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Market Risk: Exponential Weighting in the Value-At-Risk Calculation

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Abstract

When measuring market risk, credit institutions and Alternative Investment Fund Managers may deviate from equally weighting historical data in their Value-at-Risk calculation and instead use an exponential time series weighting. The use of exponential weighting in the Value-at-Risk calculation is very popular because it takes into account changes in market volatility (immediately) and can therefore quickly adapt to VaR. In less volatile market phases, this leads to a reduction in VaR and thus to lower own funds requirements for credit institutions. However, in the exponential weighting a high volatility in the past is quickly forgotten and the VaR can be underestimated when using exponential weighting and the VaR may be underestimated. To prevent this, credit institutions or Alternative Investment Fund Managers are not completely free to choose a weighting (decay) factor. This article describes the legal requirements and deals with the calculation of the permissible weighting factor. As an example we use the exchange rate between Euro and Polish zloty to estimate the Value-at-Risk. We show the calculation of the weighting factor with two different approaches. This article also discusses exceptions to the general legal requirements.

JEL classification: C22; G18; G28

Keywords: risk management, market risk, exponentially weighted moving average, weighting scheme, Value-at-Risk.

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1. Introduction

Calculating the Value-at-Risk (VaR) requires historical data, such as the last 500 daily returns of a risk factor (e.g. Hull, 2016). This data can be incorporated into the calculation with equal weighting or exponentially weighting, so-called EWMA-models for Exponentially Weighted Moving Average and supported by RiskMetrics (Alexander, 2008). When using the exponential weighting method, changes in the present are attached greater weight than changes that are further in the past. This has the advantage that the VaR can adapt more quickly to volatile market phases in order to adequately capture risks. If, on the other hand, the volatile phase is far in the past, it is taken into account less and the VaR mainly reflects the less volatile market phase. The VaR begins to "forget" the volatile time. This raises the question of which of the two approaches (equal weighting vs. exponential weighting) should be chosen to calculate the market risk, or how the weighting factor should be selected.

Example Exchange Rate Euro/Polish zloty – Value-at-Risk

Fig. 1 shows the daily log return rates of the exchange rate Polish Zloty/Euro. From this data, using the standard deviation, the Value-at-Risk was calculated for various historical time horizons (125 days, 250 days, 500 days and 1,000 days) with a level of significance of 99 percent each. The longer the historical time horizon chosen is, the lower the variation of the Value-at-Risk. A shorter historical time horizon (125 days) leads to stronger fluctuations of the VaR. In Fig. 01, the historical data was weighted equally. The legal requirements for the VaR calculation of banks are set out in Article 365 Regulation (EU) No 575/2013 (CRR, Capital Requirements Regulation): “1. The calculation of the value-at-risk number referred to in Article 364 shall be subject to the following requirements:

(a) daily calculation of the value-at-risk number;

(b) a 99th percentile, one-tailed confidence interval;

(c) a 10-day holding period;

(d) an effective historical observation period of at least one year except where a shorter observation period is justified by a significant upsurge in price volatility;

(e) at least monthly data set updates.

The institution may use value-at-risk numbers calculated according to shorter holding periods than 10 days scaled up to 10 days by an appropriate methodology that is reviewed periodically."

