TECHNISCHE UNIVERSITÄT DRESDEN

CEPIE Working Paper No. 02/20

Center of Public and International Economics

ENDOGENOUS SIGMA-AUGMENTING TECHNOLOGICAL CHANGE: AN R&D-BASED APPROACH

August 2020

Alexander Kemnitz
Michael Knoblach

Editors: Faculty of Business and Economics, Technische Universität Dresden.

This paper is published on the Open Access Repository Qucosa.
The complete Working Paper Series can be found at the CEPIE Homepage | EconStor | RePEc

ISSN 2510-1196

Endogenous Sigma-Augmenting Technological Change: An R&D-Based Approach*

Alexander Kemnitz†
Technische Universität Dresden

Michael Knoblach‡
Technische Universität Dresden

August 11, 2020

Abstract There is now increasing evidence that for the U.S. economy, the elasticity of substitution between capital and labor, σ, is rising over time. To account for this, we propose a microfounded model, where the evolution of σ and, hence, the shape of the aggregate production function occur endogenously. We develop a Schumpeterian growth model in which firms can undertake R&D activities that stochastically lead to the discovery of production technologies characterized by a higher elasticity of substitution between capital and labor. Improved possibilities for factor substitution mitigate the diminishment of the marginal product of capital and spur capital accumulation. Due to successful innovations, the steady state of the economy entails higher levels of the capital stock and the output good. Moreover, our numerical simulations show that the timing of innovations is important: two economies with the same steady-state elasticity of substitution between capital and labor can differ in terms of their steady-state levels of the capital stock and the output good.

JEL classification: E24; J24; J31; O33; O41

Keywords: Monopolistic competition; Endogenous elasticity of substitution; Functional normalization; Schumpeterian growth model

*We thank Fabian Stöckl, Patrick Zwerschke and the participants of the Brown Bag Seminar at the TU Dresden for their valuable comments and intensive discussions that significantly improved the paper.

†Faculty of Business and Economics, Technische Universität Dresden, 01062 Dresden, Germany. E-mail: alexander.kemnitz@tu-dresden.de

‡Corresponding Author. Faculty of Business and Economics, Technische Universität Dresden, 01062 Dresden, Germany. E-mail: michael.knoblach@tu-dresden.de

1 Introduction

The neoclassical growth model, originating from the seminal works of Solow (1956) and Swan (1956), highlights the role of capital accumulation in economic growth. One of its most important attributes is the neoclassical aggregate production function. This function describes the production of a homogeneous aggregate output good as a combination of two inputs capital, K, and labor, L. It is characterized by constant returns to scale, allows for substitution between capital and labor, and exhibits positive but diminishing returns to each input. These diminishing returns imply that leaving population growth or factor-augmenting technological progress aside, the growth rates of capital accumulation and output decelerate over time. However, their levels are the higher, the slower the decrease in returns to capital.

The degree to which the increase in the capital stock diminishes its marginal product depends crucially on the elasticity of substitution between capital and labor, σ. The higher this elasticity, the easier incremental capital can substitute labor, and thus, the lower the minimum values of both factors required to produce a certain amount of output. Consequently, an increase in the elasticity of substitution between capital and labor increases the efficiency of both factors in production in a manner similar to factor-augmenting technological progress (de La Grandville, 2007). Thus, as demonstrated in the seminal works of de La Grandville (1989) and Klump and de La Grandville (2000), an increase in the degree of capital-labor substitutability can spur growth in the neoclassical growth model.

Typically, the elasticity of substitution between capital and labor is considered an exogenously given, time-invariant deep parameter. However, this assumption is at odds with recent empirical evidence summarized by Knoblach and Stöckl (forthcoming), indicating that for the U.S. economy, σ rises over time. This finding is corroborated by the data underlying the recent meta-regression analysis on σ of Knoblach et al. (2020). On the basis of $2,419$ estimates of the elasticity of substitution between capital and labor for the U.S. economy, gathered from 77 studies published between 1961 and 2017, the authors estimate a long-run meta-elasticity for the aggregate economy in the range of 0.45 - 0.87.

As the dataset comprises estimates of σ based on various samples reflecting different time periods, it can be used for testing for a time trend in the size of the reported estimates. For this purpose, we extend the estimation in Knoblach et al. (2020) by including midyear, i.e., the mean year of the time span covered by the respective estimation, as an additional regressor. The corresponding coefficient value thus denotes the change in the expected estimate of σ, induced by an increase in the variable midyear. Table 1 provides the meta-regression results for this coefficient for weighted least squares (WLS), random effects (RE), and fixed effects (FE) models. Models (1), (3), and (5) pool all observations, i.e., industry-specific and aggregate economy estimates of σ, whereas Models (2), (4), and (6) utilize only aggregate economy estimates. As seen from Table 1, consistent over all model specifications, the regression coefficient for midyear is statistically significant at least at the 10% level and ranges from 0.00292 to 0.00475. Our results thus indicate that studies utilizing more recent data tend to yield higher estimates of σ. This finding provides

further evidence that the elasticity of substitution between capital and labor for the U.S. economy is rising over time.

Table 1: Results of coefficient midyear for WLS, RE, and FE models; inverse standard error weighting

	(1) WLS	(2) WLS	(3) RE	(4) RE	(5) FE	(6) FE
midyear	0.00387**	0.00475***	0.00413***	0.00313**	0.00331*	0.00292**
	(0.00162)	(0.00130)	(0.00153)	(0.00132)	(0.00190)	(0.00148)
Observations	2,419	853	2,419	853	2,419	853

Note: The results for other variables are not shown. WLS with clustered standard errors in parentheses; *** p<0.01, ** p<0.05, and * p<0.1.

Motivated by this evidence, this paper formalizes the notion of "sigma-augmenting technological change" (Klump et al., 2012, p. 793) by presenting a novel microfoundation approach to endogenize an increase in the elasticity of substitution between capital and labor.[1] To study the evolution in σ and its impact on growth and transitional dynamics of the economy, we develop a model of "creative destruction" in the spirit of Schumpeter (1934, 1942) and Aghion and Howitt (1992). It comprises a final-good, an intermediate good, and an R&D sector. The final good is produced fully competitively and requires intermediate goods as inputs. Intermediate goods in turn are produced under monopolistic competition as a combination of capital and labor based on a normalized constant elasticity of substitution (CES) production function, as developed in de La Grandville (1989) and Klump and de La Grandville (2000).

Striving for temporary monopoly profits, research firms can invest resources in R&D which stochastically leads to the discovery of new intermediate production technologies featuring higher elasticities of substitution between capital and labor. Thus, the shape of the aggregate production function results endogenously in our model.[2] These increases in σ follow a quality-ladder approach in the spirit of the seminal works of Grossman and Helpman (1991a,b), Segerstrom (1991), and Aghion and

[1]To the best of our knowledge, only a few approaches addressing changes in substitution possibilities over time exist. Miyagiwa and Papageorgiou (2007) and Xue and Yip (2013) develop a two-sector growth model, in which the aggregate elasticity of substitution between capital and labor changes as a by-product of sectoral reallocation. León-Ledesma and Satchi (2019) obtain an increase in σ over time by imposing monetary frictions on the choice of technology. Moreover, the literature on the variable elasticity of substitution (VES) production function suggests a parametric relationship between σ and the capital-labor ratio. See Karagiannis et al. (2005) for an extensive review of the related literature. Investment-induced changes in the substitutability between productive factors are also considered in energy and resource economics; see Fenichel and Zhao (2015) and Stöckl (2020). However, these approaches lack proper microfoundations.

[2]Therefore, our model is in contrast to other approaches to the microfoundation of aggregate production functions within the idea-based endogenous technology choice framework (see Jones, 2005, Growiec, 2008a,b, 2013, 2018, Matveenko, 2010, 2011, and Matveenko and Matveenko, 2015) or the mechanization framework (see Nakamura and Nakamura, 2008 and Nakamura, 2009, 2010).

Howitt (1992). To model the transition between intermediate production technologies, we build upon Antony (2009b,a, 2010) and provide a novel application of the functional normalization of CES production functions. The introduction of a new intermediate production technology with a higher σ mitigates the diminishment of the marginal product of capital as the economy grows and spurs capital accumulation. However, the incentives to innovate change during the course of development, subsiding eventually when the capital stock is sufficiently large. As a consequence, the total number of periods with positive R&D investments and thus successful innovations is finite, and the economy reaches a steady state. Each increase in the elasticity of substitution results in higher steady-state levels of the capital stock and the output good. In this respect, our multisectoral Schumpeterian growth framework with endogenous sigma-augmenting technological change is in line with the seminal works of de La Grandville (1989) and Klump and de La Grandville (2000).

To illustrate the effect of an endogenous increase in the substitutability between capital and labor on growth and the transitional dynamics of the economy, we conduct numerical simulations. We show that although a higher σ is typically associated with a higher steady-state level of the output good and the capital stock, two economies characterized by the same elasticity of substitution between capital and labor in the steady state can be characterized by different steady-state levels of the capital stock and the output good. Since successful innovations are more beneficial when the marginal product of capital is higher, the (stochastic) timing of innovations becomes an important element in our model.

The remainder of this paper is organized as follows. In Section 2, we set up a simple benchmark model of monopolistic competition in the spirit of Spence (1976) and Dixit and Stiglitz (1977). In this model, intermediates are produced as a combination of capital and labor. While technology and, hence, σ are constant in the benchmark model presented in Section 2, in Section 3, we introduce an R&D sector, in which firms can undertake research activities to discover production functions with a higher elasticity of substitution between capital and labor. In Section 4, we present a quantitative analysis of the model. We end with some concluding remarks and suggestions for future research in Section 5.

2 The benchmark model

In this section, we introduce a simple benchmark model of monopolistic competition in the spirit of Spence (1976) and Dixit and Stiglitz (1977). In this model, a homogeneous final good is produced as a combination of differentiated intermediate goods. We consider a closed economy in an infinite discrete-time horizon. To simplify the analysis, we abstract from a household sector and solely focus on the production structure of the economy. The production technology of the economy is assumed to be exogenously given and constant over time.

In what follows, we first introduce our model framework followed by a solution of the temporary and the dynamic equilibria. This serves as a benchmark for the subsequent section, where we introduce an additional competitive R&D sector, in

which research firms can invest resources to discover new intermediate production technologies.

2.1 Model setting

Consider a closed economy in an infinite sequence of periods $t \in \{0, 1, 2, ..., \infty\}$. In each period, a homogeneous final good, X_t, is produced under perfect competition as a combination of a countably finite number of N horizontally differentiated intermediate goods, indexed by i. Analogous to the Dixit and Stiglitz (1977) specification for preferences, we consider the following aggregate production function:

$$X_t = \left(\sum_{i=1}^{N} Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

where $Y_{i,t}$ measures the amount of intermediate i used in final good production. Intermediates can be substituted imperfectly for one another. Here, the parameter $\epsilon > 1$ is a measure of the elasticity of substitution between the differentiated intermediates.

Following Seegmuller (2009), each intermediate i is produced by a single monopolistically competitive firm as a combination of capital and labor.[3] In the benchmark model, each intermediate firm has free access to the intermediate production technology, as specified by the following "normalized" CES production function:[4]

$$Y_{i,t} = F(K_{i,t}, L_{i,t}) = Y_{i,0} \left[\alpha_{i,0} \left(\frac{K_{i,t}}{K_{i,0}} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_{i,0}) \left(\frac{L_{i,t}}{L_{i,0}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

In (2), the quantities of capital and labor used in the production of intermediate i are denoted by $K_{i,t}$ and $L_{i,t}$, respectively. Moreover, σ refers to the elasticity of substitution between capital and labor.[5] Based on the results provided by Knoblach et al. (2020), we posit the following:

Assumption 1. Within the production of intermediates, capital and labor are gross complements, that is,

$$\sigma \in (0, 1).$$

In this section, we adopt the standard normalization approach developed in the works of de La Grandville (1989) and Klump and de La Grandville (2000).[6] As emphasized by Klump and Preissler (2000), the basic idea behind normalization is to

[3] As a result, N also refers to the exogenous and constant number of intermediate firms.

[4] A derivation of production function (2) can be found in Appendix A.1. For a comprehensive survey of the concept of normalization, see Klump et al. (2012).

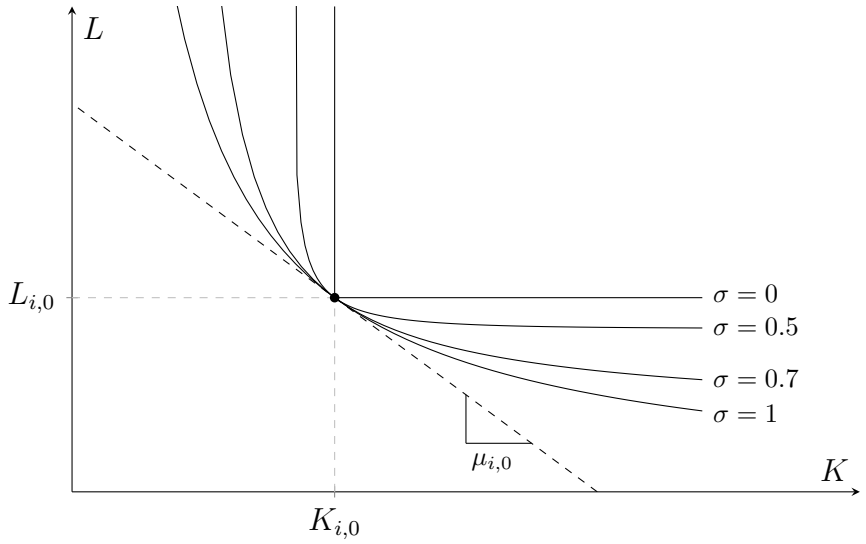
[5] Similar to all standard CES functions, (2) nests a Cobb-Douglas function for $\sigma \to 1$, a Leontief function for $\sigma \to 0$ and a von Neumann production function for $\sigma \to \infty$.

[6] This approach will be extended to functional normalization in the subsequent section.

form families of CES production functions, the members of which are distinguished only by different values of σ by providing a common baseline point. This baseline point can be a certain point in time, e.g., $t = 0$, characterized by arbitrary chosen values for the quantities of capital, $K_{i,0}$, and labor, $L_{i,0}$, used in intermediate production, the intermediate output level, $Y_{i,0}$, and the marginal rate of technical substitution, $\mu_{i,0}$. In the following, we refer to this point as the initial point of normalization. As visualized by Figure 1, in this point, isoquants of intermediate CES production functions with different values of σ but the same baseline values are tangent and thus belong to the same family. Consequently, while independent of σ at the initial point of normalization, both the intermediate output and the marginal rate of technical substitution are functions of the elasticity of substitution when the economy departs from the baseline values $K_{i,0}$ and $L_{i,0}$.[7] Letting $F_{K,t} \equiv [\partial Y_i/\partial K_i]_t$ and $F_{L,t} \equiv [\partial Y_i/\partial L_i]_t$ denote the marginal products of capital and labor in intermediate production at period t, the parameter $\alpha_{i,0} = \frac{F_{K,0}K_{i,0}}{F_{K,0}K_{i,0}+F_{L,0}L_{i,0}}$ denotes the capital share in intermediate production at the initial point of normalization.

