculated
150	140	not-corroded	12	410	420
150	130	corroded	11	330	325
250	220	not-corroded	12	340	350

(a) Rail 4-24 with an actual minimal stress level of $\sigma_{\min} = 140$ MPa (b) Rail 3-12 with an actual minimal stress level of $\sigma_{\min} = 220$ MPa^{*1}**Fig. 22.** Stress-strain behaviour of rails.

Note *1: It was found out that Fig.22 (b) was not the correct diagram. A revision has to be made. The correct one can be found on the page after this publication.

Fig. 23. Evaluated E-Modulus E_s of the rail samples after the mean stress is reached. **Fig. 24.** Evaluated E-modulus E_d during fatigue tests of the run-out rails. **Fig. 25.** Evaluated E-modulus E_d during fatigue tests of the failed rails.

tence of a crack. On the other side, the absolute values of strain gauge 2 increased due to the decrease of the intact rail cross-section. As a result, a small decrease (the maximum decrease was less than 10 GPa) of E_d is observed for most fractured rails. But the influence is very limited. The decrease is more significant (such as 6-22 and 6-26) when the strain gauges are closer to the final fractured cross-section. When a strain gauge is further than 100 mm from the fracture cross-section, almost no influence (such as 4-11 and 4-13) could be identified. In the authors' opinion, this phenomenon was more of a measurement error instead of real material behaviour; the rail steel material experienced no damages or a decrease of the Young's Modulus shortly before fracture. This should be noted when analysing the results of such tests. As the maximum influenced value is 10 GPa of rail 6-22, which is only 4.7% of its initially calculated result, it can be concluded that the influence is very slight and acceptable for this situation. Additionally, most of the dynamic E-moduli of the rails are within the range of 200 GPa to 220 GPa, which are very close to the assumption to $E = 210$ GPa, regardless of different minimum stress levels.

3.3. Staircase method analysis

The staircase method (or up-and-down method) is primarily used for the determination of fatigue resistance [45–47]. In staircase tests, a broken rail is recorded as a 'failure'. If it did not break, it is recorded as a 'run-out'. Both failures and run-outs were obtained to carry out the staircase method analysis. The test results are summarized in sequence and shown in Fig. 26.

decline shortly before failure (within the last 5% of the loading period). The strain gauges were installed in the middle of the rail span, where the fracture occurred. The fatigue crack propagated faster and larger when the rail was close to the final total brittle fracture. During this final stage, the control value $\Delta\epsilon$ of train gauge 1 remained unchanged. On one side, the absolute strain values of strain gauge 1 decreased because of strain release due to the exis-

Fig. 26. Staircase method testing results for 5·10⁶ cycles.

The mean fatigue strength was calculated as (see EN 14730–1 [30]):

$$\Delta\sigma_{D,m} = \sigma_0 + d \cdot \left(\frac{A}{N} \pm 0.5 \right) \quad (3)$$

Where σ_0 is the lowest stress range at which tests with the less frequent result were conducted, $d = 10$ MPa, $A = \sum i \cdot n_i$ (i ranges from 0 to z), $N = \sum n_i$ = total number of less frequent events, n_i = number of less frequent events at the i -th stress level above σ_0 , i = coded stress level ($i = 0$ for σ_0), z = number of stress levels above σ_0 at which testing has been undertaken.

The standard deviation was calculated with:

$$s = 1.62 \cdot d \cdot \left(\frac{B \cdot N - A^2}{N^2} + 0.029 \right) \quad (4)$$

where $B = \sum i^2 \cdot n_i$.

Fig. 27. Graphical demonstration of rail fatigue strength.

To apply the fatigue strength from the test results to the design values, the quantile value of fatigue strength must be determined for a given survival probability. According to EN 1990 [48], the lower characteristic value of building material strength should correspond to 5% quantile for limit state verification. For constructions, the confidence level of 75% has prevailed. Based on the aforementioned information, the factors of 2.048 and 2.073 can be thus determined for 12 and 11 specimens, respectively, according to [49]. As a result, the 5% quantile fatigue strength is calculated as:

$$\Delta\sigma_{D,0.05} = \Delta\sigma_{D,m} - 2.073 \cdot s \cdot (n = 11) \quad (5)$$

$$\Delta\sigma_{D,0.05} = \Delta\sigma_{D,m} - 2.048 \cdot s \cdot (n = 12) \quad (6)$$

The results are summarized displayed in Fig. 27, which shows that the fatigue strength of rails increases when the loaded stress level increases under the same corrosion state. Besides, the not-corroded rails have higher fatigue strength than the corroded rails.

3.4. Corrosion effects

The surface conditions of the tested rails have an impact on fatigue resistance [50]. For the planned minimum stress level $\sigma_{\min} = 150$ MPa experimental series, both not-corroded rails and corroded rails were tested. The fatigue test results of these two specimen types (with and without corrosion) are presented in Fig. 27 and also in the S-N diagram Fig. 28. The fatigue strengths of the corroded specimens were obviously lower than those of the not-corroded rails.

3.5. Rail behaviour after fatigue test

The maximum strains were measured at the position of strain gauge S1 (see Fig. 16). The measured strains and the calculated

Fig. 28. S-N diagram of tested rails.

Fig. 29. Stress-strain behaviour at the position of strain gauge S1.

Fig. 30. Deflection development in the middle of the rails.

stresses (with equation (2)) are presented in Fig. 29. It can be seen the rails that had been tested for five million load cycles show an almost linear stress-strain behaviour within the previous loaded stress range. When the loaded maximum stress level is exceeded, the rails soon show a non-linear stress-strain behaviour. In comparison, the new rail 8-8 shows a non-linear stress-strain behaviour from about a stress level of 300 MPa.

During the slow bending test, the deformations of the rails were also carefully measured. Laser 1 and Laser 2 measured the maximum deflections of the rail foot and head (see Fig. 30). The rails show no big deformation deviations; even the applied load introduced much larger stress than 600 MPa (corresponds to a load of 93.4 kN for this case), which must not be exceeded during operation. The largest deformation deviation of all tested rails is less than 1 mm within the 93.4 kN level. However, the load-deformation behaviour is much more non-linear when the load is larger. The deformation deviation slowly gets larger when the load increases as well. When the ultimate designed strength of the rail material 900 MPa is reached, the maximum deformation difference between the new rail and the previously loaded rails is within 2 mm, which is smaller than 8% of the absolute deformation. This proves that the stiffness and behaviour of the rails are only slightly influenced by the loading history of five million load cycles when no fatigue failure occurs.

3.6. Comparison of the results

Twenty-three not-corroded rail specimens covering two different minimum stress levels of 140 MPa, 220 MPa, and eleven corroded rail specimens were successfully tested in the transverse direction of the rail cross-section under a minimum stress level of 130 MPa. The comparison of the fatigue resistance with test results in the vertical direction is listed in Table 7. The tests were carried out with a strain-controlled procedure; the surface roughness of the tested rails are of a similar corrosion state.

Table 7
Comparison of stress results [3]

Planned $\sigma_{\min}$ [MPa]	Test direction [-]	Corrosion [-]	R_t [μm]	Validated $\sigma_{\min}$ [MPa]	Validated σ_{mean} [MPa]	$\Delta\sigma_D; 0.05$ [MPa]
150	Vertical	Not-corroded	60–100 ^{*2}	130	268	271
		Corroded	170–250	130	258	199
	Transverse	Not-corroded	35–176	140	333	362
		Corroded	131–300	130	310	342
250	Vertical	Not-corroded	60–100 ^{*2}	210	343	250
	Transverse	Not-corroded	46–167	220	393	331

Note: In Table 7, the mean values of R_t are listed. However, it was later found that here the maximum values of R_t should be written. Therefore, a correction has to be made: ^{*2}: 92–190.

Fig. 31. Smith-diagram of the tests results for $N = 5 \cdot 10^6$ load cycles in vertical and transverse direction [3,8]

However, the whole rail fatigue strength also depends on the residual stresses in the rail. A residual stress magnitude of 80 MPa was determined according to historic research [8,14,15,51]. With this assumption, Fig. 31 provides a comparison of Smith-diagrams based on both historic results in the literature [8] and new test results [3]. The stress range under different loads can be directly observed. Additionally, the number of load cycles to failure is compared in Table 8.

The rail profile UIC 60 (60E2) shows a better fatigue resistance in the transverse direction than in the vertical direction. It shows very good fatigue behaviour beyond the defined yielding point of rail steel. According to [8,51], the main factors that have a large influence on the fatigue resistance of rails are residual stress, temperature, corrosion state, load conditions (such as frequency, direction) and welding condition. Table 8 shows, however, the rails loaded in the transverse direction endure much fewer load cycles than the rails loaded in the vertical direction. For the rails under planned $\sigma_{\min} = 150$ MPa, the mean number of load cycles of the rails loaded in the transverse direction was only 22.8% and 57.2% of the rails loaded in the vertical direction for not-corroded and corroded rail, respectively. For the not-corroded rails under the planned $\sigma_{\min} = 250$ MPa, this value was 65.0%.

The two major factors account for the different fatigue resistances of rails within the tested stress range in vertical and transverse directions are the residual stress distribution in the rail cross-section and the load direction (or the form of the cross-

section perpendicular to the load direction), as the temperature variations are small and all within the limits of DIN 50100, and the tested rails were not welded.

First, the aforementioned 80 MPa tensile residual stress was determined for the centre of the rail foot in historic research [8]. However, due to the uneven cooling and rolling process during the manufacture of rails, compressive stresses instead of tensile stresses occur at the rail foot edge [16]. This leads to better fatigue resistance of the rails loaded in the transverse direction. Besides, with the rapid development of rail manufacturing technology, the residual stress statues in rails have changed. New investigations to determine the residual stresses in modern rails and their influences on rail fatigue behaviour need to be carried out.

Second, the rails were loaded in different directions. In the vertical direction, the most fatigue cracks initiate from the central part of the rail foot surface; the cracks then propagate with a half ellipse profile until total failure [3]. In contrast, in the transverse direction, the fatigue cracks develop along the rail foot edge. Under this situation, there is less material restricting the propagation of the cracks. This leads to a worse fatigue behaviour of the rails loaded in the transverse direction. However, due to the restrictions of the experimental conditions, this effect cannot be quantified in this paper.