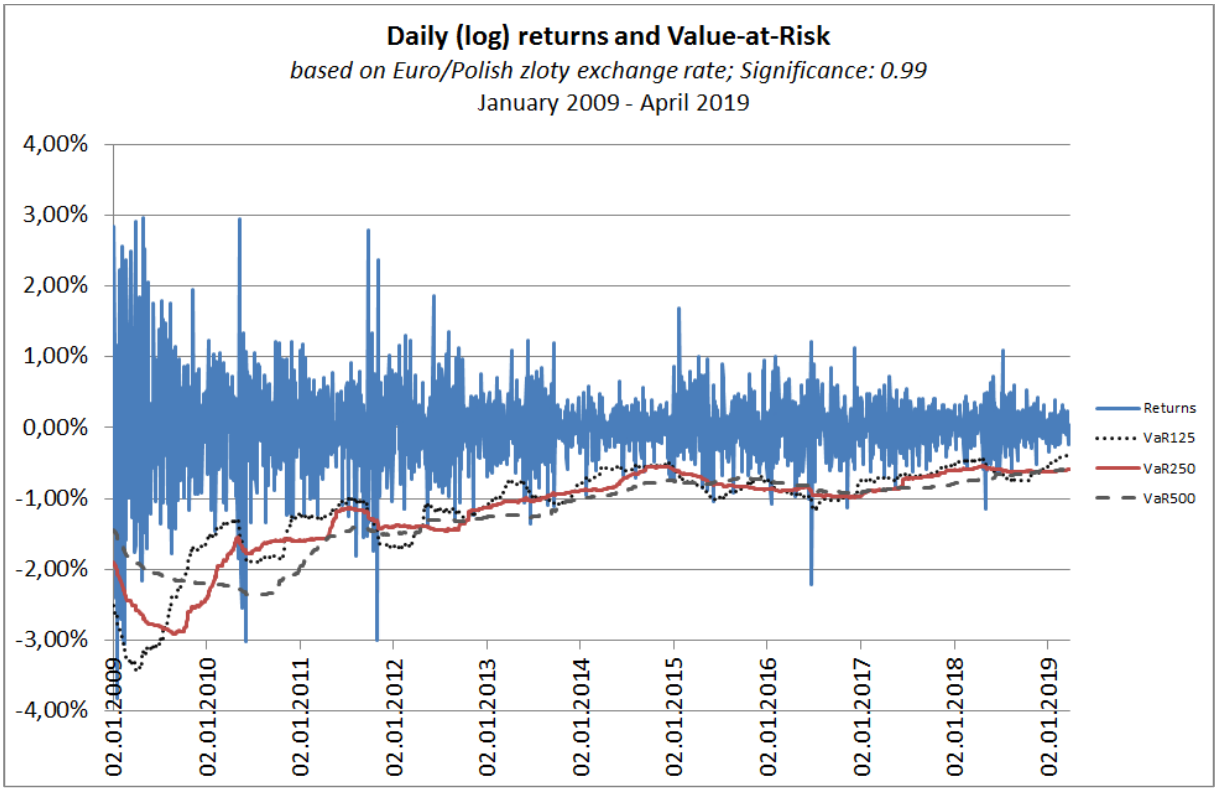

Fig. 1 Daily Log-Return for exchange Rate Euro/Zloty and Value-at-Risk (at 99 % Level of Significance), from September 2008 to January 2018, Data: ECB, 2018, own calculation.

The confidence level (99 %) is thus fixed by law and cannot be varied (99th percentile). This guarantees a level playing field for banks with regard to the VaR calculation and the associated capital requirements (Broll, Sobiech & Wahl, 2012). The VaR calculation is relevant for the

"Own Funds Requirements for Market Risk" (Title IV (EU) No. 575/2013) and especially Article 364 Regulation (EU) No 575/2013. A higher VaR leads to higher own funds requirements, so banks want to minimize own funds requirements and thus the VaR. However, the VaR cannot be calculated arbitrarily low. A wrongly chosen VaR leads to many outliers (see log-return under the VaR in Fig. 1). Too many regulatory back-testing outliers lead to "multiplication factors" for the VaR according to Article Article 366 (Tab. 1), that is, the outliers create a higher VaR and thus higher capital requirements. When calibrating the market risk model, banks face a trade off between reducing the VaR (low equity requirements), but also keeping the number of outliers low (for example by a high VaR) to avoid a (higher) multiplication factor for the VaR calculation. At a 99 % level of significance, 2.5 outliers are expected in 250 trading days.

Number of overshootings (Backtesting	**addend**
Fewer than 5	0,00
5	0,40
6	0,50
7	0,65
8	0,75
9	0,85
10 or more	1

Tab. 1 Number of overshootings and multiplication factors for VaR, Article 366 Table 1 Regulation (EU) No 575/2013.

Banks can freely choose the underlying historical observation period. However, it has to cover at least the last year (see Article 365). A shorter period, on the other hand, would be more flexible in responding to shocks (see Fig. 1 on the left hand side). A long observation period leads to a very inflexible VaR, but to stable forecasts regarding own funds requirements.