In what follows, we impose symmetry among intermediate production technologies. Thus, we set $K_{i,0} = K_0/N$, $L_{i,0} = L_0/N$, $Y_{i,0} = Y_0$, and $\alpha_{i,0} = \alpha_0$, for all i, where K_0 and L_0 refer to the economy-wide endowment of capital and labor at the initial point of normalization.[8]

Figure 1: Family of isoquant curves for the normalized intermediate CES production function (2)



[7]This feature of normalization has motivated a growing strand of theoretical literature on the comparative statics of growth models relying on different values of σ. See, e.g., Klump and de La Grandville (2000), Klump and Preissler (2000), Miyagiwa and Papageorgiou (2003), Papageorgiou and Saam (2008), Turnovsky (2008), Mallick (2010), Irmen (2011), Xue and Yip (2012), and Gómez (2020). Empirical applications of the normalized CES production function can be found, among others, in Klump et al. (2007a,b), Young (2013), Herrendorf et al. (2015), León-Ledesma et al. (2015), Cantore et al. (2017), Stewart (2018), and Stewart and Li (2018).

[8]In the following, we focus on the economically relevant cases $K_0 > 0$ and $L_0 > 0$.

Labor is supplied inelastically. As the main focus of this paper is on the development of the relation between the marginal product of capital, the aggregate capital stock, and σ, we treat the economy-wide endowment of labor, L_t, as constant over time. Furthermore, along the lines of Solow (1956), we impose a constant rate of capital depreciation, $\delta \in (0,1)$, and a constant, exogenous fraction of final output, $s \in (0,1)$, to be saved and invested every period. The law of motion for the aggregate capital stock, K_t, is thus as follows:

$$K_{t+1} = sX_t + (1-\delta)\,K_t. \tag{3}$$

The aggregate resource constraint of the economy requires at each period of time the total amount of the final good, X_t, must be equal to consumption, C_t, gross investment, I_t, and aggregate R&D expenditures, Z_t (which are zero in this benchmark model but become positive later, so we introduce them here to abbreviate later exposition):

$$X_t = I_t + C_t + Z_t \tag{4}$$

Finally, market clearing for capital and labor requires the following:

$$\sum_{i=1}^{N} K_{i,t} = K_t \qquad \text{and} \qquad \sum_{i=1}^{N} L_{i,t} = L \tag{5}$$

In the following, capital and labor are fully mobile across sectors. Thus, the factor prices for capital, $r_{i,t}$, and labor, $\omega_{i,t}$, are equalized across intermediate sectors, i.e., $r_{i,t} = r_t$ and $\omega_{i,t} = \omega_t$, respectively.

Having presented the structure of the benchmark model, in the following, we can turn to the solution of the temporary and dynamic equilibrium of the benchmark economy.

2.2 Temporary and dynamic equilibrium

We start with the definitions of the temporary and dynamic equilibrium of the benchmark economy.

Definition 1. A temporary equilibrium of the benchmark economy, given the state variable K_t, is a set of intermediate demands $Y_{i,t}$ that maximizes the profit of the final good producer and results in aggregate output X_t, intermediate good prices $p_{i,t}$ that maximize the profit of each intermediate monopolist, and factor demands $K_{i,t}$ and $L_{i,t}$, with factor prices r_t and w_t, that clear factor markets.

Definition 2. A dynamic equilibrium of the benchmark economy is given by a sequence of its temporary equilibria, where K_t develops according to (3).

In the following, we suppress time subscripts for legibility when this causes no ambiguity. Based on (1), the profit maximization of final good producers leads to the following ratio of first-order conditions between any pair of intermediates $i \neq j$:

$$\left(\frac{Y_i}{Y_j}\right)^{-\frac{1}{\epsilon}} = \frac{p_i}{p_j} \qquad \forall\ i, j \in N, \tag{6}$$

where p_i is the price of intermediate Y_i, and p_j is the price of intermediate Y_j. If N is sufficiently large so that each intermediate producer acts as though his/her behavior does not influence that of other monopolists, then Y_j and p_j in equation (6) can be replaced by aggregate output X and the price index of the final good P, respectively.[9] We choose the latter as the numeraire, such that

$$P \equiv 1 = \left(\sum_{i=1}^{N} p_i^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{7}$$

holds in each period.[10] The profit of each intermediate good producer i is then given by

$$\pi_i = \left(\frac{1}{p_i}\right)^{\epsilon} X(p_i - \psi) \qquad \forall\ i \in N, \tag{8}$$

where ψ denotes the marginal cost necessary to produce one unit of intermediate good i according to production function (2). For given factor prices of capital, r, and labor, ω, the marginal costs result in the following:[11]

$$\psi = \frac{1}{Y_0 N}\left[\alpha_0\left(\frac{r}{\alpha_0}\right)^{1-\sigma} K_0^{1-\sigma} + (1-\alpha_0)\left(\frac{\omega}{1-\alpha_0}\right)^{1-\sigma} L_0^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{9}$$

Subsequently, maximizing (8) with respect to p_i implies that the price of each intermediate i corresponds to a constant mark-up over marginal costs:

$$p_i = \left(\frac{\epsilon}{\epsilon - 1}\right)\psi, \qquad \forall\ i \in N. \tag{10}$$

Due to the symmetry of the intermediate production function (2), all intermediate producers face the same marginal costs. It follows that all intermediates are offered

[9] This simplification goes back to Dixit and Stiglitz (1977). See Yang and Heijdra (1993) for a detailed discussion related to its justification.

[10] See Appendix A.2 for a derivation of the price index.

[11] The derivation of marginal costs can be found in Appendix A.3.

at the same price, which, according to (7), is: $p_i = p = N^{\frac{1}{\epsilon-1}}$ for all i. Moreover, equation (6) implies that the producers of the final good demand the same quantity of each intermediate $Y_i = Y$ for all i. Then, from the market-clearing condition (5), we obtain that factor inputs of capital and labor are equally distributed across intermediate firms according to the following:

$$K_i = \frac{K}{N} \qquad \text{and} \qquad L_i = \frac{L}{N} \tag{11}$$

Due to this symmetry, equation (11) implies that for a given aggregate capital stock K and total population L, the equilibrium intermediate and aggregate output levels are unequivocally determined by the following:

$$Y_i = Y = Y_0 \left[\alpha_0 \left(\frac{K}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{L}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \qquad \forall\ i \in N \tag{12}$$

$$X = G(K,L) = N^{\frac{\epsilon}{\epsilon-1}} Y_0 \left[\alpha_0 \left(\frac{K}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{L}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{13}$$

In equation (13), the factor $N^{\frac{\epsilon}{\epsilon-1}}$ measures the degree of the returns to specialization, that is, the benefits from distributing the available factors capital and labor to a larger number of intermediate firms.[12]

Combining equation (11) with the relative factor demand of intermediate producers, we obtain the following:

$$\left(\frac{r}{w} \right) = \frac{\alpha_0}{1-\alpha_0} \left(\frac{L_0}{K_0} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{K}{L} \right)^{-\frac{1}{\sigma}} \tag{14}$$

According to (14), cost minimization in intermediate production requires relative factor prices to be equal to relative marginal products, i.e., the marginal rate of technical substitution.[13] Combining this result with the price index (7) and the mark-up equation (10), we can derive an explicit expression of the factor prices r and ω:

[12] Conceptually, this is identical to the "love for variety" preferences that result from a CES utility function as applied in the seminal work of Dixit and Stiglitz (1977). Note that for $N, \epsilon > 1$, the factor $N^{\frac{\epsilon}{\epsilon-1}}$ is always greater than or equal to unity, as we have $\frac{\partial N^{\frac{\epsilon}{\epsilon-1}}}{\partial \epsilon} = -\frac{1}{(\epsilon-1)^2} N^{\frac{\epsilon}{\epsilon-1}} \ln N < 0$, $\forall N > 1$, with $\lim_{\epsilon \to 1} N^{\frac{\epsilon}{\epsilon-1}} = \infty$, $\forall N > 1$ and $\lim_{\epsilon \to \infty} N^{\frac{\epsilon}{\epsilon-1}} = N$, $\forall N$. Furthermore, we have $\frac{\partial N^{\frac{\epsilon}{\epsilon-1}}}{\partial N} = \frac{\epsilon}{\epsilon-1} N^{\frac{1}{\epsilon-1}} > 0$, $\forall N$. See Benassy (1996, 1998) and Koeniger and Licandro (2006) for further discussion.

[13] See Appendix A.3 for a derivation.

$$(15)\quad r = \frac{Y_0 N^{\frac{\epsilon}{\epsilon-1}}}{\xi}\left[\alpha_0^{\sigma} K_0^{1-\sigma} + (1-\alpha_0)^{\sigma}\left(\frac{1-\alpha_0}{\alpha_0}\left(\frac{L_0}{K_0}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{K}{L}\right)^{\frac{1}{\sigma}}\right)^{1-\sigma} L_0^{1-\sigma}\right]^{\frac{1}{\sigma-1}}$$

$$(16)\quad \omega = \frac{Y_0 N^{\frac{\epsilon}{\epsilon-1}}}{\xi}\left[\alpha_0^{\sigma}\left(\frac{\alpha_0}{1-\alpha_0}\left(\frac{L_0}{K_0}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{L}{K}\right)^{\frac{1}{\sigma}}\right)^{1-\sigma} K_0^{1-\sigma} + (1-\alpha)^{\sigma} L_0^{1-\sigma}\right]^{\frac{1}{\sigma-1}}$$

where $\xi = \frac{\epsilon}{\epsilon-1}$ is the price markup, as derived in equation (10). As seen from (15) and (16), factor prices vary with the economy-wide endowment of capital and labor. Let $G_K \equiv \partial X/\partial K$ and $G_L \equiv \partial X/\partial L$ denote the marginal product of capital and labor in the final good production, respectively. Then, we have $r = \left(\frac{\epsilon-1}{\epsilon}\right) G_K = \left(\frac{\epsilon-1}{\epsilon}\right) N^{\frac{\epsilon}{\epsilon-1}} F_K$ and $\omega = \left(\frac{\epsilon-1}{\epsilon}\right) G_L = \left(\frac{\epsilon-1}{\epsilon}\right) N^{\frac{\epsilon}{\epsilon-1}} F_L$ with $\frac{\epsilon-1}{\epsilon} < 1$. Since G_L and F_L as well as G_K and F_K differ only by the scalar $N^{\frac{\epsilon}{\epsilon-1}}$, the properties of G_K and G_L laid out in the following proposition apply equally with respect to F_K and F_L.

Proposition 1. *(Properties of marginal products G_K and G_L)*

1. *In the final good production, the marginal product of capital, G_K, (labor, G_L,) is positive and diminishing (increasing) in K.*

2. *At the initial point of normalization, the marginal products of capital, G_K, and labor, G_L, are independent of the value of σ since*

$$(17)\qquad G_{K,0} = \left[\frac{\partial X}{\partial K}\right]_0 = \alpha_0 N^{\frac{\epsilon}{\epsilon-1}} \frac{Y_0}{K_0},$$

$$(18)\qquad G_{L,0} = \left[\frac{\partial X}{\partial L}\right]_0 = (1-\alpha_0) N^{\frac{\epsilon}{\epsilon-1}} \frac{Y_0}{L_0}.$$

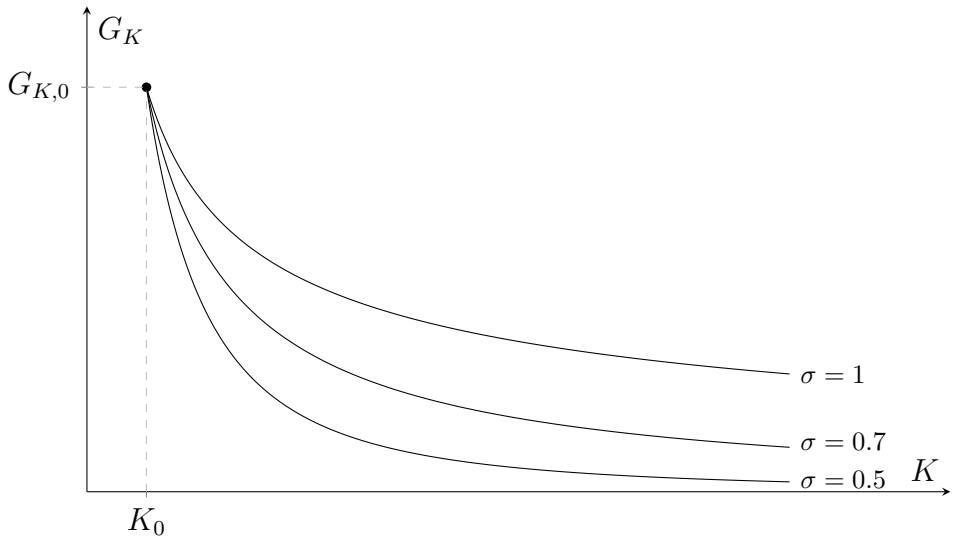
3. *For all $K > K_0$ ($K < K_0$), G_K (G_L) is increasing in the value of σ.*

Proof: See Appendix A.4 ■

Statement 1 of Proposition 1 describes two neoclassical properties of aggregate production function (13). Both properties hold for any constant returns to scale CES production function and a strictly positive value of σ. Given these conditions, $\partial G_K/\partial K > 0$ and $\partial G_L/\partial L > 0$ follow directly from (15) and (16). Moreover, the marginal product of capital (labor) is downward (upward) sloping in the aggregate capital stock since $\partial^2 X/\partial K^2 < 0$ and $\partial^2 X/\partial L \partial K > 0$. Statement 2 says that at the initial point of normalization, a change in σ has no effect on the marginal products of capital or labor. It holds irrespective of the chosen benchmark values $K_{i,0}$, $L_{i,0}$, $Y_{i,0}$, and $\mu_{i,0}$. The last statement of Proposition 1 provides an element important in the subsequent analysis. As seen from Figure 2, when the economy grows out of the

normalization point ($K > K_0$), the diminishment of the marginal returns to capital is less pronounced the higher σ is. Consequently, an economy with a higher elasticity of substitution between capital and labor will have a strictly higher marginal product of capital for any $K > K_0$.[14] Thus, as pointed out by de La Grandville (2017), a higher degree of factor substitution increases the productivity of capital in production in a manner similar to factor-augmenting technological progress.

Figure 2: Development of G_K for different values of σ



In summary, a temporary equilibrium in the benchmark economy is a set of factor prices, r and ω, satisfying (15) and (16); intermediate good prices, p_i for all i, that satisfy (7); factor demands for capital and labor that satisfy (11); intermediate production levels given by (12); and aggregate output levels given by (13). Moreover, for the given sequences of capital, $[K]_{t=0}^{\infty}$, as specified by (3), a dynamic equilibrium of the benchmark economy is given by a sequence of temporary equilibria over all $t \in \{0, 1, 2, ..., \infty\}$.

We can now turn to the characterization of the steady-state properties of the benchmark model.

2.3 Steady state

The steady-state equilibrium for the benchmark model is defined as follows.

Definition 3. A steady-state equilibrium is an equilibrium path in which $K = K^*$ for all t.