4. Summary

The presented research involves experimental investigations including 36 four-point bending fatigue tests of the typical rail profile 60E2 (former UIC 60) with R260 steel in the transverse direction. The tests were carried out strain-controlled and evaluated via the staircase method. The test program has taken different minimum stress levels and corrosion tests into account. For the tests, a resonant test facility was used. The loading frequency was about 15 Hz. The temperature of the rails increased between 4.7 K and 12.8 K, which are the maximum increases for individual rails, in the first 10 h. Under transverse load, the fatigue crack initiated at the rail foot edge on the opposite side of the load application. Subsequently, the cracks propagated along the edge of the rail foot. After a certain propagation area was reached, the crack immediately expanded to the whole rail cross-section and resulted in brittle failure. The crack area is smaller when the load level is larger. The rail material shows plastic shakedown behaviour under cyclic loading. The secant modulus of the rails remains very constant during fatigue tests for a maximum of five million load cycles. Reliability analyses with a 5% quantile of the rail fatigue strength were additionally evaluated to provide scientific design values. The test results show that the rails loaded in the transverse direction have a much higher fatigue strength but endure fewer load cycles than those loaded in the vertical direction. Moreover, the rail foot edge is under higher stresses than in the centre of rail foot under wheel loads. The magnitude of this difference should be investigated in future work.

Table 8
Comparison of load cycles to failure [3]

Planned $\sigma_{\min}$ [MPa]	Test direction [-]	Corrosion [-]	Number of load cycles till failure [million]			
			Mean	Min.	Max.	Standard deviation
150	Vertical	Not-corroded	2.28	1.25	3.64	1.06
	Transverse	Not-corroded	0.52	0.31	0.66	0.12
250	Vertical	Not-corroded	1.43	0.98	2.35	0.57
	Transverse	Not-corroded	0.93	0.56	1.76	0.47
150	Vertical	Corroded	0.96	0.62	1.38	0.30
	Transverse	Corroded	0.55	0.37	0.82	0.19

These experiments provide new results for railway design and bending fatigue research in the transverse direction of the rail cross-section. To achieve a comprehensive evaluation of the rail resistance under loads in both vertical and transverse directions, further investigations are necessary regarding the residual stress distribution and the effect of loading direction.

CRediT authorship contribution statement

Chongjie Kang: Investigation, Writing - original draft, Visualization, Data curation. **Sebastian Schneider:** Investigation, Validation, Writing - review & editing. **Marc Wenner:** Project administration, Resources. **Steffen Marx:** Supervision, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Correction *1: The following dirgram is the correct form of Fig.22(b).

2.5 Experimental investigation on the residual stress distribution and its influence on the bending fatigue resistance of rails

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Chongjie Kang was responsible for the data curation, visualization and original draft writing. Marc Wenner was responsible for the investigation of the batch A rails and the initial test program. Chongjie Kang was responsible for the investigation of the batch B rails and the further development of the test program. Steffen Marx was responsible for the acquisition of the funding for this investigation. The successful completion of this investigation was achieved due to the cooperation and exchange of all the co-authors. All co-authors reviewed this paper and provided meaningful comments during the process.

Experimental investigation on the rail residual stress distribution and its influence on the bending fatigue resistance of rails

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HIGHLIGHTS

- Sectioning and X-ray diffraction methods are carried out on the rail residual stress distribution.
- Residual stress distributions along the rail cross-section are achieved.
- A direct connection of rail fatigue resistance and residual stress distribution is established.
- A new Smith-diagram of rail considering the residual stress distribution is proposed.

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Smith-diagram

ABSTRACT

This paper presents an experimental study on the residual stress distribution of 60E2 (UIC 60) rails, and its influence on the rail bending fatigue resistance. First, a brief introduction concerning the rail bending fatigue resistance and the rail residual stress distribution is given. Second, an experimental investigation using the sectioning method on the residual stress distribution over the height and in the foot of the 60E2 rails is presented. Third, an experimental investigation on the residual stresses in the rail foot by applying the X-ray diffraction method is elaborated. Afterwards, the results of these two methods are compared. The residual stresses determined with the sectioning method show better consistency with those in literature. Subsequently, the residual stress results determined with the sectioning method are considered with the four-point-bending fatigue results and integrated into a new comprehensive Smith-diagram. As a result, the residual stress distribution is found out to be decisive when determining rail fatigue resistance. It is further concluded that rail bending fatigue resistance shows a good mean-stress dependency when the actual residual stress distribution is considered.

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1. Introduction

As vital parts of a track system, the rails function to transmit wheel loads to track bed, and together with the tread and flange of the rail, they guide vehicles [1]. The avoidance of rail failure is a primary goal of track safety [2]. However, the superposition of the tensile stresses due to temperature, cyclic train wheel loads, manufacturing process (residual stresses), and track/bridge interaction often leads to rail fatigue failure. To correctly evaluate the fatigue resistance of rail profiles, the residual stresses in rails need to be considered. It is well known that compressive residual stress has a beneficial effect on the fatigue life, crack propagation and stress corrosion of materials. In contrast, tensile residual stress

reduces their performance capacity [3–5] and thus leads to a strong reduction in bending fatigue strength and finite fatigue life [6]. The accumulation of tensile residual stresses to other stresses in rails during operation contributes to crack initiation and fracture, in the end, could result in serious accidents [7]. However, the existence of tensile stresses in rails cannot be avoided because of the rail production process that consists of three steps: hot rolling, cooling and straightening [8]. At first, the steel material experiences a hot rolling process to form a required rail profile. After the hot rolling, the temperature in the rail is higher than 900 °C. Subsequently, the rail is placed on a cooling bed until it acquires a temperature close to room (ambient) temperature. During this process, the longitudinal shape of the rail changes due to the different cooling speeds of its different parts. Therefore, the rail bends. Next, the rail is fed into a straightening machine, where it is subjected to a set of bending rollers in order to remove the displacements and curvatures. In this whole process, residual stresses

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appear in each of the three steps [9]. However, a typical residual stress pattern develops mainly due to the straightening process, and is nearly independent of the initial rail distortion in the first two steps [10]. To achieve a full evaluation of rail fatigue resistance, studies on the rail residual stress distribution after straightening, the rail fatigue resistance, and their connections are carried out in this work.

In the last 50 year, many investigations concerning the first two aspects were carried out.

On the one hand, in the 1970s and 1980s, Eisenmann/Leykauf et al [11,12] carried out four-point-bending fatigue tests on S 49, S 54 and UIC 60 rail profiles under a minimum stress level of 50 MPa up to two million load cycles in the vertical direction. Afterwards, these fatigue test results and a value of 80 MPa residual stress, which based on historic investigations on S 49 rails [13], were integrated to construct Smith-diagrams [14]. These results were the basis of the rail additional stress verification in the directive documentation 'Oberbauberechnung' [15,16], UIC 774-3 [17], Eurocode EN 1991 [18] and German guideline 'Richtlinie 804' [19]. In the last 30 years, these values have provided important guidance and reference values for the design of railway tracks and railway bridges. However, this kind of four-point-bending fatigue tests requires a large amount of time and economic investment under the condition of traditional experimental test setups with hydraulic cylinders. Therefore, they have been rarely carried out for scientific studies from the 1990s to the 2010s. As a consequence, these values have remained unchanged. However, due to the rapid development of the railway industry, the rail steel material, rail profile and manufacturing process have been largely improved. New knowledge of these rails is now necessary [20]. Since 2014, with the help of a newly developed resonant fatigue testing facility [21], Kang et al. [20,22] have carried out multi-stress-level tests on 60E2 rails up to five million load cycles in both vertical and transverse directions, respectively. They found that the new rails have a much higher fatigue resistance than those determined by Eisenmann/Leykauf.

On the other hand, finite element modelling (FEM) and experimental methods have been often applied to investigate the residual stresses in rails. FEM methods were used by many researchers like Betegón Biempica et al. [9], Tian et al. [23], Schleitze/Fisher. [24], Srimani/Basu. [25] and Kaiser et al. [26] to analyse and improve the rail manufacturing process, especially the configuration of the straightening process. They all concluded that the unfavourable tensile stresses can be largely reduced or even eliminated at the foot of rail by adjusting the geometry and arrangement of the rollers. Besides, for the determination of existing residual stresses in rails, different measurement methods have been applied, e.g. the X-ray diffraction (XRD) method [27,28], the hole-drilling method [29], the sectioning (saw cutting) method [30], the neutron diffraction method [31,32], the contour method, the transverse/oblique slicing method [33] and the ultrasonic technology [34]. A typical 'C' shape of distribution at the height of the rail cross-section, which is characterized by axial tensile stresses in the head and foot of the rail counterbalanced by compressive stresses in the rail web, has been presented in many of the aforementioned literature. However, the magnitudes of these measured stresses, which range from 0 to 400 MPa at the rail foot centre, showed huge scattering. This scattering depends on the slightly different steel material properties, rail manufacturing technologies and other conditions of different manufacturers. As a result, rails from different manufacturers or even different batches of the same manufacturer show different residual stress situations [35]. Moreover, different measurement technologies and methods often measured different residual stresses of same rail specimens [30,36]. To specifically limit the maximum residual stress, it is regulated in the current CEN norm EN 13674-1 [37] that the tensile stress

measured by the sectioning method should not exceed 250 MPa at the centre of the rail foot.

In general, the rail bending fatigue resistance is of great importance for safe operation and infrastructure design; it depends heavily on the residual stresses. However, a direct connection of the rail residual stresses and rail bending fatigue resistance has not been established in literature, therefore, remains unclear and needs to be investigated. In this work, a detailed investigation on the residual stress distribution of 60E2 rails has been carried out. Two different methods, which are the sectioning method and the XRD method, have been applied. In addition, the bending fatigue test results of the rails from the same batches are obtained from the authors' other work [20,22]. As a result, a direct connection of the residual stresses and rail fatigue resistance in the vertical direction as well as in the transverse direction is established. In the end, a new comprehensive Smith-diagram of 60E2 rails up to five million load cycles is presented.

2. Experimental investigation on the residual stress distribution of rails

2.1. Test specimen

The rail specimens are obtained from the new high-speed railway line Erfurt-Leipzig/Halle (part of the line Berlin-Munich) project in Germany. The rails have a 60E2 profile (UIC 60) with a steel grade of R260. This is the most commonly used type of rail on high-speed railway lines at present. The related basic properties of this rail profile are: Young's modulus $E = 210$ GPa, cross-sectional area of an entire rail $A = 76.48$ cm², line weight $G = 60.03$ kg/m, the bending inertias for the entire rail in the vertical and transverse directions are $I_y = 3021.5$ cm⁴ and $I_z = 510.5$ cm⁴, separately. The specimens were produced in 2010. The chemical composition of the R260 steel is listed in Table 1. More detailed information of the profile can be found in EN 13674-1 [37]. According to the norm, "the microstructure shall be pearlite but grain boundary ferrite may occur in these grades. There shall be no martensite, bainite or grain boundary cementite".

To determine the residual stresses in the rail with the sectioning method, a specimen should be longer than 1.00 m, and the thickness of the slice for the sectioning method should be 20 mm according to EN 13674-1 [37]. In order to avoid a mutual influence of the results by sample preparation, a distance of 100 mm between the investigated cross-sections for the two used experimental methods was selected. Fig. 1 shows a scheme of a rail specimen; the section S was investigated with the sectioning method and the section X was investigated with the XRD method. In the meantime, to guarantee a distance of 500 mm from the specimen's ends, the length of each sample was in this case determined to be 1100 mm. For the test program, six specimens were cut out from different positions of different 60 m long rails. Three of these rails were produced by a Czech manufacturer (Batch A) and the other three were produced by a Polish manufacturer (Batch B). After manufacturing, the rails had been in direct contact with the ambient environment but not put into operation. The corrosion and the roughness developed slowly before the tests.