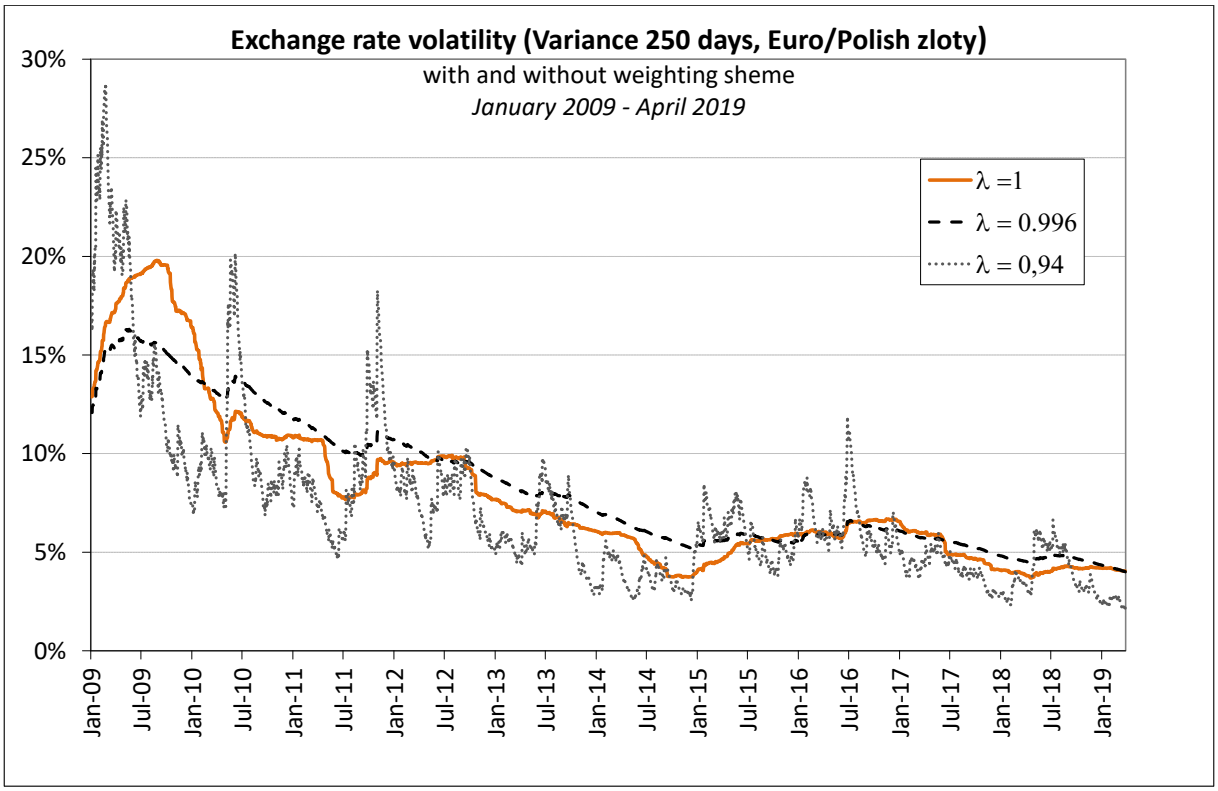

Fig. 2 Exchange rate volatility (Variance), based on the exchange rate Euro/Zlyoty, from September 2008 to January 2018, with and without a weighting scheme, Data: ECB, 2018, own calculation.

For banks and asset management companies (AIFM), weighting historical time series as part of market risk measurement is an adjustment parameter (in Germany e.g. KAGB, 2013). It is possible to deviate from an equal weighting of the historical data and instead use an exponential weighting scheme (in Germany e.g. DerivateV, 2013 and BaFin, 2013). This weighting means that the present fluctuation will have a higher impact on calculating volatility than past fluctuations. If the exchange rate fluctuates more strongly, this is immediately reflected in the market risk model with an exponential time series weighting (see Fig. 2). A risk increase (higher volatility) increases the Value-at-Risk and banks' capital requirements rise. In contrast, the outliers can be reduced by using an exponential weighting with an increase in volatility if necessary. Heightened risks can easily be taken into account when calculating the VaR in volatile market phases. Also, volatile market phases with an exponential time series weighting

become negligible. The risk can also be estimated to be lower than with an equal weighting of the time series, see Fig. 2.