We use an asterisk to denote steady-state variables. In this benchmark model, without technological change and population growth, a steady-state equilibrium requires K^* to be constant. From the law of motion for the capital stock, given

[14] In contrast, the effect of an increase in σ on the marginal product of labor, G_L, is ambiguous for $K > K_0$ and only unambiguously positive for $K < K_0$.

by differential equation (3), we see that a stationary state with $K = K^*$ is reached whenever aggregate savings are equal to depreciation, i.e., $sX = \delta K$. The following proposition characterizes the steady state in the benchmark model.

Proposition 2. *(Steady state in the benchmark model)*

1. *The benchmark model has a unique steady-state capital stock characterized by*

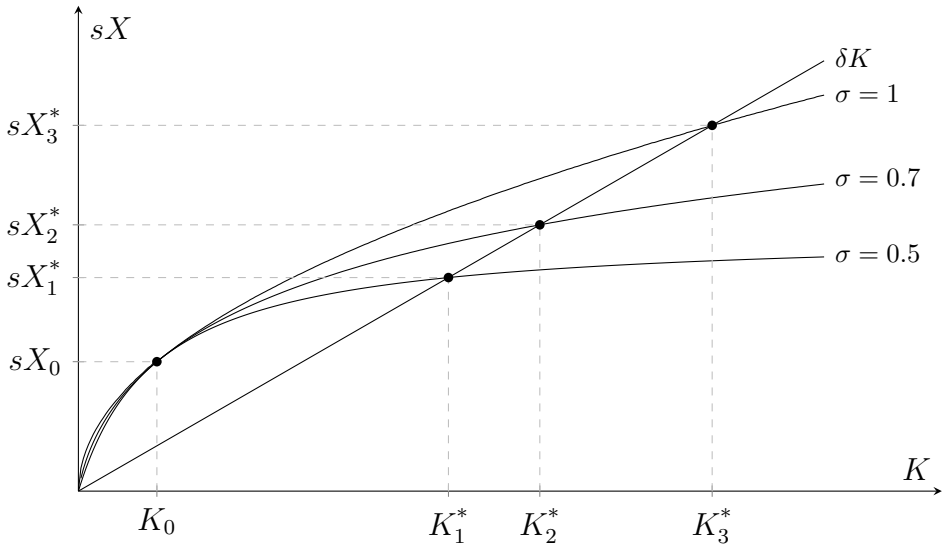
$$K^* = \frac{L}{L_0}\left(\frac{1}{1-\alpha_0}\left(\left(\frac{\delta}{sN^{\frac{\epsilon}{\epsilon-1}}Y_0}\right)^{\frac{\sigma-1}{\sigma}} - \alpha_0 K_0^{\frac{1-\sigma}{\sigma}}\right)\right)^{\frac{\sigma}{1-\sigma}} \tag{19}$$

2. *For all $K^* > K_0$, the steady-state capital stock K^* is increasing in the value of σ.*

Proof: See Appendix A.5 ■

As shown by equation (19), in the benchmark model, the steady-state capital stock K^* depends solely on exogenous parameters. Statement 2 of Proposition 2 is a representation of a theorem developed in Klump and de La Grandville (2000), which also holds in our symmetric multisector Solow (1956) growth model. According to this theorem, all other things being equal, an economy with a higher elasticity of substitution between capital and labor will have a higher capital stock and a higher level of aggregate output in the steady state. The intuition for this result follows from the discussion of Statement 3 of Proposition 1. As visualized by Figure 3, a higher value of σ decelerates the reduction in the marginal product of capital as the economy grows. This leads to a higher steady-state capital stock and a higher output of the final good.

Figure 3: Development of sX and the steady-state capital stock K^* for different values of σ



Having analyzed the benchmark model, we can now turn to the implementation of an additional R&D sector.

3 The R&D model

In this section, we endogenously derive an increase in σ by introducing a competitive R&D sector. Here, firms can engage in costly R&D, which stochastically leads to the discovery of a new intermediate production technology. This technology is characterized by a higher elasticity of substitution between capital and labor. To utilize a technology, intermediate firms need to buy a license from the R&D firm that holds a fully enforced perpetual patent on its invention. This patent allows the research firm to charge a license fee that extracts all technology-induced profits from intermediate firms.

Discovering a new intermediate production technology is open to competition. At the end of each period, research firms can invest resources in R&D. A successful research firm owns a new intermediate production technology with a higher elasticity of substitution between capital and labor and obtains a perpetual patent for its use. Through the superiority of the new technology, the successful research firm will drive the former leading firm out of the market and earns monopoly rents. Our model thus features a process of "creative destruction" in the spirit of Schumpeter (1934, 1942) and Aghion and Howitt (1992). However, although the patent of the successful research firm lasts forever, the profit flow lasts only until the next research success. Then, a new intermediate production technology with an even higher elasticity of substitution between capital and labor will replace the current technology. This process of creative destruction and replacement continues in our model and ends at the latest as the economy reaches its steady state.

In what follows, Section 3.1 outlines the research sector and solves its temporary equilibrium. Subsequently, Section 3.2 analyses the dynamic equilibrium and the steady state of the complete R&D economy.

3.1 Temporary equilibrium

In what follows, we will not repeat the entire set of equilibrium conditions from the benchmark economy but concentrate on the novel elements of this model due to the existence of the research sector.

The temporary equilibrium of the research sector is defined as follows.

Definition 4. A temporary equilibrium of the research sector, given the state variables K_t and σ_z, where z is the number of successful innovations up to period t, is a set of license fees f_t that maximizes the profit of the current leading R&D firm, as well as the equilibrium level of aggregate research expenditures Z^{**} and the corresponding probability $\phi(Z^{**})$ that result from the profit maximization of individual research firms.

To explore the incentives that research firms have to engage in R&D, we split the description of the research sector into several parts. In Section 3.1.1, we develop the functional form of the improved intermediate production technology by introducing a novel application of "functional normalization". After a discussion of its properties,

we consider the determinants of the probability of successful research in Section 3.1.2. The expected profit flow of a successful research firm is derived in Section 3.1.3. Finally, in Section 3.1.4, we solve for the equilibrium level of aggregate research expenditures.

3.1.1 Improved intermediate production technology and properties

We start with a presentation of the new intermediate production technology characterized by a higher elasticity of substitution between capital and labor and with a discussion of its properties. We assume that a successful research firm discovers a new intermediate production technology that can be applied in all intermediate sectors simultaneously. The corresponding production function takes the following form:

$$Y_i = F(K_i, L_i; z) = Y_{i,z}\left[\alpha_{i,z}\left(\frac{K_i}{K_{i,z}}\right)^{\frac{\sigma_z-1}{\sigma_z}} + (1-\alpha_{i,z})\left(\frac{L_i}{L_{i,z}}\right)^{\frac{\sigma_z-1}{\sigma_z}}\right]^{\frac{\sigma_z}{\sigma_z-1}} \tag{20}$$

where subscript $z \geq 1$ refers to the zth successful innovation.[15] At the time of the zth successful innovation, the economy-wide endowment of capital and labor is denoted by K_z and L_z, respectively, while $\sigma_z > \sigma_{z-1}$ denotes the increased elasticity of substitution between capital and labor.[16] The baseline values $Y_{i,z}$ and $\alpha_{i,z}$ will be specified below. As in the previous section, we impose the following symmetry conditions: $K_{i,z} = K_z/N$, $L_{i,z} = L_z/N$, $Y_{i,z} = Y_z$, and $\alpha_{i,z} = \alpha_z$, for all i.

To specify (20) further, we build upon the concept of the "functional normalization" of CES production functions first introduced by Jones (2003) and subsequently generalized by Antony (2009a,b, 2010). Building on the concept of normalization, as in de La Grandville (1989) and Klump and de La Grandville (2000) utilized in Section 2, functional normalization also relies on the same set of baseline values, $K_{i,0}$, $L_{i,0}$, $Y_{i,0}$, and $\mu_{i,0}$, but specifies them as being dependent on another CES production function. As demonstrated by Antony (2009a,b, 2010), the concept of functional normalization allows for a flexible modeling of the shape of the aggregate production function and especially enables σ to interact with the level of economic development. This property makes functional normalization particularly relevant to account for the empirical evidence of an increase in the elasticity of substitution between capital and labor as presented in the introduction.

In what follows, we introduce a novel application of functional normalization, where the transition between different CES production functions is endogenously derived as a result of R&D. We assume that whenever a new intermediate production technology is discovered, i.e., for all $z \geq 1$, the corresponding production function, as

[15] Before the first innovation takes place, the intermediate production technology is specified by the initial intermediate production function (2).

[16] To ensure consistent notation in terms of the benchmark model, we denote by $\sigma \equiv \sigma_0$ the elasticity of substitution between capital and labor of the initial intermediate production function (2).

specified by (20), is normalized in terms of the previous intermediate production function as follows:

$$Y_z = Y_{z-1}\left[\alpha_{z-1}\left(\frac{K_z}{K_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}} + (1-\alpha_{z-1})\left(\frac{L_z}{L_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\right]^{\frac{\sigma_{z-1}}{\sigma_{z-1}-1}}, \tag{21}$$

$$\alpha_z = \frac{K_z}{K_z + L_z\mu_z} \tag{22}$$

where the marginal rate of technical substitution, μ_z, is given by the following:

$$\mu_z = \left[\frac{F_L}{F_K}\right]_z = \frac{1-\alpha_{z-1}}{\alpha_{z-1}}\left(\frac{K_{z-1}}{L_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\left(\frac{K_z}{L_z}\right)^{\frac{1}{\sigma_{z-1}}} \tag{23}$$

Equations (21) and (23), in combination with $K_{i,z}$ and $L_{i,z}$, define the zth point of normalization. This point, which is endogenously derived as a result of R&D, defines a new family of intermediate CES production functions whose members differ solely with respect to their elasticity of substitution, as given by σ_z.[17]

Equation (21) ensures that at the zth point of normalization, the old and the new intermediate production technology produce the same amount of output. Moreover, (22) and (23) ensure that at this point, both intermediate production technologies share the same marginal rate of technical substitution and consequently the same capital and labor shares in intermediate production. The choice of this way of normalization, which implies no immediate productivity effects from the innovation, can be justified as follows. While the introduction of new technologies, and particularly general-purpose technologies (GPTs), plays a central role in explaining long-term economic growth, several authors have documented that its implementation does not necessarily have an immediate impact on productivity and growth.[18] For instance David (1990), Crafts (2004), Jovanovic and Rousseau (2005), Ristuccia and Solomou (2014), and Bekar et al. (2018) show that major historical GPTs such as the steam engine, electricity, the dynamo, computers, and information technology typically did not provide large productivity gains immediately after their introduction. Rather, the full potential of a newly introduced GPT on overall productivity and growth evolves over time. This occurs as the new technology spreads through the economy and is adopted by a broad range of sectors. In light of this evidence,

[17] As a proof of consistent normalization, at the zth point of normalization, we have $K = K_z$ and $L = L_z$, and thus, $Y = Y_z$, for all σ_z.

[18] According to Bresnahan and Trajtenberg (1995), GPTs are characterized as radical technological innovations that potentially affect the dynamics of productivity and innovation in a wide range of economic sectors. As will become clear in the following, any new intermediate production technology fulfills these characteristics and can therefore also be regarded as a GPT.

our choice of the normalization values Y_z and μ_z seems reasonable: a new intermediate production technology has no immediate effect on output and productivity but becomes increasingly beneficial as the economy grows.

In line with the empirical evidence presented in the introduction, we assume that innovations lead to increases in the elasticity of substitution between capital and labor. In what follows, we build upon the seminal work of Grossman and Helpman (1991a,b), Segerstrom (1991), and Aghion and Howitt (1992) and impose the following quality-ladder framework for the increase in the elasticity of substitution between capital and labor:

$$\sigma_z = \gamma(1 - \sigma_{z-1}) + \sigma_{z-1}, \tag{24}$$

for all $z \geq 1$ and σ_0 as the initial value. According to (24), a successful innovation leads to a discrete jump in the elasticity of substitution. The increase in size is taken as exogenous and dependent on both γ and σ_{z-1}. The parameter $\gamma \in (0, 1)$ can be interpreted as the productivity of R&D. For any previous stage of technology, as measured by σ_{z-1}, a higher level of γ allows for a greatesr increase of the elasticity of substitution.[19] However, the increase in size, as measured by $\sigma_z - \sigma_{z-1}$, is decreasing in the previous stage of technology: $\partial(\sigma_z - \sigma_{z-1})/\partial\sigma_{z-1} = -\gamma < 0$. That is, the higher σ_{z-1} is, the less "radical" innovations tend to emerge from R&D (Acemoglu and Cao, 2015). Furthermore, the above function is chosen to ensure that as z goes to infinity, the elasticity of substitution between capital and labor converts to unity. Thus, (20) reduces to Cobb-Douglas.[20] However, as we will see in the following, this level will never be attained as the economy converges to a steady-state equilibrium characterized by $\sigma < 1$.

This completes the discussion on the improved intermediate production function and its properties. We now turn to the specification of the probability of successful research.

3.1.2 Probability of successful research

We assume that there is free entry into the R&D sector and that all research firms have access to the same R&D technology. In each period, any research firm can spend τ units in terms of the final output good to engage in R&D.[21] Consequently,

[19] Note that for $\gamma = 0$, the old and new intermediate production functions have identical isoquant curves, while for $\gamma = 1$, successful research would immediately lead to a unitary elasticity of substitution implying a Cobb-Douglas production function.

[20] We choose this assumption, as it ensures the empirically plausible conjecture that even in the long run, labor cannot be fully substituted by capital in production. This is because we have $G(0, L) = 0, \forall\sigma \in [0, 1]$. Furthermore, an upper bound of the long-run elasticity of substitution between capital and labor equal to unity gets some support from the theoretical literature, e.g., Jones (2003), Jones (2005).

[21] An alternative approach, e.g., applied in Aghion et al. (2014), considers heterogeneous labor where unskilled workers, L_P, are engaged in producing intermediate goods, while skilled workers or scientists, L_R, are devoted to research. The probability of successful research is then dependent on

Z/τ measures the number of researching firms. Research expenditures stochastically lead to the invention of a new intermediate production technology. We assume that at the aggregate level, a new intermediate production technology is discovered with probability $\rho(Z, \sigma_{z-1})$. This depends on both the aggregate amount of research expenditures, Z, and the previous stage of the technology as specified by σ_{z-1}. The functional form is as follows:

$$\rho(Z, \sigma_{z-1}) = \phi(Z)\eta(\sigma_{z-1}), \qquad \text{where} \tag{25}$$

$$\phi(Z) = Z/(1+Z), \qquad \in [0,1] \tag{26}$$

$$\eta(\sigma_{z-1}) = 1 - \sigma_{z-1}^{\beta}, \qquad \in [0,1]. \tag{27}$$

Equations (26) and (27) capture two important externalities associated with R&D activities: the duplication of research expenditures at a certain point in time and intertemporal spillovers with respect to the previous stage of technology. First, consider equation (26). Research expenditures are essential for production since $\phi(0) = 0$. Furthermore, $\partial\phi(Z)/\partial Z > 0$ implies that higher aggregate R&D expenditures strictly increase the overall likelihood of discovering a new intermediate production technology. The success probability of an individual research firm, $\eta(\sigma_{z-1})\tau/(1+Z)$, is, however, strictly decreasing in Z, as $\partial^2\phi(Z)/\partial Z^2 < 0$. This property of (26) reflects the duplication and overlap of research efforts. According to this "stepping on toes" effect (see, e.g., Jones, 1995, Stokey, 1995), doubling aggregate research expenditures less than doubles the aggregate probability of discovering a new intermediate production technology.[22] Second, equation (27) implements externalities across time with respect to the previous stage of technology. Recent empirical evidence for the U.S. shows that both for several industries and the aggregate economy, "ideas are getting harder and harder to find" as the total factor productivity rises (Bloom et al., 2020, p. 1138).[23] Accounting for this, we build on the seminal work of Jones (1995), Kortum (1997), and Segerstrom (1998) and assume that the probability of discovering a new intermediate production technology strictly declines as σ_{z-1} rises. The shape parameter $\beta > 0$ governs the extent of this externality. For any previous stage of technology, σ_{z-1}, and given aggregate research expenditures, Z, the lower β is, the more difficult it is to discover a new intermediate production technology.