2.2. Sectioning method

2.2.1. Test principle

Residual stresses are internal stresses which can exist in a member not subjected to external loads. Hence, residual stresses are always in static equilibrium [38]. During the cutting process, the residual stresses relax and a new equilibrium state develops. At the positions where the measurements are set, the component

Table 1

The chemical composition of the R260 rail specimens [37].

Element	C	Si	Mn	P max.	S max.	Cr	Al max.	V max.	N max.	O	H
	[%]	[%]	[%]	[%]	[%]	[%]	[%]	[%]	[%]	[ppm]	[ppm]
Value	0.60–0.82	0.13–0.60	0.65–1.25	0.030	0.030	≤0.15	0.004	0.030	0.010	0.002	0.00025

Fig. 1. Specimen geometry for residual stress measurement (unit: mm).

is separated from the surroundings to assure a fully relaxation of stresses. During the sectioning process, the strains on the surface are measured. Subsequently, the relaxation of stresses are evaluated by applying Hooke's law. Therefore, the original residual stress state can be derived.

2.2.2. Test program

Six specimens were obtained from two manufacturers for the experimental investigation. According to EN 13674-1, a 20 mm thick segment was isolated from the specimen by means of two cuts, which is named procedure I in this work as Fig. 2 displays. To obtain a clear effective range of residual stresses in the longitudinal direction of the rail due to strain relaxation, a procedure II was developed by introducing more cuts as Fig. 3 shows. Table 2 lists the arrangement of the specimens for the two procedures.

Aim to investigate further stress releases in the centre of the rail foot, an additional procedure named Cut C (see Fig. 4) was introduced at the rail foot after Cut B for the rails of batch B. During this process, the strain gauges at the position of S_MP15 and S_MP16 (see Fig. 5) were destroyed as there were too many cables, and there was not enough space.

2.2.3. Measurement plan

Based on EN 13674-1 [37], only one strain gauge at the rail foot centre is required to determine the maximum tensile stress. In addition, according to the German Railway standard DBS 918 [39], 16 strain gauges are required to determine the residual stress distribution of a UIC60 rail profile. In this work, to obtain the

residual stress distributions not only in the foot centre but at the full height and foot of the rail, a more comprehensive measurement plan (see Fig. 5) was developed. In total, 20 stain gauges (seven on rail foot) were aligned along the rail longitudinal direction to measure the relaxation of strains. In addition, seven temperature sensors were installed to monitor the temperature changes during each test.

The strains were measured with electrical strain gauges '3/120 LY41-4L' from HBM. Fig. 6 shows the influence of temperature on the strain gauge itself. It can be observed that there is almost no temperature influence when the temperature is from 20 °C to 23 °C. The strain gauges were aligned along the rail longitudinal direction (a 1/4 bridge) as only the strains in this direction are of interest for this investigation. To eliminate the temperature effects, special procedures were applied (see Section 2.2.4).

2.2.4. Test procedure

In order to obtain reliable results, a specific test procedure was developed (Fig. 7).

Step 1: Before the strain gauges were installed on the rail surface, the rusts on it had been carefully removed with the help of a small belt sander. It was ensured that the removals took place very slowly, so that the residual stress state was not heavily disturbed. Subsequently, suitable protection (against coolant, sawdust, etc.) was applied to the strain gauges. In order to stabilize the glue and protective materials applied to the strain gauges, a time gap of at least 24 h was kept before any further steps.

Fig. 2. Sectioning procedure I according to EN 13674 [37].

Fig. 3. Sectioning procedure II.

Table 2

Specimen arrangement for the sectioning method.

Batch	Rail specimen No.	Sectioning procedure	Final Cut
A	3–9	I	B
	1–2	I	B
	2–14	I	B
B	10–1	II	C
	10–2	II	C
	10–3	II	C

Fig. 4. Additional Cut C (unit: mm).

Step 2: It can be seen from literature [40] that a permanent measurement change occurs to strain gauges when it is loaded for the first time. Besides, the strain gauges could also be incorrectly installed. To avoid these possible sources of errors, the strain gauges were pre-loaded, therefore, the plausibility of the strain gauges could be checked. Fig. 8 and Fig. 9 show the test principle and the four-point-bending test setup, respectively. The measured strain of each sensor was compared with both the calculated strains of a finite element model and equation (1).

$$\varepsilon = \frac{M \cdot y}{I \cdot E} \quad (1)$$

where M [Nmm] = $175 \cdot F$ [N] · 1000, which is calculated based on the configuration of the test setup; y is the height of the measurement position to the centroid axis; $I_y = 3021.5 \text{ cm}^4$ (60E2 [37]); $E = 210 \text{ GPa}$. The maximum applied load $F_{\max}$ was 70 kN.

In addition, a FEM is developed to double validate the results as equation (1) cannot take the exact form and the deformation of the rail cross-section into consideration. Fig. 10 shows the FEM with the same geometry configuration, loading and boundary condition of the test setup. The rail is simulated with solid elements with the assumption of the Young's modulus $E = 210 \text{ GPa}$.

As results, the measured and the calculated strains had very good agreements. The maximum deviations of all the measured strains of the six specimens in comparison with results of the FEM and equation (1) were smaller than 8% and 10%, respectively. Especially at the positions of the strain gauges S_MP11, S_MP12 and S_MP19, which are the decisive positions of our investigation, the deviations were smaller than 5%. This indicated that the measured strains were plausible.

Step 3: As for this $\frac{1}{4}$ bridge setup, the temperature had to be compensated. For this purpose, the ambient temperature and the specimen temperature were controlled to 21°C before each test. The measured strains were recorded under this temperature.

Step 4: A metal band was used to execute the cutting process, the cutting zone was cooled with cooling liquid during the whole process. To avoid unnecessary heating of the rail steel material, the feeding rate was controlled to be very slow. The temperature deviation during the cutting was kept less than 5 K. Within this range, the strain gauge was rarely affected (see Fig. 6). The strains after each cut were recorded when the temperature reached 21°C again so that the influence of the temperature could be automatically excluded.

2.2.5. Results

2.2.5.1. Necessary sample length. Taking the evaluated stresses after cut B as references, the rates of stress release after each cut are evaluated; see Fig. 11. In this figure, the evaluated results are the mean stresses of the rail specimens 10–1, 10–2 and 10–3. It can be observed that the rates are very close to zero ($<1.0\%$) after cut 1 and cut 2. This indicates that the stress states in the rails are not influenced when cutting the first 30 cm length. After cut 3, the rates at different MPs (measurement positions) increased to certain magnitudes, most of which are less than 10%. Afterwards, the rates show sharp increases from Cut 4 to Cut A with huge scatterings. Most rates are larger than 80% after Cut A. Based on these results, it can be concluded that an additional 30 cm of rail length at each side of the 20 mm slice is necessary to ensure an unaffected raw stress state before cutting. In other words, a rail length of about 60 cm instead of 100 cm can ensure an unaffected residual state at the centre of the rail.

2.2.5.2. Test results after cut B. Table 3 concludes the tests results after cut B according to EN 13674-1 [37]. The mean values and standard deviations of the three rail specimens of each batch are calculated. Most standard deviations are less than 10%. However,

Fig. 5. Measurement section of a 60E2 specimen for the sectioning method.

Fig. 6. Strain gauge behaviour against temperature.

relative high deviations exist at a few points, especially at the positions of MP11, MP12, MP19 and MP20, where the residual stresses are essential for rail fatigue resistance. These deviations are regarded as the actual stress states in the specimens as most of the deviations are within reasonable range and the sensors have been checked before each test. The mean residual stresses at the rail foot centre of batch A and batch B are 153.4 MPa and 152.4 MPa, which are almost the same. In comparison, the stresses at the edges of the foot and rail head are much more different.

To obtain the distribution of residual stresses at the height in the vertical direction and at the foot in the transverse direction of the rails, the stresses are further evaluated and shown in

Fig. 12. In the vertical direction, the stresses at each side (L = left, R = right) of the rails are separately presented. To achieve a complete profile in the vertical direction at both sides, the stresses of S_MP19 and S_MP20 are twice used. The mean stresses of two MPs at the same height (except for the rail top and rail bottom) are calculated and displayed. At the rail foot in the transverse direction, the stress of each MP is shown separately. Fig. 12 shows typical 'C' shapes of the residual stress distribution in the vertical direction. The rails from batch A show larger stress scattering due to the larger compressive stresses at rail head and larger tensile stresses at rail web. Similarly, Fig. 12 shows 90° rotated 'C' shapes of the residual stress distribution at the foot in the transverse direction as well. The stresses of batch A rails change more linearly from the edge to the centre in comparison with those of batch B. Besides, the stresses of batch B rails show larger scatterings at the foot due to larger compressive stresses at the rail foot edge.

2.2.5.3. *Stresses after Cut C.* After Cut C, the mean stresses at the positions of the remaining strain gauges are evaluated. The mean stress values are calculated and compared with the stresses after Cut B. Table 4 lists the mean stresses and related changes due to Cut C, it shows that the further cut leads to a negative change of stress for all MPs. This is due to the Poisson effect of the steel material. The further changes are within the range of $\pm 10\%$ for all MPs, with the exception of S_MP14, whose absolute value is very small. Under the condition of a four-point-bending test, the rail centre experience a pure bending state. Therefore, the stresses after Cut

Fig. 7. Procedure flow chart.

Fig. 8. Scheme of the four-point-bending test.

Fig. 9. Photo of the experimental facility for performing the plausibility test.

Fig. 10. FEM for the verification of the plausibility test.

B are more representative and critical when considering their effects on the bending fatigue behaviour of rails.

2.3. X-ray diffraction method (XRD)

2.3.1. Test principle

When a monochromatic X-ray beam irradiates a solid material, it is scattered by the atoms composing the material. Because of the regular distribution of atoms, the scattered waves lead to interferences. The intensities of scattered waves sum up into a constructive interference when the Bragg's law is fulfilled [41]:

$$n \cdot \lambda = 2 \cdot d \cdot \sin\theta \quad (2)$$

where n is an integer, λ is the wavelength of the X-rays, d is the spacing of the lattice planes, and θ is the glancing angle at which the X-rays are reflected. The strain of the material can be expressed in relation to a stress-free state. In this state, the lattice spacing of the planes equals to d_0 . Then, if elastic stress occurs, the lattice spacing changes to d . Then, the strain can be expressed as:

Fig. 11. Mean rates of stress release after each cut.

Table 3

Residual stress of the rails evaluated after cut B (Unit: MPa).