2. Legal requirement

Capital management companies (AIFMs) and banks are not completely free to choose the weighting factor in market risk measurement. Article 365 Regulation (EU) No 575/2013 requires for market risk measurement: "1. The calculation of the value-at-risk number referred to in Article 364 [Regulation (EU) No 575/2013] shall be subject to the following requirements: […] (d) an effective historical observation period of at least one year except where a shorter observation period is justified by a significant upsurge in price volatility; […]". The European Banking Authority (EBA) explains what is meant by this (2015, p. 72.) in Article 38 (Observation period):

> "Where competent authorities verify that the VaR numbers are computed using an effective historical observation period of at least one year, in accordance with point (d) of Article 365(1) of Regulation (EU) No 575/2013, competent authorities shall verify that a minimum of 250 business days is used. Where institutions use a weighting scheme in calculating their VaR, competent authorities shall verify that the weighted average time lag of the individual observations is not less than 125 business days."

The EBA equally requires an effective historical observation period of at least one year, or alternatively 250 working days (equal to 250 observations). Of course, asset management companies or banks remain free to use a longer observation period. The requirement also allows the use of different weighting schemes. In the event of a deviation from the equilibrium, the weighted average must not fall below the period of six months (Balance Point, BP). This is achieved if no weighting is applied at all: in this case, all historical observations are included

in the calculation in the same way and with full weight. Therefore, the "balance point", which divides the weighted observations into two equal parts, is at the time of six months. The same requirement applies when using a weighting scheme. In practice, weighting schemes of a more current date are used. Recent observations have a higher impact on the VaR than observations in the past. Accordingly, the higher weightings for the more recent observations must be compensated by a greater number of more recent observations to ensure the six month balance point.

In summary, this leads to the following conclusions: a historical observation period of at least 250 days is required; within the context of exponential weighting, the balance point may not fall below the duration of six months (or 125 trading days), meaning that the most recent trading days must at least cover the last six months. If the weighting factor is set too low (i.e., the recent past is attached too much weight to), the balance point shifts too much in the direction of recent history, and eventually falls below the legal required length of at least 125 trading days.

3. Calculating the balance point

For exponential weighting, a weighting factor (decay factor) λ is determined. This factor λ is less than one ($\lambda = 1$ means equal weighting). Assuming a historical observation period of 501 trading days (hence 500 rates of change), the most recent rate of change will be indexed with zero and the oldest with 499, so that 500 observations will be included in the calculation. For the weighting factor of a given day t applies t^{λ}, thus the latest rate of change is included one hundred percent into the risk calculation: $0^{\lambda} = 1$. The smaller the decay factor chosen, the more the changes in the past are "discounted" and the less they are included in the risk calculation. Fig. 3 illustrates this with different decay factors. With a decay factor of 0.94, historical trading days which are more than four months (80 days) in the past, become virtually irrelevant for the

VaR calculation. To prevent this from happening, the legislator has issued the above requirements.