---

the employment of skilled workers or scientists in research. Our approach, in contrast, builds upon the "lab-equipment" specification of R&D, as introduced in Rivera-Batiz and Romer (1991), and implicitly assumes that both capital and labor are used as inputs in the R&D sector. We choose this specification to ensure that the marginal product of capital is not disturbed by a reallocation of labor between production and research but solely depends on both K and σ.

[22] There exists a considerable number of studies that document diminishing returns induced by duplicative research expenditures. Early contributions include Griliches (1990), Kortum (1993), and Griliches (1994). For more recent evidence, see, e.g., Kogan et al. (2017).

[23] See Bond-Smith (2019) for a comprehensive review of further empirical literature.

3.1.3 Expected profits of successful research

We can now turn to the characterization of the expected profits of a successful research firm. As mentioned above, we assume that research takes place at the end of each period and that a new intermediate production technology can subsequently be applied in all sectors simultaneously. Combining (7), (8), (10), and (13), in symmetric equilibrium, each intermediate producer generates the same profit, which is given by $\pi_t = (Y_t/\epsilon)\, N^{\frac{1}{\epsilon-1}}$. Due to the increase in the elasticity of substitution, for all $K_t > K_z$, a new intermediate production technology allows for a more efficient way to combine capital and labor in production and thus for an increase in the production of the intermediate good.

We denote the additional output an intermediate firm is able to produce when applying the new instead of the old intermediate production technology at period t by ΔY_t.[24] Regarding the demand for production technologies, we assume that all intermediate producers choose the license of the intermediate production technology with the highest elasticity of substitution between capital and labor if it offers the same net profits as those from buying the license of other production technologies. As a result, whenever a new intermediate production technology is introduced, Bertrand competition between successful research firms entails that the price of the patent of the former research firm falls to zero for all subsequent periods. Meanwhile, the patent holder of the leading-edge intermediate production technology charges a price of the license equal to the following:

$$f_t = \left(\frac{\Delta Y_t}{\epsilon}\right) N^{\frac{1}{\epsilon-1}}, \tag{28}$$

which is just the increase in intermediate profits. In symmetric equilibrium, the successful research firm gains profits from patent licensing across all N intermediate producers simultaneously. Consequently, once successful, the profit of the leading research firm is given by $Nf_t = (\Delta Y_t/\epsilon)\, N^{\frac{\epsilon}{\epsilon-1}}$ for every subsequent period until replaced by a new successful innovator. As the probability of successful research is given by ρ_t, the probability of the incumbent research firm staying in the market is given by $1-\rho_t$. Discounting future profits by the corresponding interest rate r_t, the present value of the expected profits in the case of a successful innovation, $E_t(\pi)$, is given as follows:

$$E_t(\pi) = \frac{f_{t+1}}{(1+r_{t+1})} + \sum_{k=2}^{\infty} \frac{\prod_{f=1}^{k-1}(1-\rho_{t+f})}{\prod_{m=1}^{k}(1+r_{t+m})} f_{t+k}. \tag{29}$$

As seen from (29), a correct prediction of $E_t(\pi)$ requires perfect foresight about

[24] See Appendix A.7 for a derivation of ΔY_t. Based on the previous discussion, ΔY_t is equal to zero for $K_t = K_z$ but strictly increases in both K_t and σ_z for all $K_t > K_z$.

the development of f_t, r_t, and ρ_t. The former two variables depend on the aggregate capital stock, K_t, and require information about the productivity of existing intermediate production technologies. The latter variable is a function of aggregate research expenditures, Z_t, which in turn depends on the expectations about future profits of all competing research firms.

Although the concept of perfect foresight is in the tradition of many R&D-based growth models, it contradicts the factual impossibility of research firms building well-founded expectations about the profitability of an innovation over its entire lifecycle; this spreads to a large number of sectors and potentially covers many periods. Therefore, the literature provides several alternative approaches to address this problem. For example, Carlaw and Lipsey (2006, 2011) developed a model of endogenous growth driven by a sequence of GPTs. In their model, research firms can neither foresee the development of factor prices nor assign probabilities to the range of possible outcomes of a newly introduced GPT due to Knightian uncertainty. Confronted with these limitations, research firms maximize only the current profit, taking factor endowment and prices of the present period as given. Another approach is applied in Schaefer et al. (2014). Adopting an alternative suggestion proposed by Carlaw and Lipsey (2006), the authors assume that research firms are forward-looking, but cannot predict changes in the profit determining variables of a newly introduced GPT. Consequently, to derive their expectations about future profits, research firms treat all relevant variables as constant at the level currently observed and maximize accordingly over an infinite time horizon. However, in the context of our model, the constancy of f_t, r_t, and ρ_t would mean that the economy is in its long-run stationary state where all variables remain constant. With respect to the R&D sector, however, the constancy of the aggregate capital stock implies that expected profits of a newly introduced intermediate production technology are zero for all subsequent periods due to Bertrand competition.[25]

Therefore, in the following, we build upon Carlaw and Lipsey (2006, 2011) and suppose that research firms have only limited foresight about the development of future profits. We assume that research firms invest resources in R&D to discover a new intermediate production technology at point t, they consider only the potential profits of period $t+1$. Consequently, their expectations about future profits in the case of a successful innovation are given by the following approximation:[26]

$$E_t(\pi) = \frac{N^{\frac{\epsilon}{\epsilon-1}} \Delta Y_{t+1}}{\epsilon \left(1 + r_{t+1}\right)} \tag{30}$$

We also choose this specification because it does not affect the qualitative results of the paper but greatly simplifies the calculations.

[25]Note that this is a direct result of equation (21), which ensures that for $K^* = K_z$, a new intermediate production technology produces the same amount of output as the previous one for all subsequent periods.

[26]Note that this assumption corresponds to a situation in which research firms have perfect foresight but assume $\rho_{t+1} = 1$.

3.1.4 Equilibrium research expenditures

We are now in the position to solve for the equilibrium level of aggregate research expenditures Z^{**} and the corresponding probability $\phi(Z^{**})$.

In what follows, we assume that research firms are risk-neutral. Any research firm takes Z as given and decides whether to spend τ units of the final output good to engage in R&D or not. Due to free entry into the research sector, aggregate research expenditures Z^{**} are characterized by the zero-profit condition:

$$\phi\left(Z^{**}\right)\eta(\sigma_{z-1})E(\pi) - Z^{**} = 0, \tag{31}$$

$$\text{s.t.} \quad Z^{**} \leq B,$$

where $B \equiv (1-s)X$ measures the maximum amount of resources that can be spent on research according to the budget constraint (4).

The following proposition characterizes the temporary equilibrium level of aggregate research expenditures Z^{**} and the corresponding probability $\phi(Z^{**})$:

Proposition 3. *(Temporary equilibrium level of aggregate research expenditures Z^{**} and corresponding probability $\phi(Z^{**})$)*

Suppose that the R&D expenditures of individual research firms are sufficiently small. Then, at each period of time, the temporary equilibrium level of aggregate research expenditures, Z^{**}, is given by the following:

$$Z^{**} = \begin{cases} 0 & \text{, if} \quad \eta(\sigma_{z-1})E(\pi) < 1 \\ \eta(\sigma_{z-1})E(\pi) - 1 & \text{, if} \quad 1 \leq \eta(\sigma_{z-1})E(\pi) - 1 < B \\ B & \text{, if} \quad \sigma^{a}_{z-1}E(\pi) - 1 \geq B \end{cases}, \tag{32}$$

with the corresponding probability $\phi(Z^{**})$, given by

$$\phi(Z^{**}) = \begin{cases} 0 & \text{, if} \quad \eta(\sigma_{z-1})E(\pi) < 1 \\ 1 - (\eta(\sigma_{z-1})E(\pi))^{-1} & \text{, if} \quad 1 \leq \eta(\sigma_{z-1})E(\pi) - 1 < B \\ B\,/(1+B) & \text{, if} \quad \sigma^{a}_{z-1}E(\pi) - 1 \geq B \end{cases}. \tag{33}$$

The intuition for Proposition 3 is the following. For $\eta(\sigma_{z-1})E(\pi) < 1$, research firms do not engage in R&D since the net expected profits of research investments are negative. For $\eta(\sigma_{z-1})E(\pi) > 1$, research firms spend a nonnegative amount of aggregate research expenditures Z^{**}. This leads to a nonnegative probability $\phi(Z^{**})$. The corresponding equilibrium number of researching firms is given by Z^{**}/τ. As long as the budget constraint is not binding, then Z^{**}, $\phi(Z^{**})$, and Z^{**}/τ are strictly increasing in both $\eta(\sigma_{z-1})$ and $E(\pi)$. For $1 \leq \eta(\sigma_{z-1})E(\pi) \leq B$, equation (32) has

$0 \leq Z^{**} \leq B$ as a unique solution, with the corresponding probability $0 \leq \phi(Z^{**}) \leq \phi(B) \leq 1$. In contrast, for $\eta(\sigma_{z-1})E(\pi) > B$, the budget constraint becomes binding. In that case, more research firms want to invest in R&D since the net expected profit of research investments is positive. However, the budget constraint permits more firms to enter the R&D sector, as aggregate research expenditures Z^{**} are bounded from above by B.

As a result of the above depiction, a temporary equilibrium of the research sector is a set of license fees f_t that satisfy (28), expected profits $E_t(\pi)$ of research firms that satisfy (30), aggregate research expenditures Z^{**} and probability $\phi(Z^{**})$ satisfying (32) and (33). This completes the description of the temporary equilibrium of the research sector. We can now turn to the characterization of the dynamic equilibrium and the steady state of the complete R&D economy.

3.2 Dynamic equilibrium and steady state

We start with a definition of the dynamic equilibrium of the R&D economy.

Definition 5. A dynamic equilibrium of the R&D economy is given by a sequence of its temporary equilibria where K_t develops according to (3), and σ_z develops along the quality ladder (24) according to the stochastic process (25).

As in the benchmark model, input factors capital and labor are equally distributed across intermediate producers in the symmetric equilibrium of the R&D economy, according to $K_i = K/N$ and $L_i = L/N$, at all points in time. Moreover, (28) ensures that all intermediate producers always utilize the intermediate production technology with the highest elasticity of substitution between capital and labor. Consequently, the aggregate production technology in the R&D economy can be expressed by the following set of $z \in \{0, 1, 2, ..., z^*\}$ piecewise defined production functions:

$$X = G(K, L; z) = N^{\frac{\epsilon}{\epsilon-1}} Y_z \left[\alpha_z \left(\frac{K}{K_z} \right)^{\frac{\sigma_z - 1}{\sigma_z}} + (1 - \alpha_z) \left(\frac{L}{L_z} \right)^{\frac{\sigma_z - 1}{\sigma_z}} \right]^{\frac{\sigma_z}{\sigma_z - 1}}, \tag{34}$$

where z^* refers to the total number of successful innovations until the steady state is reached. As will be outlined in the following, the shape of the aggregate production function results endogenously from R&D. For any $z \geq 0$, aggregate production function (34) is applied for all $K > K_z$ until a new intermediate production technology is introduced. Within this realm, the marginal product of capital in the final good production, G_K, is given by the following:

$$G_K = \left(N^{\frac{\epsilon}{\epsilon-1}} Y_z \right)^{\frac{\sigma_z - 1}{\sigma_z}} \alpha_z \left(\frac{X}{K} \right)^{\frac{1}{\sigma_z}} K_z^{\frac{1-\sigma_z}{\sigma_z}}, \tag{35}$$

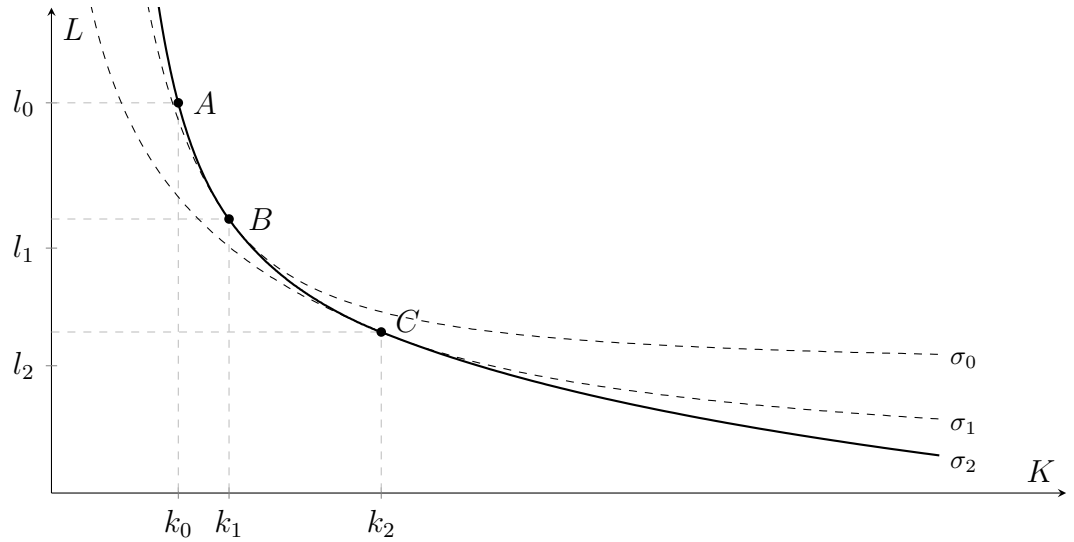
and the unit isoquant curve,

$$K = \left(\frac{\left(N^{\frac{\epsilon}{\epsilon-1}} Y_z \right)^{\frac{1-\sigma_z}{\sigma_z}} - (1-\alpha_z) \left(\frac{L}{L_z} \right)^{\frac{\sigma_z-1}{\sigma_z}}}{\alpha_z K_z^{\frac{1-\sigma_z}{\sigma_z}}} \right)^{\frac{\sigma_z}{\sigma_z-1}}, \tag{36}$$

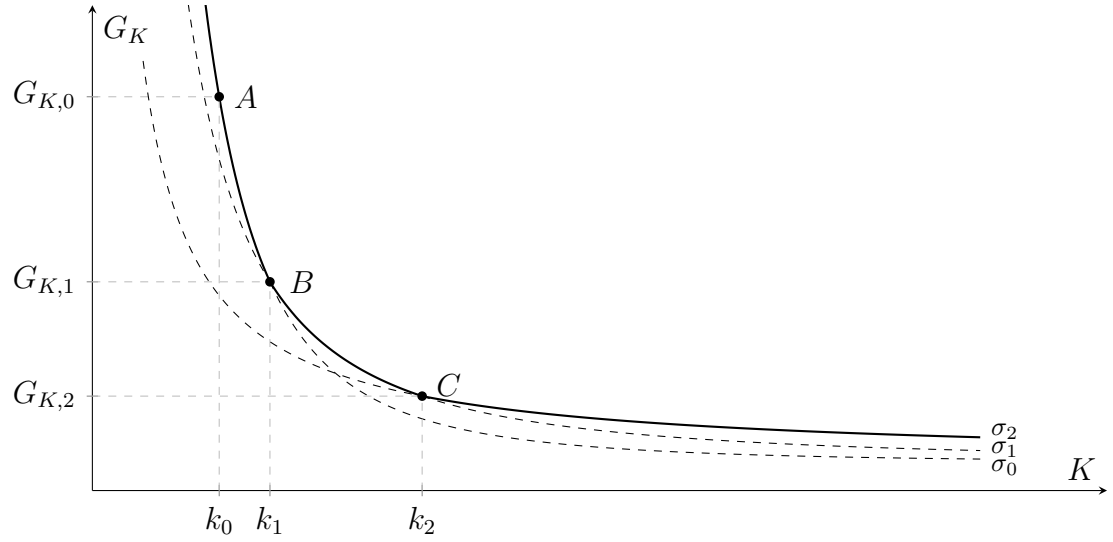
represents the set of different combinations of capital and labor required to produce one unit of the final output good.