Measurement position	Batch A					Batch B				
	1–2	2–14	3–9	Mean value	Standard deviation	10–1	10–2	10–3	Mean value	Standard deviation
S_MP20	359.1	362.9	352.1	358.0	5.5	241.9	241.6	282.2	255.2	23.3
S_MP1	–39.1	–62.4	–72.0	–57.8	16.9	9.6	31.9	38.6	26.7	15.2
S_MP2	–42.4	–48.6	–55.8	–48.9	6.7	11.9	9.3	2.4	7.8	4.9
S_MP3	–37.4	–68.6	11.1	–31.6	40.2	20.5	–27.5	28.6	7.2	30.3
S_MP4	–17.2	–32.0	38.9	–3.4	37.4	–33.1	–36.0	–48.4	–39.2	8.1
S_MP5	–115.2	–114.5	–127.0	–118.9	7.0	–96.3	–78.3	–91.2	–88.6	9.3
S_MP6	–107.5	–102.6	–113.2	–107.8	5.3	–98.9	–93.2	–102.2	–98.1	4.6
S_MP7	–171.1	–181.7	–164.1	–172.3	8.9	–139.9	–130.6	–145.3	–138.6	7.5
S_MP8	–172.7	–169.9	–163.7	–168.8	4.6	–140.7	–143.3	–131.7	–138.5	6.1
S_MP9	34.3	52.3	32.6	39.7	10.9	37.4	29.7	33.1	33.4	3.9
S_MP10	26.5	29.3	35.2	30.3	4.4	8.8	8.6	40.9	19.4	18.6
S_MP11	–53.8	–63.9	–43.4	–53.7	10.3	–90.2	–98.5	–29.7	–72.8	37.5
S_MP12	–37.8	–42.2	–32.4	–37.4	4.9	–44.0	–46.9	–90.3	–60.4	25.9
S_MP13	–30.5	–34.8	–13.1	–26.2	11.5	–75.5	–83.9	–29.0	–62.8	29.5
S_MP14	–22.7	–20.2	–11.1	–18.0	6.1	–29.8	–46.6	–49.2	–41.9	10.6
S_MP15	5.4	8.1	15.8	9.8	5.4	–19.8	–24.6	–2.2	–15.5	11.8
S_MP16	5.5	7.2	10.7	7.8	2.7	–12.4	–19.7	9.2	–7.6	15.0
S_MP17	92.4	96.1	53.4	80.6	23.6	154.6	181.3	85.2	140.4	49.6
S_MP18	74.4	75.8	59.0	69.7	9.3	80.6	90.5	129.4	100.1	25.8
S_MP19	157.0	178.9	124.2	153.4	27.6	139.6	161.8	155.8	152.4	11.5

Fig. 12. Measured mean residual stress distribution of the rail cross-section.

Table 4

Further release of the stresses due to Cut C.

MP	Mean stress of batch B after Cut B	Standard deviation	Mean stress of batch B after Cut C	Standard deviation	Change of stress	Stress change (Cut C – Cut B)/ Cut C
	[MPa]	[MPa]	[MPa]	[MPa]	[MPa]	[%]
S_MP11	-72.8	37.5	-74.9	39.6	-2.1	2.9%
S_MP12	-60.4	25.9	-65.4	28.8	-5.0	8.0%
S_MP13	-62.8	29.5	-63.5	28.9	-0.7	1.0%
S_MP14	-41.9	10.6	-47.8	13.8	-5.9	14.0%
S_MP17	140.4	49.6	126.9	50.9	-13.5	-9.6%
S_MP18	100.1	25.8	97.2	21.9	-2.9	-3.0%
S_MP19	152.4	11.5	139.7	12.5	-12.7	-8.0%

$$\varepsilon = \frac{d - d_0}{d_0} \quad (3)$$

When the material is under tension, the d-spacing increases and when the material is under compression, the d-spacing decreases. The shift of diffraction peak can be converted to a residual stress value. During the measurement, the specimen can be radiated from different tilted angles ψ in various direc-

tions. The diffraction peak is recorded for each ψ position. Once the peak positions are known, they are plotted versus $\sin^2\psi$. In the case of homogeneous isotropic samples, the plot of the peak position θ vs $\sin^2\psi$ is a straight line, of which the slope is proportional to the stress in the material. Specific theoretic calculations and parameter determinations can be found in standards [42–44] and other literature [45].

Fig. 13. Arrangement of measuring points on the rail cross section by XRD method.

2.3.2. Measurement plan

The measurements are set to five points which are distributed over the width on the bottom surface of the rail foot (Fig. 13) in accordance with the positions of the sectioning method sensors (Fig. 5).

To obtain the residual stresses under the surface of the rail, a depth profile was set at each measuring point. Table 5 lists the different depth profiles for the measurement positions of all six specimens. Depth profile C was to obtain the residual stress on the shallow surface; depth profile D was to obtain residual stresses in deeper depth and determine the necessary depth for further measurement; depth profile E was decided after profile D to obtain residual stresses in deeper depth.

The individual depths were generated by electrolytic polishing of the surface. For material removal, a special electric polisher was used. The depths were determined with a dial indicator to $\pm 3 \mu\text{m}$.

2.3.3. Test procedure

Before the measurement, each rail specimen was carefully prepared. The rust on the rail foot surface was removed manually with sandpaper by grinding along the longitudinal direction of the rail axis. Afterwards, the measurement points were marked with a scribe. Subsequently, the cross lines were drawn within an area of about 1.5 cm diameter around the measuring point, so that the residual stresses would not be disturbed by the scribing. Then, the measurement points were numbered from 1 to 5 as Fig. 14 shows. In the end, the specimen was mounted in the X-diffraction test setup, see Fig. 15.

Fig. 14. Specimen preparation (rail 2-14).

The tests were carried out in the laboratory of Stresstech in Rennerod, Germany according to the EN 15305 [42]. For the measurements, Planes {211} with a wavelength $\lambda = 0.22911 \text{ nm}$ were used for the ferritic rail steel, so that the results could be analysed in good condition as this particular wavelength is easy to produce from a standard chromium anode X-ray tube. The measurement conditions are listed in Table 6.

2.3.4. Results

For the evaluation of the measurements, the Young's modulus was $E = 210 \text{ GPa}$ and the Poisson's ratio was 0.3. Fig. 16 displays the XRD measured stress of the Batch A rails. The connected lines of the measured stresses in the transverse direction in the rail foot at different depths are relatively flat. The results of the three rail specimens show a similar trend. First, compressive residual stresses were measured on the surface of the rail at all measurement points (depth = $0 \mu\text{m}$). Then, the stresses increased (with the tendency to tensile stress) very fast in the first $20 \mu\text{m}$ depth. Subsequently, the residual stresses increased relatively slower to further depths $70 \mu\text{m}$ and $120 \mu\text{m}$. At the depth of $120 \mu\text{m}$, tensile stresses were measured at all points except for the MP5 of rail 2-14. To investigate the residual stress in deeper depth. Measurements were carried out at MP3 of rail 2-14. The results are illustrated in Fig. 17, which shows a small further growth of the tensile stress up to $520 \mu\text{m}$.

Due to the abovementioned experience, a new depth profile with a maximum depth of $500 \mu\text{m}$ was determined for the three rail specimens 10-1, 10-2 and 10-3. The corresponding results are shown in Fig. 18.

Fig. 18 shows that the measured residual stresses developed remarkably in the shallow surface from $0 \mu\text{m}$ to $100 \mu\text{m}$. Subsequently, the residual stresses develop slowly from $100 \mu\text{m}$ to $300 \mu\text{m}$ then to $500 \mu\text{m}$. The measured stresses in the depths $300 \mu\text{m}$ and $500 \mu\text{m}$ of Rail 10-3 are very close to each other. In general, MP1 and MP5 show compressive stresses, MP2, MP3, and MP4 show tensile stresses at the $500 \mu\text{m}$ depth for all rail

Table 5
Depth profiles of the specimens.

Batch	Rail No.	Measurement position	Depth profile	Depth [μm]
A	1-2	MP1 – MP5	C	0, 20, 70, 120
	3-9	MP1 – MP5	C	0, 20, 70, 120
	2-14	MP1 – MP4	C	0, 20, 70, 120
	2-14	MP5	D	0, 10, 20, 30, 40, 50, 70, 120, 220, 520
	10-1	MP1 – MP5	E	0, 100, 300, 500
B	10-2	MP1 – MP5	E	0, 100, 300, 500
	10-3	MP1 – MP5	E	0, 100, 300, 500

Fig. 15. Experimental investigation (a) measuring; (b) electrolytic polishing.

Table 6

Measurement conditions for residual stress measurements.

Measurement conditions	Parameter
X-rays (wavelength)	Cr (0.2291 nm)
Tube voltage / current	30 kV / 8 mA
Collimator	Ø 3 mm
Calibrated distance	18.69 mm
Number of pixels / size	512 / 50 μm
Angular range	146° – 164°
Step size	0.06°
Reflection hkl	211
Diffraction angle 2θ	156.4°
Absorption coefficient μ	89.7 / mm
Tilt angle χ (9)	0°, $\pm 20.7^\circ$, $\pm 30.0^\circ$, $\pm 37.8^\circ$, $\pm 45.0^\circ$
Exposure time	8 s

specimens. The three connected lines of the measured stresses at the depth of 500 μm have similar 90° turned 'C' shapes.

2.4. Comparison of the two methods

In general, the penetration depth of X-rays usually does not exceed 30 μm or 40 μm [45]. Hence, the XRD method concerns a certain volume of the material, called the irradiated volume. This means the measured residual stress is much localized. With an electrolytic technique, it is able to determine the residual stress up to more than 500 μm in the depth of the rail samples. In comparison, the sectioning method determines an integrated residual stress in a relatively greater area not only concerns the length of the thickness of the final slice but also the dimension of the strain gauges. The residual stresses determined by both methods are based on the assumption of the Young's Modulus $E = 210 \text{ GPa}$.

Fig. 19(a) shows the stresses, which are determined with the sectioning method and the stresses determined with the XRD method at 120 μm depth of the batch A rails. No obvious consistent is observed, the results are very different and thus almost incomparable. The distributions of the stresses on rail foot determined with the XRD method are flat. Besides, the stresses are much less than those determined with the sectioning method. It was later found out that the mean value of the maximum peak-to-valley roughness R_t of those rails were about 100 – 150 μm . It can be thus concluded that these results were not the actual residual stresses in the rail material but only false reflections of the characteristic of the rail foot surface. In comparison, Fig. 19(b) shows more comparable results. It can be seen that the distributions of the

Fig. 16. XRD measured stresses of the rails manufactured in Czech.

Fig. 17. XRD measured stress of the MP3 of rail 2-14.

Fig. 18. XRD measured stresses of the rails manufactured in Poland: (a) Rail 10-1; (b) Rail 10-2; (c) Rail 10-3.