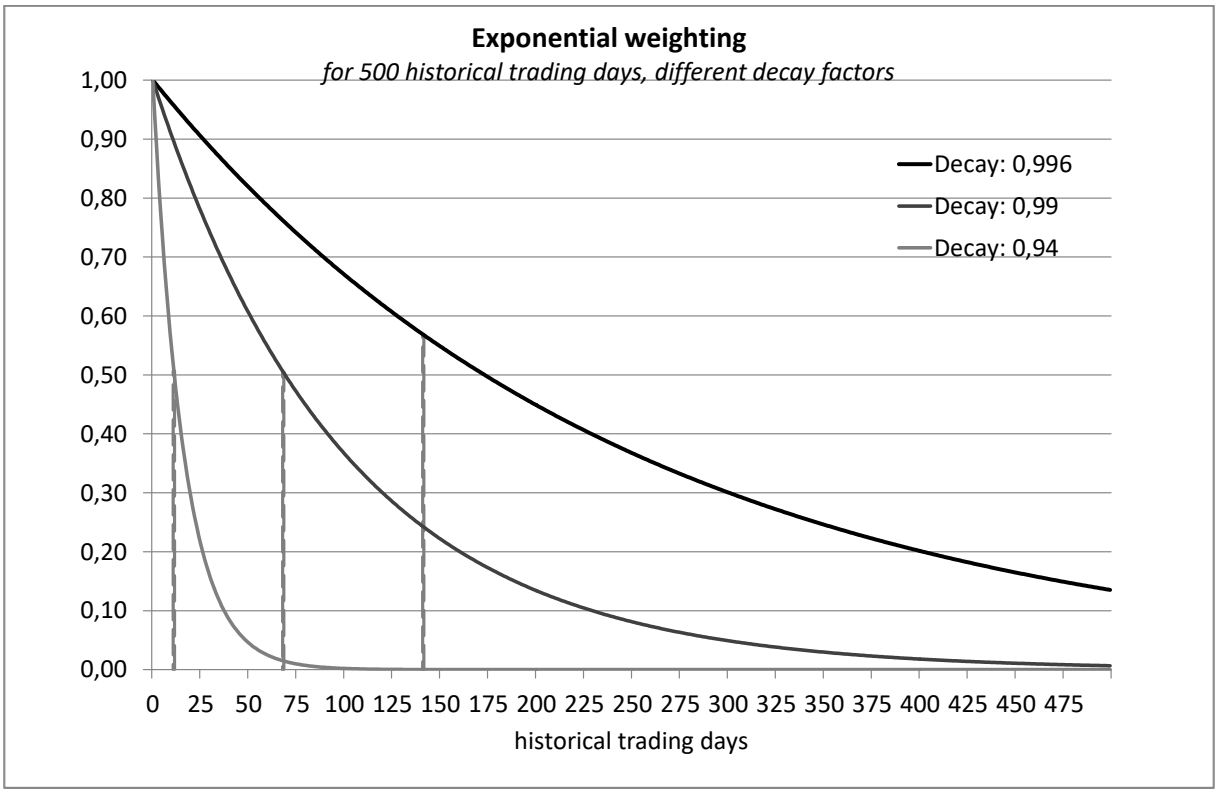

Fig. 3 Exponential weighting of historical trading days for different weighting factors, own calculation.

The balance point (BP) is the day on which the weighted observations are separated into two parts, whereas the more recent observations (e.g. trading days 0 to 141) equals the older observations (trading days 142 to 499 in Fig. 3) for a total of 500 historical data points. Since the younger observations are have a higher impact on VaR than the older ones, more trading days in the distant past must be included in the calculation. Expressed mathematically, the definite integral of data point 0 up to the balance point (BP) must correspond to the definite integral of trading days from the balance point (BP) to the last observation data point (499). For calculating the definite integral of the total period:

$\int_0^{499} e^{-(1-\lambda)*t} dt$,

applies, where *t* represents the trading day and λ the decay factor. The result is divided by two and the area is obtained from day 0 to the balance point (or from BP to the oldest observation). If, for example, a decay factor of λ = 0.996 is used, this results in a value of approx. 108 (or a total surface area of approx. 216). Now the day of the balance point (BP) can be calculated:

$108 = \int_0^{BP} e^{-0{,}004*t} dt$;

The balance point (BP) is at a decay factor of 0.996 on trading day *t* = 141 (and with 500 historical trading days). Since the recent past is more than a half year (125 trading days), the choice of this decay factor is permissible. With λ = 0.99, the balance point is reached after 68 trading days; this decay factor would therefore be inadmissible. The same applies to the decay factor of 0.94 (as proposed by RiskMetrics, see J.P.Morgan/Reuters 1996), where the balance point would be on the eleventh trading day and would therefore be completely unacceptable.

According to the legal requirements, an exponential weighting for a historical observation period of 250 days is not allowed for the VaR calculation since the balance point would always be less than 125 days. Longer (than 500 days) historical observation periods allow only a slightly lower decay factor than 0.996. In addition, it would not make sense to outweigh a smaller decay factor with a longer time series.