In the following, we discuss the transitional dynamics of the R&D economy by means of the development of its unit isoquant curve (36) and the marginal product of capital, G_K, in the final good production, as specified by (35). To provide an illustration, Figure 4 plots the unit isoquant curve and the marginal product of capital for an exemplary economy with three different technologies: an initial technology exhibiting an elasticity of substitution between capital and labor equal to σ_0 and two technologies characterized by $\sigma_1 > \sigma_0$ and $\sigma_2 > \sigma_1$, which are based on the discovery of a new intermediate production technology at illustrative but arbitrarily chosen capital stocks K_1 and K_2, respectively. Point A refers to the initial point of normalization. Here, the economy starts to produce with the initial technology characterized by σ_0. At point A, one unit of the final output good X is produced with $k_0 = K_0 / \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0 \right)$ units of capital and $l_0 = L_0 / \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0 \right)$ units of labor. This leads to an initial marginal product of capital in the final good production equal to $G_{K,0}$. In subsequent periods, K_t develops according to (3) leading to an increase in the capital-labor ratio. As long as the stage of technology remains constant, the economy evolves along its initial unit isoquant curve, indicated by the solid line. The marginal product of capital, G_K, strictly decreases as a result of capital deepening in production. In point B, the first new intermediate production technology is discovered. As seen from Figure 4a, this invention generates a new unit isoquant curve for the final output good characterized by $\sigma_1 > \sigma_0$. Our choice of normalization values ensures that at point B, both unit isoquant curves are tangent and share the same marginal rate of technical substitution between capital and labor. Consequently, as visualized by Figure 4b, at the point of transition, the marginal product of capital is identical for both technologies and equal to $G_{K,1}$. However, for all $K > K_1$, the new intermediate production technology with the higher elasticity of substitution permits labor to be substituted relatively more easily for capital. Thus, it allows for a more efficient way to combine both factors in production. As seen from Figure 4b, diminishing returns to capital set in less rapidly than for the initial technology. This implies a strictly higher marginal product of capital, G_K, in the final good production for all $K > K_1$. Due to (28), all intermediate producers choose the license of the intermediate production technology with the higher elasticity of substitution between capital and labor. Accordingly, the economy evolves along the unit isoquant curve characterized by $\sigma_1 > \sigma_0$ until, in point C, the next successful innovation takes place. Characterized by an even higher elasticity of substitution between capital and labor, $\sigma_2 > \sigma_1$, the new technology replaces the former intermediate production technology for all $K > K_2$.

Figure 4: Development of the unit isoquant curve of aggregate output X and the marginal product of capital, G_K, as a result of sigma-augmenting technological changes.



(a) Development of the unit isoquant curve



(b) Development of G_K

Proposition 4 summarizes the results of the above discussion.

Proposition 4. *(Properties of the unit isoquant of aggregate output and the marginal product of capital, G_K, in the final good production)*

Suppose that equations (21), (22), *and* (23) *are satisfied. Moreover,* (28) *ensures that intermediate producers always utilize the intermediate production technology with the highest elasticity of substitution between capital and labor. Then, the following holds:*

1. *The unit isoquant of the final output good is continuously differentiable.*

2. *The marginal product of capital, G_K, in the final good production is continuous but not continuously differentiable.*

3. *G_{KK} has jump discontinuities for all $z \geq 1$, according to the following:*

$$\lim_{K \to K_z^-} G_{KK} < \lim_{K \to K_z^+} G_{KK}$$

4. *The limit of the marginal product of capital, G_K, in the final good production is zero as K approaches infinity:*

$$\lim_{K \to \infty} G_K = 0 \quad , \forall \sigma_z \leq 1$$

Proof: See Appendix A.6 ■

Next, we show that the total number of successful innovations, z^*, is finite. Thus, the elasticity of substitution between capital and labor in the steady state, σ_{z^*}, is below the Cobb-Douglas value of unity. As derived in Proposition 3, at each point in time, a positive amount of aggregate research expenditures and thus the possibility of discovering a new intermediate production technology requires the following:

$$\left(1 - \sigma_{z-1}^{\beta}\right) \frac{N^{\frac{\epsilon}{\epsilon-1}} \Delta Y_{t+1}}{\epsilon \left(1 + r_{t+1}\right)} \geq 1. \tag{37}$$

Moreover, as shown in Appendix A.7, the additional output ΔY_{t+1} that an intermediate firm is able to produce when applying intermediate production technology z instead of $z-1$ is given by the following:

$$\begin{aligned} \Delta Y_{t+1} = Y_z \left[\alpha_z \left(\frac{K_{t+1}}{K_z} \right)^{\frac{\sigma_z - 1}{\sigma_z}} + (1 - \alpha_z) \left(\frac{L_{t+1}}{L_z} \right)^{\frac{\sigma_z - 1}{\sigma_z}} \right]^{\frac{\sigma_z}{\sigma_z - 1}} \\ - Y_z \left[\alpha_z \left(\frac{K_{t+1}}{K_z} \right)^{\frac{\sigma_{z-1} - 1}{\sigma_{z-1}}} + (1 - \alpha_z) \left(\frac{L_{t+1}}{L_z} \right)^{\frac{\sigma_{z-1} - 1}{\sigma_{z-1}}} \right]^{\frac{\sigma_{z-1}}{\sigma_{z-1} - 1}} \end{aligned} \tag{38}$$

where $K_{t+1} = K_z + sX_t - \delta K_t = K_z + \Delta K_t$. In a growing economy, characterized by $\Delta K_t > 0$, ΔY_{t+1} is positive for all $\sigma_z > \sigma_{z-1}$. As long as ΔY_{t+1} is sufficiently large, such that $\eta(\sigma_{z-1}) E(\pi) > 1$, then the net expected profits of research expenditures are positive. This leads to a positive amount of aggregate research expenditures Z^{**}. However, since G_K is decreasing in K with $\lim_{K \to \infty} G_K = 0$, both ΔK_t and ΔY_{t+1} decline to zero as K approaches infinity. Thus, condition (37) does not necessarily hold for some positive finite ΔK_t and ΔY_{t+1} already. Therefore, further investment in technological change is no longer profitable.

Moreover, $\delta > 0$ ensures that $\Delta K_t = 0$ is reached in finite time and thus that z^* is finite. Consequently, from $\gamma < 1$ it follows that σ_{z^*}, is below the Cobb-Douglas value

of unity. As a result of the above discussion, the following proposition characterizes the steady state in the R&D model:

Proposition 5. *(Steady state in the R&D model)*

The R&D model has a unique steady-state capital stock characterized by the following:

$$K^* = \frac{L}{L_{z^*}} \left(\frac{1}{1-\alpha_{z^*}} \left(\left(\frac{\delta}{sN^{\frac{\epsilon}{\epsilon-1}} Y_{z^*}} \right)^{\frac{\sigma_{z^*}-1}{\sigma_{z^*}}} - \alpha_{z^*} K_{z^*}^{\frac{1-\sigma_{z^*}}{\sigma_{z^*}}} \right) \right)^{\frac{\sigma_{z^*}}{1-\sigma_{z^*}}}. \tag{39}$$

Proof: See Appendix A.8 ■

This completes the description of the dynamic equilibrium and the steady state of the R&D model. We can now turn to a quantitative analysis of the model.

4 Quantitative Analysis

In this section, we conduct numerical simulations to illustrate the effect of endogenous sigma-augmenting technological change on growth and the transitional dynamics of the economy. Utilizing data from the U.S. Bureau of Economic Analysis (BEA) and Penn World Table (PWT) 9.1 (Feenstra et al., 2015), we calibrate our model to the U.S. economy on an annual basis from 1955 to 2015.

Table 2: Baseline Calibration

Parameter		Value
Production Technologies		
Number of intermediate producers	N	10,000
Elasticity of substitution between intermediate goods	ϵ	3.80
Initial capital share	α_0	1/3
Initial elasticity of substitution between capital and labor	σ_0	0.60
Initial aggregate labor stock	L_0	109 m
Initial aggregate capital stock	K_0	28 sm
Initial intermediate output	Y_0	100
Solow-Model		
Savings rate	s	0.20
Depreciation rate	δ	0.04
Research Sector		
R&D productivity parameter (R&D model)	γ	0.14
R&D productivity parameter (Benchmark model)	γ	0.00
Shape parameter of $\eta(\sigma_{z-1})$	β	0.85

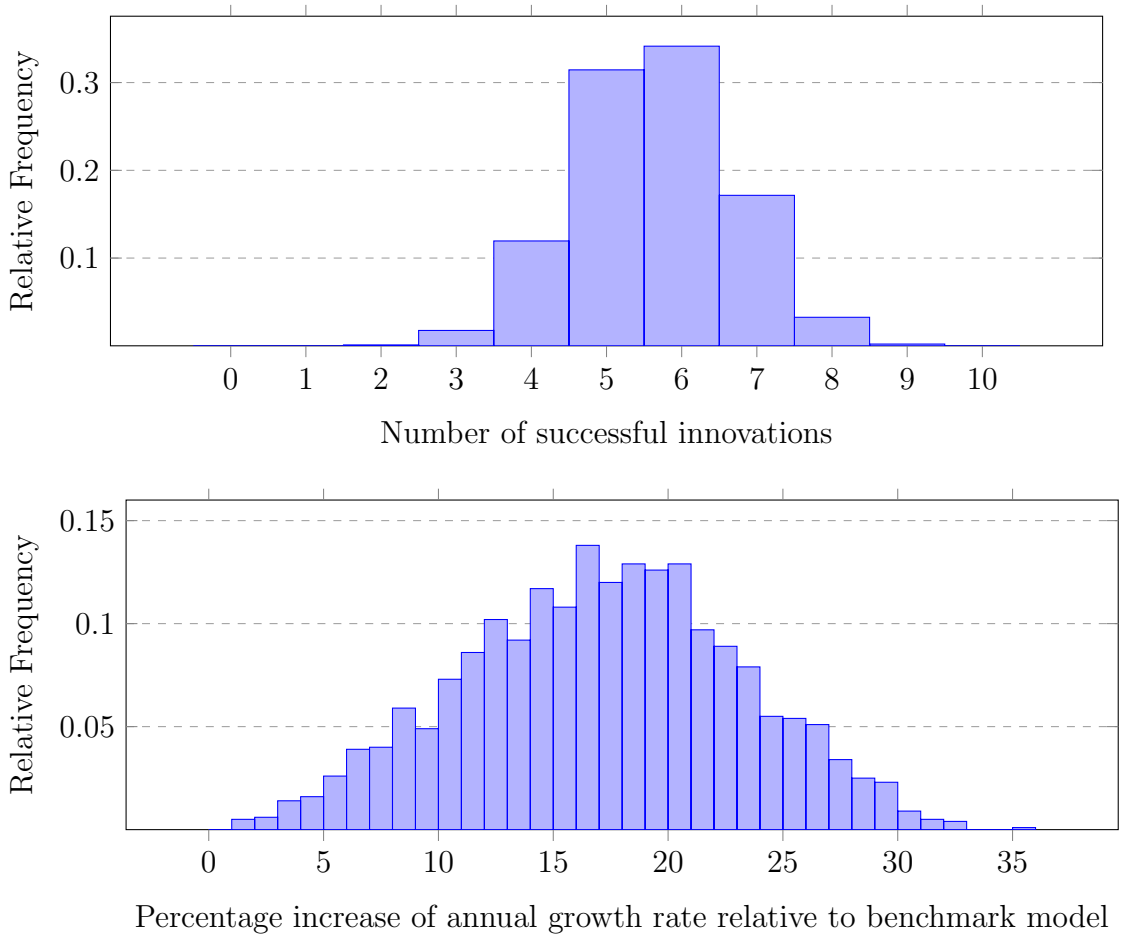
Table 2 reports the values used for the baseline calibration. Parameter and benchmark values are chosen as follows. We follow Bernard et al. (2003) and set the

elasticity of substitution between intermediate goods, ϵ, equal to 3.8. Our choice for the initial capital share, $\alpha_0 = 1/3$, is in line with the data (see, e.g., Karbarbounis and Neiman, 2014). The estimates of σ for the U.S. economy vary considerably. Knoblach et al. (2020) estimate a meta-elasticity for the aggregate U.S. economy in the range of 0.45-0.87. Given that evidence, we utilize a value of $\sigma_0 = 0.6$ for the initial elasticity of substitution between capital and labor. Furthermore, we set L_0 equal to 109 million, the average size of employment in the U.S. from 1955 to 2015 according to PWT. For the parameters of the Solow growth model, we utilize a series of gross savings as a percentage of gross national income, as available from the BEA. Over the period from 1955 to 2015, average annual savings lie slightly above 20% of GDP. Therefore, we set $s = 0.2$. Furthermore, we set $\delta = 0.04$, the average value of the depreciation rate from 1955 to 2015 according to PWT. The remaining parameters N, K_0, Y_0, γ, and β are chosen to match the following two characteristics. First, from 1955 to 2015, the PWT reports an average annual growth rate of the aggregate capital stock of approximately 3.3%. Second, according to Table 1, the average annual increase in σ for the aggregate U.S. economy lies in the range of 0.00292 to 0.00475. We set $N = 10,000$, $K_0 = 28m$, $Y_0 = 100$, $\gamma = 0.14$, and $\beta = 0.85$. These choices ensure that on average, over $2,000$ simulation runs, our results are in line with these two observations. Moreover, to allow for a comparison of the results with the benchmark model developed in Section 2, we simulate the model with $\gamma = 0.00$. This ensures that research firms do not invest in R&D since the expected profits from successful innovations are zero at each point in time. Thus, the elasticity of substitution between capital and labor remains constant over time and is equal to σ_0.