Fig. 19. Results comparison of the two methods.

stresses on rail foot determined with the XRD method at 500 μm depth show 90° rotated 'C' shapes, which agree well with those determined with the sectioning method. However, the stress magnitudes are less than those of the sectioning method. Smaller tensile stresses and larger compressive stresses are determined at rail foot centre and edges, respectively.

In general, the results determined with the sectioning method are more comparable and are more consistent with the results in literature [10,26,36]. When applying the XRD method, the surface roughness of the material should be taken into account. The penetration depth should be larger than the maximum peak-to-valley roughness. However, when measuring the stresses at different depth of the material by removing some material with the help of an electrochemical technique, the original stress distribution is already disturbed. Part of the residual stress is already released. In these aspects, the sectioning method is more appropriate in determining rail residual stresses distribution close to the rail surface. As a result, the residual stresses determined with the sectioning method are further considered for the determination of the rail fatigue resistance in Section 3.

3. Experimental investigation on the bending fatigue resistance of rails

3.1. Smith-diagram based on historical residual stresses

To determine the bending fatigue resistance of the 60E2 rail with R260 steel grade, experimental investigations had been carried out by the authors. The objective of the tests was to investigate the bending fatigue strength of the rails up to $N = 5 \cdot 10^6$

Fig. 20. Principle design for the fatigue tests: (a) in the vertical direction; (b) in the transverse direction.

load cycles in both vertical and transverse directions. The test setups are schematically shown in Fig. 20. Detailed experimental approaches in both directions and the controlling program can be found in the authors' other work [20–22].

As a result, a comprehensive smith-diagram, which is the typical diagram to present rail fatigue resistance [14], is concluded as Fig. 21 displays. The horizontal axis and vertical axis present the σ_{mean} and σ_{max} (also σ_{min}) level, respectively. The results in the coordinate system drawn with dashed grey lines are the results of the aforementioned bending fatigue tests. The 80 MPa residual stress, which was determined for the rail foot according to historic research [16], is integrated as part of the mean stresses. In this case, the residual stress at the edge of the rail foot is also considered to be 80 MPa. Therefore, the coordinate system is offset to the other coordinate system drawn with black solid lines. The final fatigue resistance with considering the residual stress at different stress levels can be read in this coordinate system. Based on these results, the fatigue strength in the transverse direction is much greater than that in the vertical direction. The fatigue strength of the rail shows great dependence on the loading direction, or in other words, on the form of the rail cross-section.

3.2. Smith-diagram based on updated residual stresses

The mean stresses of the abovementioned sectioning method tests at the rail foot centre and edge are concluded in Table 7.

To distinguish the influence of actual residual stresses on rail bending fatigue resistance, the measured mean residual stresses at rail foot and rail edge are integrated in the smith-diagram of rails tested in the vertical direction and transverse direction,

separately. Consequently, the coordinate systems are offset with different magnitudes of –56 MPa and 153 MPa for the test results in the transverse direction and vertical direction, respectively (see Fig. 22). It should be noted that the standard deviation is listed for data integrity, however, not used. The reason is that the number of samples is relatively small due to financial limitations. The statistical analysis taking into account the standard deviation leads to unreal results, therefore, not considered in this work. The range of the mean stress level σ_{mean} is calculated between 265 MPa and 488 MPa; see Table 8. In this table, the original values of σ_{min} , $\Delta\sigma$, σ_{max} are the results of the fatigue tests [20,22]. This resultant range corresponds exactly to the experimental investigation in both directions.

To achieve a better comparison, the results of each test direction are linearly extended to this range. The results show that the fatigue resistance of the 60E2 rail in the transverse direction is slightly greater than that in the transverse direction within this range. Under the mean stress levels of 265 MPa and 488 MPa, the fatigue resistance in the transverse direction is 116.0% and 102.0% of that in the vertical direction. This indicates that the fatigue resistance $\Delta\sigma$ increases slightly sharper in the transverse direction as the σ_{mean} increases. In general, the rails show an obvious mean-stress-dependent fatigue resistance in both directions when the different residual stresses are considered. The dependence on the loading direction or rather the rail form in different directions is not decisive in this case.

Moreover, the yield strength of rail steel has been regarded as the upper limit, which must not be exceeded, of a Smith-diagram [14]. As a result of numerous tension tests in the 1970s, an average yield stress value of $\sigma_{\text{S,M}} = 525$ MPa was determined. According to

Fig. 21. Smith-diagram of the tests results for $N = 5 \cdot 10^6$ load cycles in vertical and transverse direction with the assumption of $\sigma_R = 80 \text{ MPa}$ [22].

Table 7

Mean stress of the specimens at rail foot and rail edge (unit: MPa).

Foot centre Mean stress	Standard deviation	Foot edge Mean stress	Standard deviation
$\sigma_{R, \text{mean}}$		$\sigma_{R, \text{mean}}$	
152.9	18.9	-56.1	24.1

test results of the R260 steel specimens, which were produced from the rails of the same batch [20], the rail begins to show an obvious non-linear stress-strain relationship when the stress is larger than 500 MPa. The evaluated 0.2% yield strength is 599 MPa. However, Fig. 22 shows a linear development of flexural fatigue resistance, when the tested maximum stress level is up to 613 MPa. The rail fatigue resistance shows no sign of abrupt or non-linear decreasing within this stress range. This indicates that the current norm regulated upper limit is very conservative, and should be further investigated.

4. Summary

In this work, two experimental investigations concerning the sectioning method and the XRD method have been applied to measure the residual stress distribution in six 60E2 rail specimens from two different manufacturers. The results obtained from the sectioning method show a 'C' shape form and a 90° rotated 'C' form at the height and the foot of the rail profile, respectively. The mean stresses in the centre and edge of the rail foot are 153 MPa and -56 MPa respectively. In addition, to achieve an unaffected resid-

ual state at the centre of the rail, a minimum sample length of 60 cm is suggested for the sectioning method. This should be also noted when carrying out other experimental tests on rail specimens. Besides, the surface roughness should be considered, when applying the X-ray method. The measuring depth should be deeper than the maximum peak-to-valley roughness to obtain actual residual stresses in the steel material. Additionally, the initial residual stress state is already disturbed when applying an electrochemical technique to remove steel material. Considering the different residual stresses at the centre and edge of the 60E2 rail, the rail bending fatigue resistance shows very good mean-stress dependency. This indicates that the direction of loading and, accordingly, the different shapes of the rail cross-section in different directions have a limited influence on the rail fatigue resistance. In the end, a new comprehensive Smith-diagram of 60E2 rails for up to five million load cycles in both vertical and transverse direction is presented. The significant load-direction dependent differences in fatigue strength can be explained by different residual stress states. Furthermore, it shows that the transition into the plastic region of steel has almost no influence on the rail fatigue strength. This is of important reference value for railway design and railway bridge design when considering the track/bridge interaction. The aforementioned investigations provide new and up-to-date results concerning the current rail manufacturing technology, and also evidence of a mean-stress-depend rail resistance in different directions. However, due to limited resources and the limitation of the applied measurement methods, the residual stress tests and fatigue tests are carried out on different rail specimens. To obtain a more precise and direct connection

Fig. 22. Smith-diagram of the tests results for $N = 5 \cdot 10^6$ load cycles in vertical and transverse direction with measured residual stresses.

Table 8

Identification of the stress levels for the updated Smith-diagram [20,22].

Stress level	Test direction	$\sigma_{\min}$	$\Delta\sigma$	$\sigma_{\max}$	$\sigma_{R, \text{mean}}$	$\sigma_{\text{mean}} = (\sigma_{\min} + \sigma_{\max})/2 + \sigma_{R, \text{mean}}$
Minimum	Vertical	140	362	502	-56	265
Maximum	Transverse	210	250	460	153	488

between the residual stress distribution and rail fatigue resistance, further investigations on the same samples are recommended to be carried out, when proper technologies are available.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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3 Further studies and conclusions

3.1 Further studies

During the publication process, some further investigations have been carried out. To provide a more comprehensive knowledge of this topic, several new results are supplemented.

3.1.1 Fatigue tests up to 50 million load cycles on rails of the same batch (batch A)

In operation, there are rails on busy sections that undertake up to 50 million load cycles during their service life. For instance, a highly frequently loaded line, used by 56-axle-trains every 10 minutes for 15 years corresponds to $44.1 \cdot 10^6$ load cycles. To verify the fatigue resistance of the rail not only for these current extreme situations but also for a possible increase of the operating speed in the future, further tests have been carried out on 2.2 m long rail samples for up to 50 million load cycles on the same four-point bending test setup. In this program, 13 rails have been tested in the vertical direction of the rail cross-section, each run-out test needed 30 days to be completed with a load frequency of about 20 Hz. The planned and validated minimum stress levels were 150 MPa and 130 MPa, respectively. Four rails failed and nine rails ran out up to 50 million load cycles. The related characteristic cross-sectional dimensions of the failed rails are listed in Table 1. One rail failed up to 5.05 million load cycles, the other three failed within 3.5 million load cycles. All these failed rails showed fracture type 1 (see Fig. 3, also see section 2.3 [22]). The mean area S_m of the fatigue crack was 256.80 mm^2 . This value is almost the same with the $S_m = 254.72 \text{ mm}^2$ of the rails tested under the same minimum stress level up to five million load cycles. It again confirms the dependence of the fatigue crack profile area on the stress level [22].

Fig. 3: Fracture type 1 und measured crack dimensions

Table 1: Rail specimen fracture status of rails up to $5 \cdot 10^7$ cycles

Rail specimen No.	Load cycles [million]	fracture type	D [mm]	R [mm]	L_0 [mm]	Crack profile area S [mm ²]	Mean values S_m [mm ²]
5-13	5.05	1	22.0	11.0	108.0	380.1	256.80
5-11	3.04	1	17.0	8.0	78.0	213.6	
5-10	1.17	1	15.0	7.0	72.0	164.9	
5-9	2.29	1	19.0	9.0	72.0	268.6	

Of the 13 rail specimens, the first two rails ran out and showed no signs of failure. Afterwards, the load was increased by 20 MPa. Hence, the first two rails cannot be

evaluated with the other 11 rails according to the staircase method regulation [25]. The effective results are arranged and shown in Fig. 4.

Fig. 4: Staircase method testing results of not-corroded rails up to $5 \cdot 10^7$ cycles

Similarly, the results were evaluated taking into account a value of 5%-quantile and a confidence level of $\gamma = 0.75$. The corresponding results are compared with the test results of the rails loaded up to five million cycles and are listed in Table 2.