Weighting of trading days

In addition to the previous approach, weighting or - for the calculation of the balance point - cumulative weighting can be considered for each trading day. If a decay factor λ = 0.996 is chosen for 500 historical trading days, the initial weight α will be $\alpha = (1-\lambda)/(1-\lambda)^{500}) = 0.46$ %, which is well above 0.2 % and would result in equal weight. If all data are equally weighted,

each trading day is weighted at 0.2 % for a historical observation period of 500 days and a straight line is given for the presentation of the cumulative equal weighted trading days, Fig. 4.

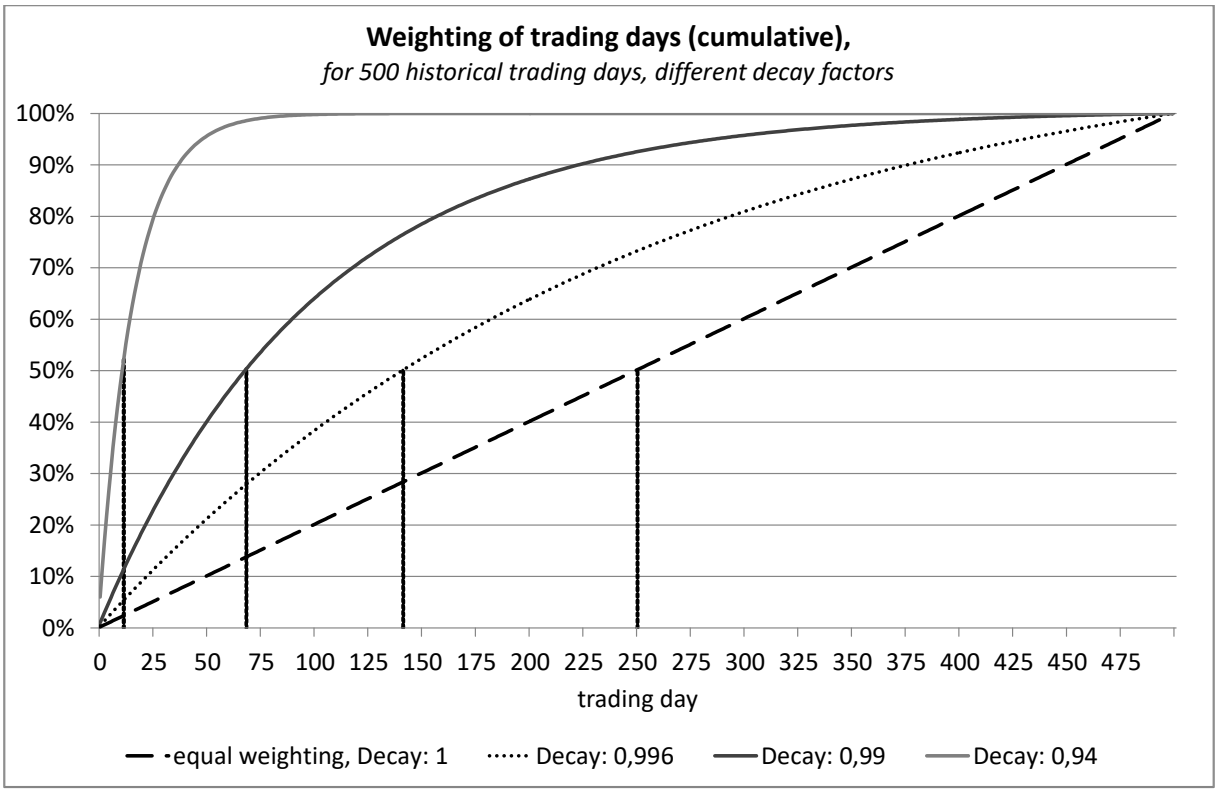

Fig. 4 Cumulative weighting of historical trading days with different weighting factors, own calculation.