Figure 5 depicts the frequency distribution for the number of successful innovations and the percentage increase in the average annual growth rate relative to the benchmark model over a sample of $2,000$ simulation runs. As seen from the upper histogram, for this sample, the smallest number of successful innovations is 2, where the elasticity of substitution between capital and labor increases from 0.6 to 0.7042. The largest number of successful innovations is 9. In this case, σ increases from an initial value of 0.6 to 0.8971. The histogram is slightly skewed to the left. The median number of successful innovations is 6, as obtained in 683 simulation runs, while the mean number is 5.632. Thus, on average, a new intermediate production function is discovered within an interval of ten years. Over the entire sample of $2,000$ simulation runs, the average increase in the elasticity of substitution between capital and labor is 0.22669, corresponding to an average annual increase of 0.00378.

Next, we investigate how sigma-augmenting technological change influences the relative growth performance of the R&D economy. For this purpose, we compute the geometric average annual growth rate of the final output good, X, separately for each simulation run. Then we compare it to the corresponding growth rate of the benchmark model. The result of this exercise is shown in the lower histogram of Figure 5. Induced by the development of new intermediate production technologies characterized by a higher elasticity of substitution between capital and labor, the average annual growth rate of final output increases by 17.12% relative to the benchmark model. The histogram is negatively skewed with a median value of 17.26%,

Figure 5: Number of successful innovations and percentage increase in average annual growth rate relative to benchmark model over 2,000 simulation runs



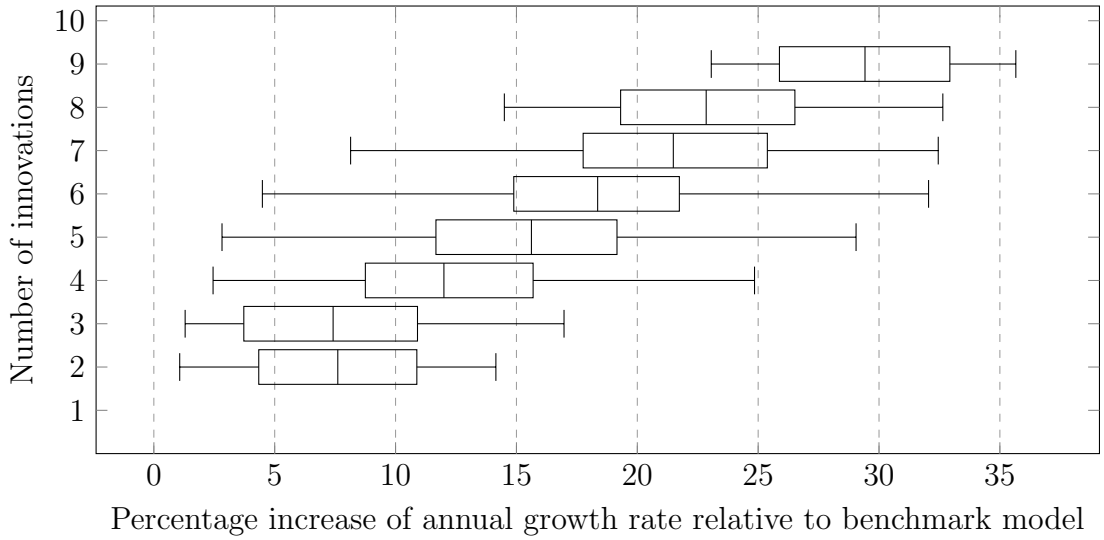
The above histograms are constructed over 2,000 simulation runs based on the parameter values reported in Table 2.

which is slightly higher than the mean. However, the impact of R&D on the relative increase in the growth rate varies considerably and ranges from 1.07% to 35.66%.

Figure 6 provides a deeper investigation of the large dispersion shown in the lower histogram of Figure 5 by presenting a separate box plot for each total number of successful innovations. In general, the figure shows a positive association between the number of successful innovations and the percentage increase in the annual growth rate relative to the benchmark model. However, the results still vary considerably even for a constant number of innovations. The median number of 6 successful innovations provides a good example. As seen from Figure 6, over 683 simulation runs, the percentage increase in the annual growth rate relative to the benchmark model ranges from 4.49% to 32.05%.

As will be seen in the following, the timing of innovations is another important factor in determining the large dispersion in the growth performance of the R&D economy.

Figure 6: Boxplot for each number of successful innovations

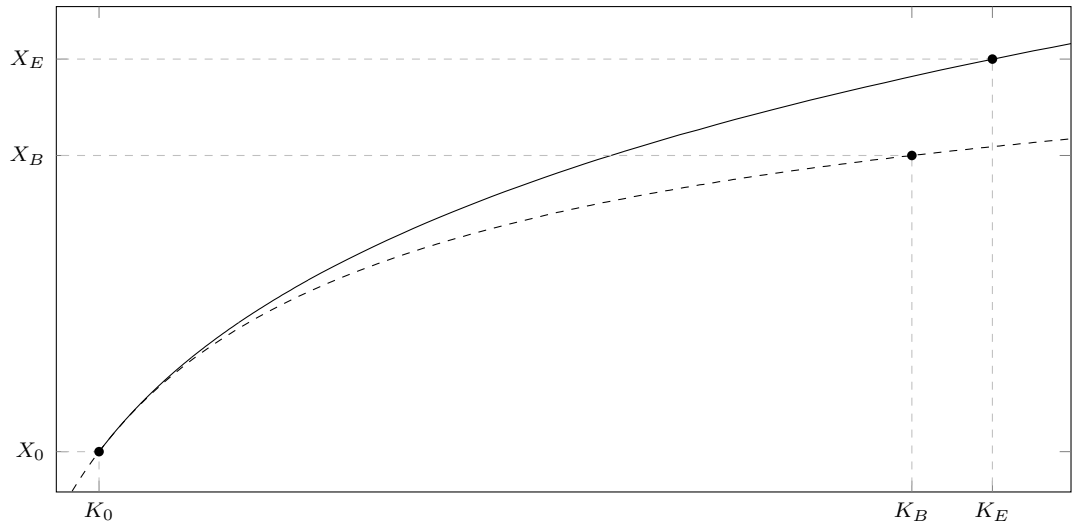


The above box plots are constructed over 2,000 simulation runs based on the parameter values reported in Figure 2.

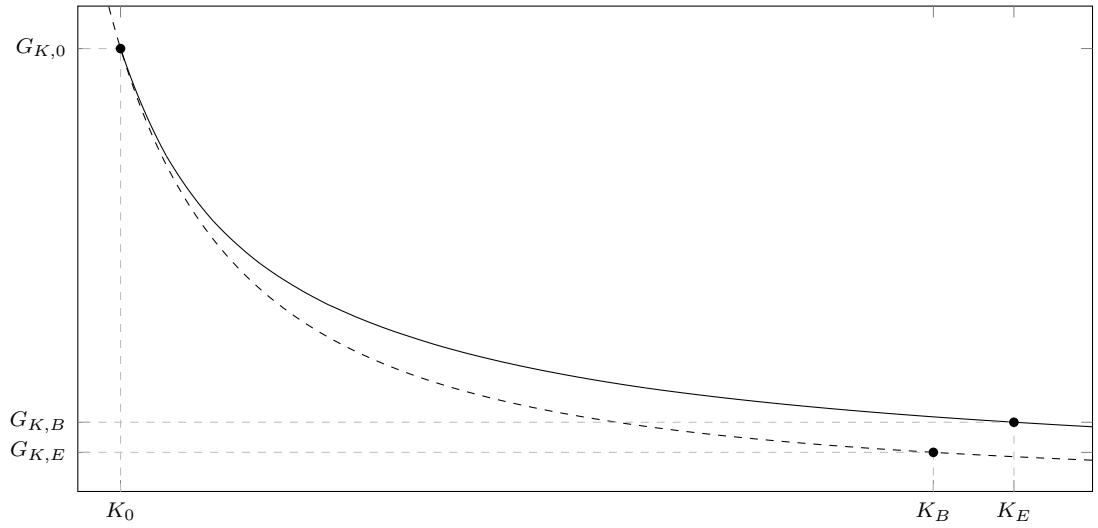
Figures 7 and 8 illustrate the impact of the timing of innovations on the development of the final output, X, and the marginal product of capital, G_K, in the final good production. We consider two economies here, each characterized by 6 successful innovations. However, innovations occur at different points in time in both economies. In the "Early R&D" case, new intermediate production technologies are discovered at the end of periods 0, 2, 4, 6, 9, and 25. In contrast, in the "Late R&D" case, innovations occur at the end of periods 5, 10, 16, 19, 20, and 26. Figure 7 illustrates the case of an economy with early R&D. In both panels, the solid line shows the development of the R&D economy, while the dashed line refers to the development of the benchmark economy characterized by $\gamma = 0.00$. Panels (a) and (b) of Figure 7 show that induced by a series of early innovations, the diminishment of the marginal product of capital is considerably less pronounced in the early R&D economy compared to the benchmark case. As a result, during the course of 60 periods, the R&D economy records a higher growth rate leading to a final output, X_E, which is considerably higher than the corresponding final output, X_B, in the benchmark model. In contrast, in the "Late R&D" case displayed in Figure 8, the first successful innovation sets in at the end of period 5, where the marginal product of capital has already declined substantially. Consequently, differences in the R&D economy compared to the benchmark economy are less pronounced, both in terms of the reduction in the diminishment of the marginal product of capital and with respect to the difference in final good production. This highlights that within our model, the timing of innovations can be of central importance with respect to the impact of sigma-augmenting technological change on economic development.

Finally, we conduct a brief sensitivity analysis by considering different values for the R&D productivity parameter, γ, the shape parameter β, and the initial elasticity of

Figure 7: Typical simulation run: Early R&D



(a) Development of final output X



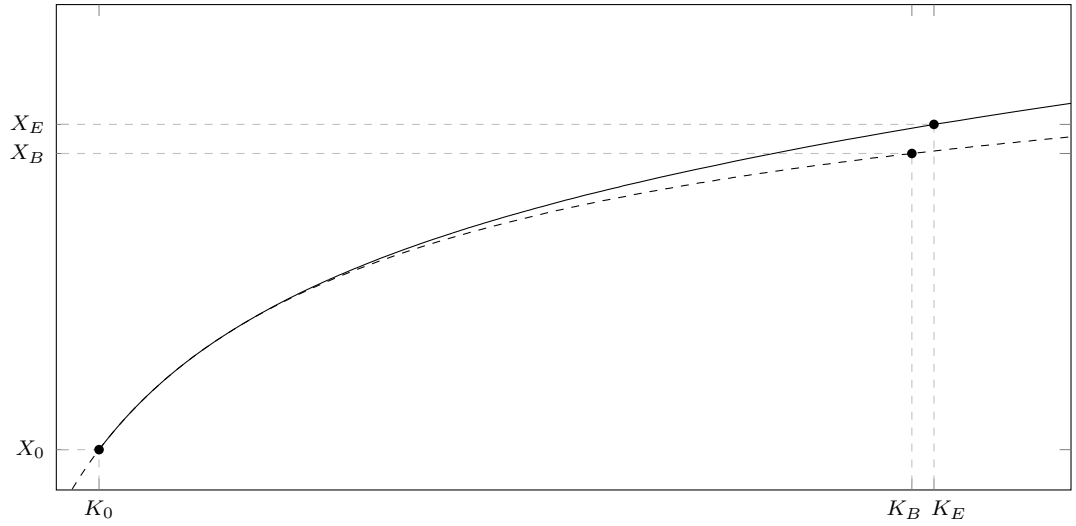
(b) Development of marginal product G_K

The above illustrations are constructed based on the parameter values reported in Table 2. In the "Early R&D" case, there are 6 successful innovations discovered by the end of periods 0, 2, 4, 6, 9, and 25.

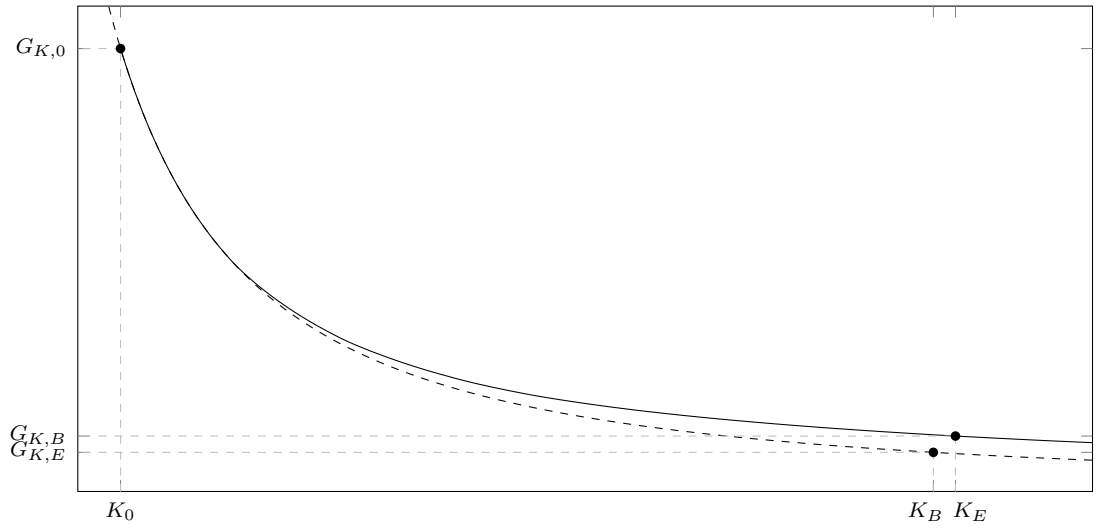
substitution between capital and labor, σ_0. All other parameter values remain unchanged. Table 3 reports the central results of the R&D model based on alternative calibrations, each constructed over 2,000 simulation runs.

First, consider a variation in the R&D productivity parameter γ. As seen from Table 3, in our example, a change in γ has only little impact on the number of successful innovations. However, following (24), the level of γ influences the increase in the elasticity of substitution between capital and labor in the case of a successful innovation. Consequently, as the number of successful innovations is approximately

Figure 8: Typical simulation run: Late R&D



(a) Development of final output X



(b) Development of marginal product G_K

The above illustrations are constructed based on the parameter values reported in Table 2. In the "Late R&D" case, there are 6 successful innovations discovered by the end of periods 5, 10, 16, 19, 20, and 26.

constant, a lower (higher) value of γ implies a lower (higher) average increase in σ and thus a lower increase in the relative growth rate.

Second, consider a change in the shape parameter β, which is inversely related to the success probability $\eta(\sigma_{z-1})$. As a result, a lower (higher) value of β induces a higher (lower) amount of aggregate research expenditures and consequently, on average, a higher (lower) number of successful innovations. This effect is also mirrored in the results provided in Table 3.