Table 2: Evaluated test results of rails up to $5 \cdot 10^6$ and $5 \cdot 10^7$ cycles

Load cycles [million]	R_t [μm]	Planned $\sigma_{\min}$ [MPa]	Validated $\sigma_{\min}$ [MPa]	$\sigma_{D; m}$ [MPa]	s [MPa]	$\Delta\sigma_{D; 0.05}$ [MPa]
5	92 – 190	150	130	276.7	2.7	271
50	106 – 224	150	130	252.5	11.6	229

R_t denotes the maximum peak-to-valley roughness, $\sigma_{D; m}$ denotes the mean fatigue strength, s is the standard deviation and $\Delta\sigma_{D; 0.05}$ is the 5%-quantile fatigue strength. It can be observed that the fatigue strength of the rails up to 50 million load cycles is 42 MPa less than that of the rails up to five million cycles. It seems easy to conclude that the rails show worse fatigue behaviour when the number of load cycles increases. However, this is not precise in this case. First, the difference of the directly evaluated $\sigma_{D; m}$ is about 23 MPa. The larger difference of $\Delta\sigma_{D; 0.05}$ is due to the larger standard deviation of the rails loaded up to 50 million load cycles. Second, the rails loaded up to 50 million cycles showed a more severe surface roughness condition. Considering these two reasons, the actual difference of the fatigue strength of rails up to five and 50 million load cycles is supposed to be less.

3.1.2 Fatigue tests up to five million load cycles on rails of a different batch (batch B)

The rails for the investigations in papers No. 3 [22] (see section 2.3) and No. 4 [23] (see section 2.4) were produced by a Czech manufacturer (batch A). To test and verify the obtained knowledge, additional comparison tests have been carried out on rails produced by a Polish manufacturer (batch B).

The planned minimum stress level was $\sigma_{\min} = 250$ MPa. Of all the 13 tested rails, seven fractured, the other six ran out up to five million load cycles. The rail cracks showed fracture type 1 (see 3.1.1) in the cross-section. The measured characteristic dimensions are listed in Table 3. All the fractured rails failed within 1.5 million load cycles. The mean crack area was $S_m = 59.95 \text{ mm}^2$. In contrast, the mean area S_m of the batch A rails tested under the same planned minimum stress level $\sigma_{\min} = 250$ MPa was 127.64 mm^2 . This indicates that the rails from batch B are more brittle than the rails from batch A.

Table 3: Rail specimen fracture status of batch B rails up to $5 \cdot 10^6$ cycles

Rail specimen No.	Load cycles [million]	fracture type	D [mm]	R [mm]	L ₀ [mm]	Crack profile area S [mm ²]	Mean values S _m [mm ²]
9-12	0.72	1	55.0	10.3	5.0	80.9	58.95
9-1	0.69	1	86.0	10.0	5.0	78.5	
9-6	1.07	1	56.0	9.0	4.3	60.8	
9-3	1.06	1	85.0	7.3	3.0	34.4	
9-9	1.19	1	59.5	10.0	5.0	78.5	
9-7	0.95	1	57.5	6.5	3.4	34.7	
9-4	0.89	1	58.0	7.5	3.8	44.8	

Based on the supporting forces, the validated minimum stress level $\sigma_{\min}$ of these batch B rails was calculated to be 230 MPa, which is 20 MPa larger than that of the rails from batch A. This also proves that the steel material of the batch B rails is more elastic than that of the batch A rails. The results according to the staircase method are arranged and shown in Fig. 5. The evaluated results of batch A and batch B are listed in Table 4. The determination of the planned and validated $\sigma_{\min}$ can be found in section 2.4/publication No. 4 [23].

Fig. 5: Staircase method testing results of corroded batch B rails up to $5 \cdot 10^6$ cycles**Table 4:** Evaluated results of rail up to $5 \cdot 10^6$ load cycles of the two batches

Batch	R _t [μm]	Planned $\sigma_{\min}$ [MPa]	Validated $\sigma_{\min}$ [MPa]	$\sigma_{D; m}$ [MPa]	s [MPa]	$\Delta\sigma_{D; 0.05}$ [MPa]
A	92 – 190	250	210	266,7	8.1	250
B	163 – 248	250	230	260.0	9.9	240

The rails from two different batches show relative similar fatigue resistance, despite the differences in the surface roughness condition and validated $\sigma_{\min}$. The deviation of the finally evaluated fatigue resistances is only 10 MPa.

3.2 Rail resistance due to TBI

3.2.1 Rail compressive resistance

Buckling has been a critical issue for rails in ballast track systems. The verifications in Richtlinie 804 [15] and Eurocode 1991 [13] are based on the stability theory [26,27]. The current permissible additional compressive stress value of 72 MPa corresponds to a

maximum axial rail force of 1.94 MN according to historical experimental tests on rails in ballast track systems [28].

Compared to ballast track systems, ballastless track systems have better integrity and much larger stiffness. According to the laboratory and field test results mentioned before, rails in ballastless track systems show abrupt deformations due to material plasticity in advance of buckling failures. In this case, the deformation and stress state of the rail become the limit state of the rail instead of buckling.

The adapted limit value of 92 MPa for ballastless track systems according to Richtlinie 804 [15] under compression based on the stability theory is very conservative [20]. Based on both numerical and experimental investigations in the laboratory, it can be concluded that the maximum rail deformation stays smaller than 2.5 mm and no indication of system instability occurs up to an axial force of 3.0 MN. Abrupt non-linear developments of rail stresses and deformations only occur when the force is larger than 3 MN according to experimental tests. At this stage, the steel reaches its yield point, thus, begins to show remarkably non-linear behaviour when the axial load exceeds 3 MN. These results are consistent with the results based on large-scale tests in the field [29]. However, if taking a closer look at the stress development when the axial force develops from 2 MN to 3.5 MN as Fig. 6 (scaled from fig 25 in publication No. 2 of section 2.2 [21]) shows, noticeable non-linear force-stress behaviour begins to occur when the load exceeds 2.5 MN for the rails tested without failed fasteners. This non-linearity develops more obvious as the load increases and becomes almost exponentially when the load exceeds 3 MN.

Fig. 6: Rail stress development of rails tested without failed fasteners [21]

As the serviceability criteria (yield stress of the rail steel) and the deformation of the rail and its impact on the riding safety of rolling stock are decisive [29], a loading capacity $F_{R, crit}$ of the 60E2 rails in the ÖBB-Porr system is recommended to be 2.5 MN as the limit state when almost no plasticity of the steel material occurs. This force corresponds to axial stress of $2.5 \text{ MN} / 7650 \text{ mm}^2 \approx 326 \text{ MPa}$ for a straight rail.

According to Eq. (18) in paper 2 [21] (also see section 2.2), the critical temperature $\Delta T_{R, crit}$ can be calculated as:

$$\Delta T_{R, crit} > \Delta T_{R, max} + \Delta T_{R, S} + \Delta T_{R, B} \quad (1)$$

In which $\Delta T_{R, max}$ presents the maximum allowed temperature, $\Delta T_{R, S}$ is a safety margin and $\Delta T_{R, B}$ denotes the equivalent temperature of the longitudinal force due to TBI. Thus,

$$\Delta T_{R, B} < \Delta T_{R, crit} - \Delta T_{R, max} - \Delta T_{R, S} \quad (2)$$

$$\alpha \cdot \Delta T_{R, B} \cdot E \cdot A < \alpha \cdot (\Delta T_{R, crit} - \Delta T_{R, max} - \Delta T_{R, S}) \cdot E \cdot A \quad (3)$$

$$F_{R, B} < F_{R, crit} - F_{R, max} - F_{R, S} \quad (4)$$

It is assumed that the minimum safety margin is $\Delta T_{R,S} = 20$ K due to eddy current brake and $\Delta T_{R,max} = 50$ K [21]. Hence:

$$F_{R,B} < 2.5 \text{ MN} - 1.15 \cdot 10^{-5} \cdot (20 \text{ K} + 50 \text{ K}) \cdot 210000 \text{ MPa} \cdot 7650 \text{ mm}^2 = 1.207 \text{ MN} \quad (5)$$

The permissible additional compressive stress $\sigma_{add,C}$ due to TBI is therefore:

$$\sigma_{add,C} = \frac{F_{R,B}}{A} \leq \frac{1.207 \cdot 10^6 \text{ N}}{7650 \text{ mm}^2} = 157 \text{ MPa} \quad (6)$$

The calculated permissible additional compressive stress is 157 MPa, which is much larger than 92 MPa according to the current EN 1991 and Richtlinie 804. The calculated stress corresponds to the longitudinal stress on the neutral axis of the rail cross-section. Even though this result is based on multiple assumptions and the condition of the static experimental investigation, it still indicates that the 92 MPa value for rails in ballastless systems in current norms is very conservative.

3.2.2 Rail tensile resistance

According to abundant experimental tests on rail fatigue resistance in different directions and residual stress distribution tests on 60E2 rail specimens, three significant new findings have been obtained.

First, the new rails tested in this work show profoundly larger fatigue resistance than the rails tested in the 1980s.

Second, the upper limit (470 MPa) of the Smith-diagram of rails, which has been applied since the 1980s, is proved to be too conservative. This value was calculated with a lower 5-percentile value and an average yield strength of 525 MPa, which were determined according to numerous static tension tests in the 1970s. Afterwards, the historical fatigue tests in the 1970s to 1990s were carried out under a minimum stress level of 50 MPa, the maximum stresses had never achieved this upper limit. Therefore, there has been no experimental evidence of rail fatigue behaviour above 470 MPa. In contrast, the newly tested rail steel R260 begins to show a non-linear stress-strain relationship above 500 MPa based on tension tests, and the 0.2% yield strength is evaluated to be 599 MPa. A linear development of the fatigue strength of the rail, on the other hand, was observed up to 613 MPa based on the fatigue tests. The evaluated rail fatigue resistance shows no sign of an abrupt or non-linear reduction up to this stress level. This is due to the plastic shakedown behaviour of the rail steel material according to paper No. 4 [23] (also see section 2.4). Furthermore, it can be concluded that an even higher upper limit, which is larger than 613 MPa, can be assumed based on the evaluated resistance and material behaviour.

Third, an apparent mean stress dependence of the rail fatigue strength is observed. There had been rare investigations concerning the rail fatigue resistance in the transverse direction. Furthermore, in existing literature, residual stress tests and fatigue tests have never been performed on rails from the same batches. Therefore, the relationship between these two properties of rails has remained unclear. Due to the investigation program in this dissertation, rails have been tested in both vertical and transverse directions. Moreover, the rail residual stresses in the rail cross-section have been studied as well. As a result, a mean-stress dependency is found when the residual stress distribution is integrated into the Smith-diagram. This provides not only new information on the rail fatigue resistance in the transverse direction but also an innovative insight into

the rail fatigue behaviour in both vertical and transverse directions. A more comprehensive understanding of rail flexural fatigue behaviour is achieved.

Based on the abovementioned findings, a new verification concept is recommended instead of only limiting the stress in the rail foot centre: the stresses in both rail foot centre and edge must be limited taken into account the residual stress distribution in the rail foot.