With a weighting of $\lambda = 0.996$, the most recent day has a weight of 0.46 % ($\alpha*\lambda^0 = 0{,}46\ \%*0{,}996^0 = 0.46\ \%$), while the oldest observation weighs only $\alpha*\lambda^{499} = 0.0046*0.996^{499} = 0.06\ \%$ assigned. The sum of all daily weights (days 0 to 499) advises one. At the balance point the younger and the older past cumulatively each weigh about 50 %. Due to the integerity problem (here a full day), the recent past is slightly larger than 50 % up to the balance point. With a decay factor of 0.996, the balance point would be on day 141 of the time series: on this day, the cumulative weight of trading days 0 to 141 (about 50.16 %) corresponds to the weight of the trading days 142 to 499 (almost 50 %), Fig. 4. If the decay

factor were chosen to be smaller, the weight of the recent past is higher ($\lambda = 0.99$ or $\lambda = 0.94$) and the balance point in Fig. 04 would continue to shift to the left. Thus, the recent past would be overweighted and the requirement of the CRR (balance point after 125 trading days) would be violated.

4. Exemption

The German Derivatives Regulation provides for an exemption for Alternative Investment Fund Manager (AIFM): it is possible to derogate from the weighting scheme due to exceptional market conditions and the prior approval of Federal Financial Supervisory Authority (BaFin, 2013). This would allow for an effective balance point of less than six months under certain circumstances when using exponential weighting. Such exponential weighting, which results in a lower balance point, may be allowed in the context of extreme market conditions characterized by exceptionally volatile markets. As recent observations become more influential, the volatility estimate will increase more rapidly in increasingly volatile markets. On the other hand, such a weighting also leads to a greater drop in the volatility estimate in calming markets. In this respect, a precise balance must be made here and the balance point at least not too short. In order to prevent the exponential weighting from being used to widen the market risk limit, the BaFin (2013) requires prior approval. The suitability of the model must be substantiated in the corresponding application. The BaFin can combine the approval with conditions.

There is also an exemption in the banking sector. No. 2 Article 38 EBA (2015, p.72) explains: “Where, according to point (d) of Article 365 (1) of Regulation (EU) No 575/2013 the calculation of the VaR is subject to an effective historical observation period of less than one year, competent authorities shall verify that the institution has in place procedures to ensure

that the application of a shorter period results in daily VaR numbers greater than daily VaR numbers computed using an effective historical observation period of at least one year."

As mentioned above, the exemption also applies in the banking sector. However, it is questionable whether banks will voluntarily make use of this exemption in stressed market phases: a stronger weighting of a recent volatile past will lead to a higher VaR and thus to increased own funds requirements (Article 364 CRR Regulation No. 575/2013). For this reason, there is no incentive to recognize the risk for banks in stressful market phases at this point. The exemption should not result in the weighted VaR being lower than the unweighted VaR (EBA 2015, p. 72).

The requirements of the German Derivatives Regulation and the CRR are intended to ensure that the past is not forgotten too quickly in the VaR calculations. A hybrid approach would also be interesting for capital management companies or banks, with the VaR calculated using equally weighted and exponentially weighted data. This means that two risk ratios would have to be determined, which would involve enormous computational effort. However, the calculation would be helpful for investment management companies to obtain a second reference point for the risk assessment; i.e. a risk assessment for a longer history (equal weighting) and an estimate for the most recent past (weighted time series).

Banks calculate the market risk potential under stress conditions as a second reference point in accordance with Article 365 (2) CRR Regulation No. 575/2013 - but here without the inclusion of weighted time series data (EBA 2012). However, there is no hybrid approach, as this would lead to leaps in the capital requirement. In addition to the increased calculation effort for a VaR with an exponential time series weighting, the VaR is "biased", as it does not simply serve as a risk indicator, but is used in the banking sector for the capital requirements: a volatile VaR is not always desirable from this point of view. However, a more flexible VaR can also help to reduce the outliers in backtesting the VaR and thus reduce the capital requirements.

5. Conclusions

In order to satisfy the regulatory requirements for market risk measurement (EWMA models), the decay factor λ should not be too small; at 500 historical rates of change, it must be around 0.996. This fulfills the legal requirement of a balance point after 125 business days. A small weighting factor of about 0.94 is not permitted by law. An exponential weighting of the historical time series over an observation period of 250 days is also inadmissible.

Even a weighted time series will not be able to replicate an increase in volatility due to behavioral risks in the risk model within seconds; however, risks following a shock can be processed more quickly and, if necessary, more adequately in the market risk model. An advantage in the risk assessment seems to be calculating the VaR with a weighted and an unweighted historical time series, as there are two reference points for the risk assessment.

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