Table 3: Sensitivity analysis

	Number of innovations			Increase in σ		Increase in growth rate		
	min	max	median	average	annual	min	max	median
Baseline	2	9	6	0.22669	0.00378	1.07%	35.66%	17.26%
$\gamma = 0.1$	2	9	6	0.19284	0.00321	1.32%	26.92%	13.54%
$\gamma = 0.2$	2	7	5	0.26129	0.00435	2.07%	42.04%	21.89%
$\beta = 0.5$	1	7	4	0.17938	0.00299	0.12%	32.05%	11.95%
$\beta = 1.3$	4	10	7	0.25969	0.00433	5.07%	34.09%	21.51%
$\sigma_0 = 0.5$	3	10	7	0.31126	0.00519	2.09%	52.61%	30.49%
$\sigma_0 = 0.7$	1	8	4	0.14438	0.00241	0.11%	19.70%	8.80%

Third, the choice of the initial elasticity of substitution between capital and labor, σ_0, considerably influences the relative growth performance of the R&D economy. If σ_0 is low, then our specification of the quality-ladder (24) indicates a greater increase in the elasticity of substitution in the case of a successful innovation compared to a case where σ_0 is high. This in turn induces a higher amount of aggregate research expenditures and thus a higher probability of discovering a new intermediate production technology. In our example, for an initial value of $\sigma_0 = 0.5$, the average increase in σ is 0.3122, leading to a median increase in the relative growth rate of 30.49%. In comparison, for $\sigma_0 = 0.7$, the average increase in σ declines substantially compared to the baseline configuration, inducing a median increase in the relative growth rate of only 8.80%.

5 Conclusions

Our model provides a first approach of endogenous "sigma-augmenting technological change" (Klump et al., 2012, p. 793), where R&D investments aim at discovering new intermediate production technologies with improved substitution possibilities between capital and labor. In light of the empirical observation that σ increases over time, we believe that our approach will prove useful in many applications, particularly as the model is technically quite manageable and extendable to several interesting directions.

To focus on the relation between the marginal product of capital, the aggregate capital stock, and σ, we have abstracted from population growth and factor-augmenting technological progress. Surely, these features are relevant for real-world growth processes, so incorporating them is suggested. In particular, allowing for factor-augmenting technological progress and exploring its interrelation with sigma-augmenting technological change would be an interesting focus. From a theoretical perspective, this would concern how incentives to invest in which kind of research

evolve over time.[27] From the empirical perspective, one might think of growth accounting in the spirit of Solow (1957) disentangling the relative contributions of factor accumulation and sigma- and factor-augmenting technological progress on output growth.

Moreover, the empirical literature has shown that the estimates of σ for the U.S. agriculture, manufacturing, and services sectors differ substantially from each other (see Knoblach and Stöckl, (forthcoming) for a review). Thus, another promising extension would be to introduce the possibility of endogenous changes in the elasticity of substitution between capital and labor into a model of structural transformation in the spirit of Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Alvarez-Cuadrado et al. (2017). By considering sectoral differences in either the R&D productivity parameter γ or the shape parameter β, these sectoral differences in the extent of capital-labor substitutability can be derived endogenously as a result of sigma-augmenting technological progress.

Motivated by the vast majority of empirical evidence and theoretical considerations, we have restricted our analysis to elasticities of substitution of at most one. Nevertheless, allowing for the possibility that capital and labor become gross substitutes would constitute an interesting direction for future research. As already shown by Solow (1956) and Pitchford (1960), perpetual growth is possible due to capital accumulation alone if σ exceeds a threshold that is necessarily above unity. However, to pass this threshold, an economy starting with some $\sigma < 1$ must experience a sufficient number of successful improvements of the elasticity of substitution. It is easy to devise a poverty trap-like situation in which some countries with little innovational success become stuck in a steady state with no output growth, whereas more successful ones keep growing. We leave the investigation of this setting, which is original to the existence of sigma-augmenting technological change, for future research.

[27] See Stöckl (2020) for a discussion of this topic in the context of energy economics.

Appendix A

A.1 Explicit normalization of the intermediate production function

In this appendix, we derive the explicit normalized intermediate CES production function (2). The derivation can start with the primal Arrow et al. (1961) specification of the CES production function:

$$Y_i = F(K_i, L_i) = C_i \left[a_i K_i^{\frac{\sigma-1}{\sigma}} + (1-a_i) L_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{A.1}$$

where σ is the elasticity of substitution between capital and labor, C_i denotes an (Hicks-neutral) "efficiency" parameter, and $0 < a_i < 1$ refers to a "distribution" parameter.[28] To guarantee a meaningful and consistent comparison of model results relying on different values of substitution elasticity, de La Grandville (1989) and Klump and de La Grandville (2000) introduced the idea of "normalizing" CES production functions. Based on a given set of arbitrarily chosen baseline values, $K_{i,0}$, $L_{i,0}$, $\mu_{i,0} = [F_L/F_K]_0$ and $Y_{i,0}$, the production function (A.1) can be transformed as follows:

$$\begin{aligned} Y_{i,0} &= C_i \left[a_i K_{i,0}^{\frac{\sigma-1}{\sigma}} + (1-a_i) L_{i,0}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &\Rightarrow \mu_{i,0} = \left[\frac{F_L}{F_K} \right]_0 = \frac{1-a_i}{a_i} \left(\frac{K_{i,0}}{L_{i,0}} \right)^{\frac{1}{\sigma}} \\ &\Leftrightarrow a_i(\sigma) = \frac{K_{i,0}^{1/\sigma}}{K_{i,0}^{1/\sigma} + L_{i,0}^{1/\sigma} \mu_{i,0}} \end{aligned} \tag{A.2.1}$$

$$\begin{aligned} Y_{i,0} &= C_i \left[a_i K_{i,0}^{\frac{\sigma-1}{\sigma}} + (1-a_i) L_{i,0}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &\Rightarrow C_i(\sigma) = Y_{i,0} \left[\frac{K_{i,0}^{1/\sigma} + L_{i,0}^{1/\sigma} \mu_{i,0}}{K_{i,0} + L_{i,0} \mu_{i,0}} \right]^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{A.2.2}$$

where $F_K \equiv \partial Y_i / \partial K_i$ and $F_L \equiv \partial Y_i / \partial L_i$ denote the marginal product of capital and labor, respectively. This procedure leads to an explicit relationship between the elasticity of substitution, σ, and both C_i and a_i. Inserting $a_i(\sigma)$ and $C_i(\sigma)$ into (A.1) provides, after some rearranging, a normalized CES production function as follows:

$$Y_i = Y_{i,0} \left[\alpha_{i,0} \left(\frac{K_i}{K_{i,0}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_{i,0}) \left(\frac{L_i}{L_{i,0}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{A.3}$$

[28] For a derivation of the CES production function based on the formal definition of the elasticity of substitution between capital and labor, see Brown and De Cani (1963), Klump et al. (2012), and de La Grandville (2017).

This is similar to (2). In this specification, the parameter $\alpha_{i,0} = \frac{F_{K,0}K_{i,0}}{F_{K,0}K_{i,0}+F_{L,0}L_{i,0}}$ denotes the capital share in intermediate production at the point of normalization $t = t_0$. At that point, (A.3) reduces to $Y_i = Y_{i,0}$ and is thus independent of σ.

A.2 Price index

In this appendix, we build upon Bergholt (2011) and derive the price index (7). We start by solving the first-order condition of final good producers (6) for Y_i:

$$Y_i = Y_j \left(\frac{p_i}{p_j}\right)^{-\epsilon}, \qquad \forall\ i, j \in N \tag{A.4}$$

Plugging (A.4) into the budget constraint $\sum_{i=1}^{N} p_i Y_i = M$, where M measures the total amount of money spent on intermediate goods, leads to the following:

$$\begin{aligned} M &= Y_j p_j^{\epsilon} \sum_{i=1}^{N} p_i^{1-\epsilon} \\ \Rightarrow Y_j &= \frac{M p_j^{-\epsilon}}{\sum_{i=1}^{N} p_i^{1-\epsilon}} \end{aligned}$$

Inserting this result into the production function of the final good (1) with index $j \in N$ obtains

$$\begin{aligned} X &= \left(\sum_{j=1}^{N} \left(\frac{M p_j^{-\epsilon}}{\sum_{i=1}^{N} p_i^{1-\epsilon}}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= M \left(\sum_{j=1}^{N} p_j^{1-\epsilon}\right)^{\frac{\epsilon}{\epsilon-1}} \left(\sum_{i=1}^{N} p_i^{1-\epsilon}\right)^{-1} \\ &= M \left(\sum_{i=1}^{N} p_i^{1-\epsilon}\right)^{\frac{1}{\epsilon-1}} \end{aligned}$$

Finally, we can define $M \equiv P$ as the budget necessary to buy one unit of aggregate output X. Choosing P as the numeraire, we obtain

$$P \equiv 1 = \left(\sum_{i=1}^{N} p_i^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{A.5}$$

A.3 Marginal costs with a CES production function

In this appendix, we derive the marginal cost function (9). Intermediate production is given by the following normalized CES production function:

$$Y_i = Y_0 N \left[\alpha_0 \left(\frac{K_i}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{L_i}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{A.6}$$

where, as already mentioned, intermediate output Y_i is produced as a combination of capital K_i and labor L_i. Furthermore, $L_{i,0}$, $K_{i,0}$, and Y_0 are baseline values, $0 < \alpha_0 < 1$ refers to the distribution parameter, and σ is the elasticity of substitution between capital and labor. Following Jiang and León-Ledesma (2018), for given factor prices for capital r and labor ω, the cost minimization problem of a representative firm to produce a fixed amount of output $\overline{Y}$ can be formalized as follows:

$$\begin{aligned} \min_{K_i, L_i} \quad & rK_i + \omega L_i \\ \text{s.t.} \quad & \overline{Y_i} = Y_0 N \left[\alpha_0 \left(\frac{K_i}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{L_i}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

From that, we can form the Lagrangian as follows:

$$\begin{aligned} \mathscr{L}(K_i, L_i, \lambda, r, \omega, Y_i) = & rK_i + \omega L_i \\ & + \lambda \left(\overline{Y_i} - Y_0 N \left[\alpha_0 \left(\frac{K_i}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{L_i}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right), \end{aligned}$$

where λ is the Lagrange multiplier. From the above equation, we can derive the following three first-order conditions:

$$\frac{\partial \mathscr{L}}{\partial K_i} = r - \lambda Y_0 N \left[\alpha_0 \left(\frac{K_i}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{L_i}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} \alpha_0 K_0^{\frac{1-\sigma}{\sigma}} K_i^{-\frac{1}{\sigma}} = 0$$

$$\frac{\partial \mathscr{L}}{\partial L_i} = \omega - \lambda Y_0 N \left[\alpha_0 \left(\frac{K_i}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{L_i}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (1-\alpha_0) L_0^{\frac{1-\sigma}{\sigma}} L_i^{-\frac{1}{\sigma}} = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda} = \overline{Y_i} - Y_0 N \left[\alpha_0 \left(\frac{K_i}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{L_i}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = 0.$$

Taking the first two equations, we can derive the following:

$$\left(\frac{r}{w}\right) = \frac{\alpha_0}{1-\alpha_0}\left(\frac{L_0}{K_0}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{K_i}{L_i}\right)^{-\frac{1}{\sigma}}, \tag{A.7}$$

which is identical to equation (14) and states that cost minimization in production requires relative factor prices to be equal to the marginal rate of technical substitution. Next, solving for L_i and raising both sides of the resulting equation to the power $\frac{\sigma-1}{\sigma}$ obtains:

$$L_i^{\frac{\sigma-1}{\sigma}} = K_i^{\frac{\sigma-1}{\sigma}}\left(\frac{L_0}{K_0}\right)^{(1-\sigma)\frac{\sigma-1}{\sigma}}\left(\frac{\omega}{r}\right)^{1-\sigma}\left(\frac{1-\alpha_0}{\alpha_0}\right)^{\sigma-1}. \tag{A.8}$$

Plugging (A.8) into (A.6) and solving for K_i gives, after some rearranging, the following:

$$K_i = \frac{Y_i K_0^{1-\sigma}\left(\frac{r}{\alpha_0}\right)^{-\sigma}}{Y_0 N\left[\alpha_0\left(\frac{r}{\alpha_0}\right)^{1-\sigma}K_0^{1-\sigma} + (1-\alpha_0)\left(\frac{\omega}{1-\alpha_0}\right)^{1-\sigma}L_0^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}}. \tag{A.9}$$

Analogously, we obtain the following:

$$L_i = \frac{Y_i L_0^{1-\sigma}\left(\frac{\omega}{1-\alpha_0}\right)^{-\sigma}}{Y_0 N\left[\alpha_0\left(\frac{r}{\alpha_0}\right)^{1-\sigma}K_0^{1-\sigma} + (1-\alpha_0)\left(\frac{\omega}{1-\alpha_0}\right)^{1-\sigma}L_0^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}} \tag{A.10}$$

For a given amount of output Y_i, equations (A.9) and (A.10) represent the cost-minimizing demand functions for capital and labor, respectively. Inserting both equations into the cost function $C = rK_i + \omega L_i$ obtains

$$\begin{aligned} C &= \frac{Y_i\left(rK_0^{1-\sigma}\left(\frac{r}{\alpha_0}\right)^{-\sigma} + \omega L_0^{1-\sigma}\left(\frac{\omega}{1-\alpha_0}\right)^{-\sigma}\right)}{Y_0 N\left[\alpha_0\left(\frac{r}{\alpha_0}\right)^{1-\sigma}K_0^{1-\sigma} + (1-\alpha_0)\left(\frac{\omega}{1-\alpha_0}\right)^{1-\sigma}L_0^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}} \\ &= \frac{Y_i}{Y_0 N}\left[\alpha_0\left(\frac{r}{\alpha_0}\right)^{1-\sigma}K_0^{1-\sigma} + (1-\alpha_0)\left(\frac{\omega}{1-\alpha_0}\right)^{1-\sigma}L_0^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \end{aligned} \tag{A.11}$$

Finally, the partial differentiation of (A.11) with respect to Y_i gives the marginal costs of producing one unit of intermediate output Y_i as follows:

$$\psi = \frac{1}{Y_0 N}\left[\alpha_0\left(\frac{r}{\alpha_0}\right)^{1-\sigma} K_0^{1-\sigma} + (1-\alpha_0)\left(\frac{\omega}{1-\alpha_0}\right)^{1-\sigma} L_0^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{A.12}$$

A.4 Proof of Proposition 1

In this appendix, we first show that the marginal product of capital, G_K, (labor, G_L,) is diminishing (increasing) in K. The marginal product of capital is given by

$$\frac{\partial X}{\partial K} = \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} \alpha_0 \left(\frac{X}{K}\right)^{\frac{1}{\sigma}} K_0^{\frac{1-\sigma}{\sigma}} \tag{A.13}$$

where X is specified by (13). From (A.13), following Brown (1966), we obtain

$$\frac{\partial^2 X}{\partial K^2} = \frac{J}{K}\left(\frac{\partial X}{\partial K} - \frac{X}{K}\right), \tag{A.14}$$

where

$$J = \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{1}{\sigma}\right) \alpha_0 \left(\frac{X}{K}\right)^{\frac{1-\sigma}{\sigma}} K_0^{\frac{1-\sigma}{\sigma}} > 0. \tag{A.15}$$

Combining (A.13), (A.14), and (A.15) yields the following after rearranging:

$$\frac{\partial^2 X}{\partial K^2} = \frac{JX}{K^2}\left[\frac{1}{1 + \frac{1-\alpha_0}{\alpha_0}\left(\frac{K}{K_0}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{L}{L_0}\right)^{\frac{\sigma-1}{\sigma}}} - 1\right] \tag{A.16}$$

Since $\frac{1-\alpha_0}{\alpha_0}\left(\frac{K}{K_0}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{L}{L_0}\right)^{\frac{\sigma-1}{\sigma}} > 0$, the first term in the square brackets is less than unity. Consequently, we have $\partial^2 X/\partial K^2 < 0$, and thus, the marginal product of capital, G_K, is downward-sloping in the aggregate capital stock. Next, we need to show that the marginal product of labor, G_L, is increasing in K. The marginal product of labor, G_L, is given by the following:

$$\frac{\partial X}{\partial L} = \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} (1-\alpha_0) \left(\frac{X}{L}\right)^{\frac{1}{\sigma}} L_0^{\frac{1-\sigma}{\sigma}}. \tag{A.17}$$

where X is given by (13). From (A.17), we can derive the following:

$$\frac{\partial^2 X}{\partial L \partial K} = \frac{\left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-2}{\sigma}} (1-\alpha_0)\alpha_0 \left(K_0 L_0\right)^{\frac{1-\sigma}{\sigma}} (KL)^{-\frac{1}{\sigma}}}{\sigma \left[\alpha_0 \left(\frac{K}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{L}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}} > 0, \tag{A.18}$$

and thus, the marginal product of labor, G_L, is increasing in the aggregate capital stock K.