Usually, rail tracks are under both vertical and transverse actions in operation. Furthermore, the transverse load becomes larger when the rail track is on curves or in the area of bridges, where additional stresses are introduced and superimposed in the rail foot. As a result, both the rail foot centre and the rail foot edge become critical locations in different situations. In addition, the residual stresses in the rail foot and rail edge are very different; large tensile stresses and compressive stresses are commonly measured at the rail foot centre and edge, respectively. These residual stresses are superimposed on the other stresses due to external actions. Hence, both locations must be verified.

Fig. 7: Smith-diagram of the tests results for $N = 5 \cdot 10^6$ load cycles in the vertical and transverse direction with measured residual stresses (figure 22 in section 2.5 [24])

According to the experimental test results, the upper Smith-diagram (see Fig. 7, refer to Fig.22 in paper No. 5 [24], also see section 2.5), in which the measured residual stresses are integrated, is obtained. By offsetting the coordinate system, the rail fatigue resistance in this Smith-diagram contains a residual stress value of 0 MPa [19]. Therefore, the residual stresses, which have relatively huge scatterings and depend on various factors,

can be separated and regarded as external effects. This profoundly extends the applicability of this Smith-diagram.

The core part (show as a blue area in Fig. 7) of all the results in the vertical direction and horizontal direction are considered as the initial part. Subsequently, these boundary lines on the upper side and lower side are linearly extended (the dark dash lines in Fig. 8) to the coordinate system. Finally, the closed grey area (see Fig. 8) is recommended as the valid area for 60E2 (UIC60) rails.

Fig. 8: Recommended Smith-diagram for 60E2 (UIC60) rails up to five million load cycles

To determine the permissible additional tensile stress, other stresses due to temperature and train traffic shall be pre-determined. As the investigation of these actions is not the focus of this work, the related stresses due to these actions refer to the literature [13,16].

(1) In the rail foot centre

The stresses due to temperature and train loads are $\sigma_{T, \max} = 120$ MPa and $\sigma_{Q, \text{centre}} = 158$ MPa, respectively, according to EN 1991 [13] and Oberbauberechnung 1992 [16]. The residual stress is $\sigma_{R, \text{centre}} = 153$ MPa [24] (also see section 2.5) for the tested rail. Then the permissible additional tensile stress in rail foot centre ($\sigma_{\text{add}, T, \text{centre}}$) due to TBI can be calculated as:

$$\sigma_{\text{add}, T, \text{centre}} = (613 - 120 - 158 - 153) = 182 \text{ MPa.}$$

In contrast to the value of 92 MPa in current EN 1991 [11] and the value of 112 MPa in the new FprCEN/TR 17231 EN 1991 [30], the evaluated value it is larger and indicates that the rail resistance still has a greater residual capacity than the expected values according to the guidelines.

(2) In the rail foot edge

The stress due to temperature is also set to $\sigma_{T, \max} = 120$ MPa. However, the stress at the rail foot edge due to train loads must be adjusted, as the value of 158 MPa was only

determined for the stress in the rail foot centre. To estimate the permissible additional tensile stress at the rail foot edge, two conditions (see section 2.4 [23]) are set for rails on bridges: $Y/Q = 0.6$, which corresponds to a ratio of $\sigma_{Q, \text{edge}}/\sigma_{Q, \text{centre}} = 2.36$ [31]; 1 mm translation of the track system in the transverse direction, which induces $\sigma_{TB, \text{edge}} = 30$ MPa more stress in the rail foot edge than in rail foot centre [32]. Besides, the residual stress is $\sigma_{R, \text{edge}} = -56$ MPa [24] (also see section 2.5) for the tested rail. Then the permissible additional tensile stress at rail foot edge ($\sigma_{\text{add}, T, \text{edge}}$) due to TBI can be calculated as:

$$\sigma_{\text{add}, T, \text{edge}} = [(613 - 120 - 158 \times 2.36 - (-56) - 30)] = 146 \text{ MPa}.$$

With these assumptions, the permissible additional stress at the rail foot edge is 36 MPa lower than that in the rail foot centre. However, it is still larger than the values in the standards.

Besides, when it is assumed that $\sigma_{Q, \text{centre}} = 158$ MPa and $\sigma_{\text{add}, T, \text{centre}} = \sigma_{\text{add}, T, \text{edge}}$, it can be calculated that $\sigma_{Q, \text{edge}}/\sigma_{Q, \text{centre}} = 2.114$, under which circumstance, the permissible additional stress at both centre and edge of the rail foot is the same. In operation, the ratio $\sigma_{Q, \text{edge}}/\sigma_{Q, \text{centre}}$ depends on the condition of the characteristic of the infrastructure system such as the curvature as well as the train type, track system, the driving speed and weather condition etc. The transmission state can only be accurately determined under certain conditions. As this is not the focus of this work, it will not be further discussed.

These two above-calculated values are valid for the rails, which are tested within the scope of this dissertation. These values can be important reference values for 60E2 (UIC 60) rails. For specific conditions, when the related stresses are pre-determined, the permissible tensile stresses can be calculated by applying the Smith-diagram (see Fig. 8).

3.2.3 Summary

The values calculated in sections 3.2.1 and 3.2.2 are deterministic results for certain determined conditions. For un-determined situations, equation (7) below is proposed:

$$\begin{cases} \sigma_{\text{add}, C} = 326 \text{ MPa} - \sigma_{T, \text{max}} - \sigma_{T, S} \\ \sigma_{\text{add}, T, \text{centre}} = 613 \text{ MPa} - \sigma_{T, \text{max}} - \sigma_{Q, \text{centre}} - \sigma_{R, \text{centre}} \\ \sigma_{\text{add}, T, \text{edge}} = 613 \text{ MPa} - \sigma_{T, \text{max}} - \sigma_{Q, \text{edge}} - \sigma_{R, \text{edge}} - \sigma_{TB, \text{edge}} \end{cases} \quad (7)$$

This equation has been formulated according to the performed investigations in this dissertation. The formulas are applicable for UIC 60 rails in ballastless track systems constructed in the area of bridges. This can be applied to different situations when other stresses are determined. As extreme conditions were set for the determination of the critical values, they are conservative in the scope of this dissertation.

3.3 Other obtained knowledge

3.3.1 Four-point-bending fatigue tests

There are some basic recommendations and regulations in current standards on the four-point-bending test of rails under cyclic loads in both vertical [25] and horizontal directions [33]. The recommended span configurations vary in a certain range. The basic principles of the 2.2 m long rail specimen, the 1.9 m span and the 150 mm loading distance in this work are based on historical tests in the 1970s [34]. This configuration creates a similar bending moment distribution of the rail specimens to the rails in

operation, which are laid on sleepers with a distance of approximately 60 cm in ballast track systems. Besides, the support and load transfer rollers of the vertical tests (see Fig. 9) meet the requirements of EN 14730-1 [25]. The conditions for load application and support in the transverse direction (see Fig. 10) are based on DBS 918254-1 [33], according to which the load and load dissipation take place via fittings that correspond to the shape of the rail profile and thus introduce the load over the entire cross-sectional height.

The above-mentioned clear geometry conditions are commonly applied to the experimental investigations on rail fatigue strength nowadays in both research and industry fields. However, the control and regulating of these tests under cyclic loads vary and remain uncertain. There are two major control principles: the stress-controlled principle (or load-controlled) and the strain-controlled principle. Stress-controlled fatigue tests are typically performed on specimens under low-strain levels in the elastic range, usually relating to high cycle fatigue tests. In contrast, strain-controlled tests are commonly performed on specimens under high-strain levels that exceed the elastic range, usually relating to low cycle fatigue tests. Historical investigations in the 1970s to 1990s [35–37] were carried out based on a stress-controlled principle, however, usually controlled with stain-gauge-determined stresses. Hence, they were fundamentally strain-controlled as well. Because the minimum stress level of these tests was always 50 MPa and the maximum stress levels were smaller than 400 MPa, this method was applicable as the steel material behaves almost linearly in this stress range. However, much higher stress levels have been reached in the tests of this dissertation. Most rails were tested under stress levels over 500 MPa, which is more realistic in practice. In addition, the potential of rail fatigue strength was to be explored. As a consequence, the tested maximum stress level even exceeded 610 MPa. In this range, a pure stress-controlled method based on a linear assumption of rail material leads to inaccurate results, and is not proper. On the other hand, a pure stain-controlled procedure is not proper either as the rails show a plastic shakedown behaviour and do not show low cycle fatigue failures according to the test results. It is, therefore, important to measure and record both the stress and strain development of the rail.

To obtain a comprehensive understanding of the stress and strain behaviour of the rail in this state, some recommendations are given so that the tests can be well controlled and the results can be correctly evaluated.

(1) Rails loaded in the vertical direction

A configuration of the test setup in the vertical direction is shown in Fig. 9.

Fig. 9: Four-point-bending fatigue test in the vertical direction [22]

To measure the strains on the rail foot in the mid-span, at least three strain gauges shall be applied. On the one hand, the torsion of the rail can be reflected with the measured strains of strain gauge 2 and strain gauge 3. On the other hand, the stress-strain behaviour at different locations is slightly different under external loads due to the steep distribution of the residual stresses in the rail foot. With three strain gauges, the rail behaviour can be better recorded and evaluated. The mathematical mean value of the measured values of all three strain gauges is recommended as the representative strain in the rail foot. Besides, load cells must be placed under the supports to measure and monitor the reaction forces during the tests. This enables the stress level in the mid-span to be evaluated. With strain gauges and load cells, both strains and stresses in the rail foot can be obtained regardless of a stress-controlled or a strain-controlled procedure. The results of one resource can provide supplement and reference values to the other. This is of essential importance to achieve reliable test results due to the plasticity of the steel material.

(2) Rails loaded in the transverse direction

A configuration of the test setup in the transverse direction is shown in Fig. 10.

Fig. 10: Four-point-bending fatigue test in the transverse direction [23]

The principle of the configuration is similar to the test setup in the vertical direction. In comparison, one strain gauge is applied to each rail edge to adapt to the available surface for strain gauges, and the symmetrical rail form in this direction.

(3) Temperature monitoring

Regardless of the direction of the tests, a temperature sensor shall be applied in mid-span close to strain gauge 1. The maximum temperature increase during the test must not exceed 30 K according to DIN 50100 [38].

In general, a procedure of stress-controlled/force-controlled principle supplemented with additional strain measurements is recommended for further experimental investigations.

3.3.2 Load cycles for fatigue tests on rails

By summarizing the test results, it is concluded that 47 out of 96 batch A rails and seven out of 13 batch B rails failed. The maximum load cycles of all the failed rails are shown in Fig. 11. All rails failed in the range of 0.31 to 5.05 million load cycles. According to the results of tests in which up to five million load cycles in the vertical direction were applied, two specimens lasted three to four million load cycles; another two rail specimens lasted two to three million load cycles; the remaining specimens failed within two million load cycles, and the minimum failed load cycle was 0.62 million. In general, most rails failed in

the range of 0.8 million to two million load cycles in the tests where up to five million load cycles were applied in the vertical direction. Besides, one rail failed up to 5.05 million load cycles, three rails failed in the range of 1.17 million to 3.04 million load cycles based on the test results in the vertical direction up to 50 million load cycles. The other eight rail specimens ran out. In comparison, most failure cases for the rails tested in the horizontal direction occurred at less than one million load cycles, only two of them at up to 1.76 million load cycles. The results indicate a threshold phenomenon of fatigue development in the rails. Hence, the rails would either fail within limited load cycles or run out up to ultra-high load cycles (50 million in this case). This similar threshold phenomenon of different materials has been investigated and explained in micro-scale and meso-scale levels [39–41]. However, there has been no concrete theories and quantitative evidence on this issue.

Fig. 11: Load cycles of failed rails

Based on the abovementioned results, the following empirical recommendations are made for further laboratory studies or practice:

1. To determine the fatigue strength of 60E2 rails in the vertical direction of the rail profile, a maximum of $5 \cdot 10^6$ load cycles is recommended.
2. To determine the fatigue strength of 60E2 rails in the transverse direction of the rail profile, a maximum of $2 \cdot 10^6$ load cycles is recommended.
3. In practice, the rails need to be more intensively checked and maintained in the first five million load cycles concerning the flexural fatigue failure.

3.3.3 Corrosion test and roughness measurement

In the laboratory, the surface of the rail sample is cleaned with a soft cloth and a high-pressure air gun before measurement. This method can remove most of the rust and other external debris without damaging the rail surface. It is important to keep the surface clean when using a stylus instrument to measure the roughness of the rail sample surface as the rust and other external debris on the rail surface could influence the measurement results. However, these physical treatments do not always work efficient. On the one hand, these treatments have their limitations; some tiny powders can stick to the surface and become undetectable. On the other hand, some larger rust particles or deposits cannot

be removed by these two methods without damaging the surface of the rail steel. The remaining materials can influence the results of the roughness measurement. In addition, these two treatments also depend on the operator. Therefore, there is a large scatter.

To evaluate their influence, three rail samples were electrolysed for 24 hours. Electrolysis is a method of removing rust (iron oxide) by passing a small electrical charge from a battery or battery charger through the rusty metal to stimulate ion exchange while the rusty metal is immersed in an electrolyte solution. For this purpose, sacrificial steel is required to accommodate the rust. In addition, washing soda, which is sodium carbonate, is used. Sodium and carbonate ions are dissolved in water. This accelerates the chemical reaction. Pure steel material does not get involved in this chemical reaction. Therefore, the rust can be removed without damaging the rail steel material. The configuration of this method is schematically represented in Fig. 12.

Fig. 12: Schematic representation of the electrolysis

The roughness of the rail surface is measured before and after the treatment. Afterwards, the differences are evaluated. To ensure good statically reliability, multiple measurement lines are used during the investigation (see Fig. 13).

Fig. 13: Measurement positions (unit: mm)

The maximum peak-to-valley roughness R_t is analysed as it is decisive for the fatigue resistance of the rail. The results are shown in Fig. 14, Fig. 15 and Fig. 16. It can be observed that the measured results before the treatment fluctuate considerably and are generally larger than those after the treatment. Fig. 17 shows the photographs of the surfaces of the rail foot before and after the electrolysis treatment.

Fig. 14: R_t of the rail 4-27

Fig. 15: R_t of the rail 8-2

Fig. 16: R_t of the rail 5-13

Surface before the treatment

Surface after the treatment

Fig. 17: The rail foot surface condition before and after the electrical treatment

The maximum measured values of all three rails are listed in Table 5. It can be seen that the maximum deviations are -86.9%, -5.85% and -34.42% for each rail, respectively. Direct measurement only with physical treatments generates larger measurement values. This leads to inaccurate and unsafe evaluations of the rail fatigue resistance relating to corrosion and surface conditions. This treatment is recommended to be considered for further experiments.

Table 5: Maximum R_t of the rails

Rail	Status	Before electrolysis [μm]	After electrolysis [μm]	Difference [μm]	Deviation [%]
4-27	corroded	373.38	199.78	-173.60	-86.90%
8-2	corroded	226.24	213.73	-12.51	-5.85%
5-13	not-corroded	165.28	122.96	-42.32	-34.42%

4 Outlook

This dissertation focuses on the resistance verification of the rail track under compression and tension considering TBI. The results obtained in this work are based on certain assumptions and limited by the experimental conditions. Consequently, these results are deterministic. Some issues remain as recommended future researches to achieve more generalized knowledge within the scope of Part IV (see 1.2) regarding the resistance of rail under compression and tension, and in the scope of TBI.

4.1 On the side of rails under compression

1) Experimental and numerical investigations on rails in operation

Within the scope of this work, the experimental investigations were limited to the laboratory condition. Only half of the cross-section and 8.17 m of the length of the ÖBB-Porr slab track system were built. The boundary conditions are not identical to the rails in operation. To determine the rail resistance in operation, field tests and related simulations are necessary, taking into account material, geometry and structural non-linearities.

2) Rail resistance under compression considering dynamic effects

In this dissertation, the dynamic effects due to trains on rail resistance under compression are not considered. However, train-track interaction has negative impacts on the lateral resistance of the whole track system, and the rail resistance. First, uplift of the track due to the bending wave between the wheel loads can cause reduced lateral resistance of the track system. Second, the dynamic track vibrations caused by the passing of a vehicle can cause a loss of track lateral resistance. Third, transverse forces generated on the track due to wheel/rail interaction can increase the transverse loads on the track. These impacts have been confirmed by multiple investigations [42–45] on rails in ballast track systems. However, investigations on ballastless track systems are scarce and are necessary to be carried out in the future. For this purpose, both experimental investigations and multi-body train-track-bridge dynamic simulation methods are to be applied.

3) Rail resistance under compression in different ballastless track systems

The ballastless system used in the experimental investigation is the ÖBB-PORR slab track system, which has been widely constructed on the German new HSR line Berlin–Munich. However, there are many different ballastless track systems. These different systems have different features and thus different stiffness, although they share more similarities compared to ballast systems. In particular, their stiffness in the transverse direction of the rail, which has a significant influence on the behaviour of the rail, can vary widely. This property is decisive whether the rail buckles first or yields first. That is, it is decisive whether the stability or the stress state or the deformation is the limit state of the railway system. Hence, a systematic analysis of different ballastless track systems is proposed to be further investigated.

4.2 On the side of rails under tension

1) The upper limit of rails under cyclic loads

All the tested rails in the fatigue tests are within the elastic or elastic-plastic range of the rail material. Due to the plastic shakedown behaviour, the rail fatigue strength shows a linear development in the range of the tested load levels. The maximum upper limit (see Fig. 8) of the rail under high cycle loads has not been obtained. The rail fatigue strength behaviour from an elastic to a plastic range could not be achieved. This information can be of great value in understanding the behaviour of not only the rail material but also other steel materials under large cyclic loads. Moreover, this information can be of significant value to complete a full model of the Smith-diagram, thus, contributes to the design and calculation of steel constructions.

2) Non-destructive methods in measuring the rail residual stress distribution in the cross-section

Due to the large dimensions of rail specimens and the limitation of residual stress measurement technologies as well as financial expenses, a completely non-destructive method to determine the rail residual stress distribution has not been achieved in this dissertation. On the one hand, the residual stress state in the rail cross-section must have been more or less disturbed by the destructive methods in determining the corresponding residual stresses. On the other hand, the precise control of these destructive procedures is difficult. This unavoidably results in un-evaluable scatterings. Moreover, the destructed rails cannot be further used for the determination of fatigue strength. Consequently, a direct connection between the residual stress distribution and the fatigue strength can only be indirectly obtained. To achieve a non-disturbed residual stress state of the rail cross-section and a direct connection between the residual stress distribution and fatigue resistance, a non-destructive residual stress measurement technology should be applied. Subsequently, the fatigue resistance of the measured rails should be determined. Through this, a direct connection between the residual stress distribution and the fatigue resistance of the rail could be accomplished.

3) Rail fatigue resistance under effects in operation

The tests in this work were carried out in laboratory-based settings according to historical tests and norms. They were limited to laboratory conditions. Among these conditions, there are two of them, which shall be further studied.

First, the tests were carried out under approximately 18°C room temperature, the maximum temperature increase of the rail was less than 30 K for all tests. This met the regulations according to DIN 50100 [38] so that the test frequency shall only have negligible effects on the fatigue strength of the specimen due to heating or a change in the damage mechanism due to high-stress velocity. However, rail fracture happens usually in winter when the temperature is relatively low, e.g. the rail temperature on Grubental Viaduct in Thüringen, Germany reached -20°C in the winter of 2018 [17]. The rail temperature can be much lower in countries such as Russia, Norway etc. According to the literature [46–48], low temperature can have either negative or positive impacts on steel fatigue resistance depending on different steel materials. To obtain the rail fatigue resistance under lower temperature, further experimental studies need to be carried out.

Second, the test load level was controlled to be stable during each test. These regulations are to ensure reliable and comparable test results under specific conditions, however, not realistic. Rails are under variable mean stress levels in static situations and stress ranges under different trains. The cross effect of these loads is difficult to estimate.

How different loads influence the actual rail resistance, in reality, needs further investigation.

4) Rail fatigue investigation based on continuum and fracture mechanics

Based on the fatigue test results, typical fatigue cracks were directly observed; a plastic shakedown behaviour of the steel material can be obtained when evaluating the stress-strain results; the existence of a threshold phenomenon of fatigue development can be identified. These issues can only be explained at a micro-scale or a meso-scale level of analysis. At the meso-scale level of analysis, the continuum mechanics and fracture mechanics have been often used to study and explain these phenomena. Results have been achieved on sophisticated small specimens [49–51]. However, these results depend greatly on the specific forms, material structures of the specimens and the testing conditions. Hence, there have been no reliable results concerning the specific conditions relating to railway tracks. Further studies can be carried out on rail specimens to achieve a comprehensive understanding of the rails under cyclic loads, and finally, contributing this meso-scale level knowledge to macro-scale level engineering benefits.

4.3 In the scope of TBI

In the broader scope of the TBI field, the four parts mentioned in section 1.2 are necessary to be systematically considered. To improve the design concept of railway and railway bridges considering TBI, it is essential to take into account the realistic rail resistance, actions and the corresponding flow of forces in the behaviour model that explains the transmission mechanism between actions and resistance (nonlinear modelling of TBI). Besides the resistance of the rail itself, the integrity of all related components, such as rail fasteners, track base plates etc. have to be assured, the derailment of trains must be avoided, the travel comfort shall be guaranteed.

Moreover, long-term monitoring shall be applied to actual structures to study the TBI considering the degradation of the system. Furthermore, stochastic analysis considering the random characteristics of material properties, structural errors, actions, initial irregularities, etc. needs to be carried out. Finally, a new comprehensive concept for TBI can be achieved.

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