Second, we show that at the initial point of normalization, marginal products of capital, G_K, and labor, G_L, are independent of the value of σ. According to (A.13), at the aggregate level, the marginal product of capital is given by the following:

$$\frac{\partial X}{\partial K} = \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} \alpha_0 \left(\frac{X}{K}\right)^{\frac{1}{\sigma}} K_0^{\frac{1-\sigma}{\sigma}}$$

At the initial point of normalization, we have $K = K_0$ and $L = L_0$. Plugging these values into (13), aggregate output can be written by $X = X_0 = N^{\frac{\epsilon}{\epsilon-1}} Y_0$ and is thus independent of σ. Plugging this result into (A.13) and substituting K with K_0 obtains

$$\begin{aligned} G_{K,0} &= \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} \alpha_0 \left(\frac{N^{\frac{\epsilon}{\epsilon-1}} Y_0}{K_0}\right)^{\frac{1}{\sigma}} K_0^{\frac{1-\sigma}{\sigma}} \\ &= \alpha_0 N^{\frac{\epsilon}{\epsilon-1}} \frac{Y_0}{K_0}, \end{aligned}$$

which is identical to equation (17). Note that at the initial point of normalization, the capital share in aggregate production, as given by $\alpha_0 = \frac{F_{K,0} K_0}{F_{K,0} K_0 + F_{L,0} L_0} = \frac{K_0}{K_0 + [F_K / F_L]_0 L_0} = \frac{K_0}{K_0 + \mu_0 L_0}$, is independent of σ. It also solely depends on benchmark values. Similarly, $G_{K,0}$ is independent of σ. Analogous, at the initial point of normalization, for the marginal product of labor as specified by (A.17), we obtain the following:

$$\begin{aligned} G_{L,0} &= \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} (1-\alpha_0) \left(\frac{N^{\frac{\epsilon}{\epsilon-1}} Y_0}{L_0}\right)^{\frac{1}{\sigma}} L_0^{\frac{1-\sigma}{\sigma}} \\ &= (1-\alpha_0) N^{\frac{\epsilon}{\epsilon-1}} \frac{Y_0}{L_0} \end{aligned}$$

which is identical to (18) and independent of σ.

Finally, we build upon Mallick (2012) to show that for all $K > K_0$ ($K < K_0$), G_K (G_L) is increasing in the value of σ. We start by rearranging the marginal product of capital, G_K, as given by (A.13), according to the following:

$$\frac{\partial X}{\partial K} = \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} \alpha_0 \left(\frac{X}{K}\right)^{\frac{1-\sigma}{\sigma}} K_0^{\frac{1-\sigma}{\sigma}} \frac{X}{K}, \tag{A.19}$$

where $\left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} \alpha_0 \left(\frac{X}{K}\right)^{\frac{1-\sigma}{\sigma}} K_0^{\frac{1-\sigma}{\sigma}} = \left(1 + \frac{1-\alpha_0}{\alpha_0} \left(\frac{K/K_0}{L/L_0}\right)^{\frac{1-\sigma}{\sigma}}\right)^{-1} = \frac{\partial X}{\partial K}\frac{K}{X}$ is the capital share of the final output, and X/K is the average product of capital.

The partial derivation of $\frac{\partial X}{\partial K}\frac{K}{X}$ with respect to σ yields the following:

$$\frac{\partial}{\partial \sigma}\left(\frac{\partial X}{\partial K}\frac{K}{X}\right) = \ln\left(\frac{K/K_0}{L/L_0}\right) \frac{\frac{1-\alpha_0}{\alpha_0}\left(\frac{K/K_0}{L/L_0}\right)^{\frac{1-\sigma}{\sigma}}}{\sigma^2\left(1 + \frac{1-\alpha_0}{\alpha_0}\left(\frac{K/K_0}{L/L_0}\right)^{\frac{1-\sigma}{\sigma}}\right)^2}, \tag{A.20}$$

and thus $sgn\left(\frac{\partial}{\partial \sigma}\left(\frac{\partial X}{\partial K}\frac{K}{X}\right)\right) = sgn\left(\ln\left(\frac{K/K_0}{L/L_0}\right)\right)$. Due to our assumption of a constant endowment of labor, L/L_0 is equal to unity. As a result, for all $K > K_0$ ($K < K_0$), the capital share of final output is strictly increasing (decreasing) in σ. Analogously, we can rearrange the marginal product of labor, G_L, as given by (A.17), according to the following:

$$\frac{\partial X}{\partial L} = \left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} (1-\alpha_0) \left(\frac{X}{L}\right)^{\frac{1-\sigma}{\sigma}} L_0^{\frac{1-\sigma}{\sigma}} \frac{X}{L}, \tag{A.21}$$

where $\left(N^{\frac{\epsilon}{\epsilon-1}} Y_0\right)^{\frac{\sigma-1}{\sigma}} (1-\alpha_0) \left(\frac{X}{L}\right)^{\frac{1-\sigma}{\sigma}} L_0^{\frac{1-\sigma}{\sigma}} = \left(1 + \frac{\alpha_0}{1-\alpha_0}\left(\frac{K/K_0}{L/L_0}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1} = \frac{\partial X}{\partial L}\frac{L}{X}$ is the labor share of final output, and X/L is the average product of labor.

The partial derivation of $\frac{\partial X}{\partial L}\frac{L}{X}$ with respect to σ yields the following:

$$\frac{\partial}{\partial \sigma}\left(\frac{\partial X}{\partial L}\frac{L}{X}\right) = -\ln\left(\frac{K/K_0}{L/L_0}\right) \frac{\frac{\alpha_0}{1-\alpha_0}\left(\frac{K/K_0}{L/L_0}\right)^{\frac{\sigma-1}{\sigma}}}{\sigma^2\left(1 + \frac{\alpha_0}{1-\alpha_0}\left(\frac{K/K_0}{L/L_0}\right)^{\frac{\sigma}{1-\sigma}}\right)^2}, \tag{A.22}$$

and thus $sgn\left(\frac{\partial}{\partial \sigma}\left(\frac{\partial X}{\partial L}\frac{L}{X}\right)\right) = -\,sgn\left(\ln\left(\frac{K/K_0}{L/L_0}\right)\right)$. As a result, due to our assumption of a constant endowment of labor, the labor share of final output is strictly increasing (decreasing) in σ for all $K < K_0$ ($K > K_0$).

Next, it can be shown that both the average product of capital,

$$\frac{X}{K} = \frac{N^{\frac{\epsilon}{\epsilon-1}} Y_0 \left[\alpha_0 \left(\frac{K}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{L}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{K}, \tag{A.23}$$

and labor,

$$\frac{X}{L} = \frac{N^{\frac{\epsilon}{\epsilon-1}} Y_0 \left[\alpha_0 \left(\frac{K}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{L}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{L}, \tag{A.24}$$

are strictly increasing in σ. As already mentioned in Arrow et al. (1961), the term $\left[\alpha_0 \left(\frac{K}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{L}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ can be interpreted as a general mean of order $\frac{\sigma-1}{\sigma}$. As demonstrated by Hardy et al. (1952) and de La Grandville and Solow (2017), a general mean is an increasing function of its order. In (A.23) and (A.24) the order, in turn, is an increasing function of σ; thus, we have that $\partial\left(X/K\right)/\partial\sigma > 0$ and $\partial\left(X/L\right)/\partial\sigma > 0$. Combining this result with (A.20) and (A.22), it follows that for all $K > K_0$ ($K < K_0$), G_K (G_L) is increasing in the value of σ.[29]

A.5 Proof of Proposition 2

In this appendix, we derive the steady-state capital stock, K^*, as given by (19), and show that for all $K^* > K_0$, the steady-state capital stock is increasing in the value of σ. Without technological change and a constant endowment of labor, a steady state in the benchmark model requires $sX = \delta K$. Substituting X by (13) and solving for K gives equation (19) in the text. The uniqueness and existence of K^* follows from $G(0, L) = 0$, $\delta < \lim_{K\to 0} s\frac{\partial G}{\partial K} = \infty$, $\delta > \lim_{K\to\infty} s\frac{\partial G}{\partial K} = 0$, and $\frac{\partial^2 G}{\partial K^2} < 0$, for all $\sigma \in (0, 1)$.

Next, we build upon de La Grandville and Solow (2017) to provide a simple proof that the steady-state capital stock of the benchmark model, as specified by (19), is increasing in the value of σ.[30] Combining (3) and (13), the law of motion for the aggregate capital stock, K, can be specified by the following:

$$K_{t+1} = sN^{\frac{\epsilon}{\epsilon-1}} Y_0 \left[\alpha_0 \left(\frac{K_t}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{L_t}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \delta K_t. \tag{A.25}$$

[29] Note that for $K < K_0$ ($K > K_0$), the sign of the partial derivative $\partial G_K/\partial\sigma$ ($\partial G_L/\partial\sigma$) is ambiguous and dependent on parameter values.

[30] For an alternative proof, see Klump and Preissler (2000).

As we have seen in Appendix A.4, the term $\left[\alpha_0 \left(\frac{K_t}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{L_t}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ can be interpreted as a general mean of order $\frac{\sigma-1}{\sigma}$ and is thus increasing in σ. Consider, now, two values of the elasticity of substitution with $\sigma_1 < \sigma_2 < 1$, which guarantee the existence of the steady-state capital stock K_1^* and K_2^*, respectively. Based on the property of general means, we have the following:

$$\text{(A.26)} \quad \left[\alpha_0 \left(\frac{K_1^*}{K_0}\right)^{\frac{\sigma_1-1}{\sigma_1}} + (1-\alpha_0)\left(\frac{L}{L_0}\right)^{\frac{\sigma_1-1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1-1}} < \left[\alpha_0 \left(\frac{K_1^*}{K_0}\right)^{\frac{\sigma_2-1}{\sigma_2}} + (1-\alpha_0)\left(\frac{L}{L_0}\right)^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}},$$

and thus $sN^{\frac{\epsilon}{\epsilon-1}} Y_0 \left[\alpha_0 \left(\frac{K_1^*}{K_0}\right)^{\frac{\sigma_2-1}{\sigma_2}} + (1-\alpha_0)\left(\frac{L}{L_0}\right)^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}} > \delta K_1^*$. As a result, for K_2^* to be the steady-state capital stock corresponding to σ_2, it must hold that $K_2^* > K_1^*$. This completes the proof of Proposition 2.

A.6 Proof of Proposition 4

In this appendix, we first show that the unit isoquant curve (36) is continuously differentiable, which requires that its derivative is continuous. The derivate of the unit isoquant curve is given by the marginal rate of technical substitution μ. At the zth point of normalization, where the economy shifts from production technology $z-1$ to z, the left-hand limit of μ is given by the following:

$$\text{(A.27)} \quad \lim_{K \to K_z^-} \mu = \frac{1-\alpha_{z-1}}{\alpha_{z-1}} \left(\frac{K_{z-1}}{L_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}} \left(\frac{K_z}{L_z}\right)^{\frac{1}{\sigma_{z-1}}} = \mu_z.$$

For production technology z, we have

$$\text{(A.28)} \quad \mu = \frac{1-\alpha_z}{\alpha_z} \left(\frac{K_z}{L_z}\right)^{\frac{\sigma_z-1}{\sigma_z}} \left(\frac{K}{L}\right)^{\frac{1}{\sigma_z}}.$$

Inserting (22) into (A.28) leads to the following:

$$\text{(A.29)} \quad \mu = \mu_z \left(\frac{K_z}{L_z}\right)^{-\frac{1}{\sigma_z}} \left(\frac{K}{L}\right)^{\frac{1}{\sigma_z}},$$

where, based on the constancy of the labor supply, the right-hand limit of (A.29) is given by the following: :

$$\text{(A.30)} \quad \lim_{K \to K_z^+} \mu = \mu_z \left(\frac{K_z}{L_z}\right)^{-\frac{1}{\sigma_z}} \left(\frac{K_z}{L_z}\right)^{\frac{1}{\sigma_z}} = \mu_z$$

and thus equal to the left-hand limit of μ, as specified by (A.27). As a result, at the zth point of normalization, the marginal rate of technical substitution is continuous, and therefore, the unit isoquant curve is continuously differentiable.

Next, we show that the marginal product of capital, G_K, is continuous at the zth point of normalization. Combining (34) and (35), the left-hand limit of G_K is given by the following:

$$\begin{aligned}\lim_{K\to K_z^-}\frac{\partial X}{\partial K} &= \left(N^{\frac{\epsilon}{\epsilon-1}}Y_{z-1}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\alpha_{z-1}\left(\frac{X}{K_z}\right)^{\frac{1}{\sigma_{z-1}}}K_{z-1}^{\frac{1-\sigma_{z-1}}{\sigma_{z-1}}} \\ &= \frac{\left(N^{\frac{\epsilon}{\epsilon-1}}Y_{z-1}\right)\alpha_{z-1}K_z^{-\frac{1}{\sigma_{z-1}}}K_{z-1}^{\frac{1-\sigma_{z-1}}{\sigma_{z-1}}}}{\left[\alpha_{z-1}\left(\frac{K_z}{K_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}+(1-\alpha_{z-1})\left(\frac{L_z}{L_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\right]^{\frac{1}{1-\sigma_{z-1}}}}. \qquad \text{(A.31)}\end{aligned}$$

Moreover, at the zth point of normalization, the right-hand limit of G_K is given by the following:

$$\begin{aligned}\lim_{K\to K_z^+}\frac{\partial X}{\partial K} &= \left(N^{\frac{\epsilon}{\epsilon-1}}Y_z\right)^{\frac{\sigma_z-1}{\sigma_z}}\alpha_z\left(\frac{X}{K_z}\right)^{\frac{1}{\sigma_z}}K_z^{\frac{1-\sigma_z}{\sigma_z}} \\ &= \frac{\left(N^{\frac{\epsilon}{\epsilon-1}}Y_z\right)\alpha_z K_z^{-\frac{1}{\sigma_z}}K_z^{\frac{1-\sigma_z}{\sigma_z}}}{\left[\alpha_z\left(\frac{K_z}{K_z}\right)^{\frac{\sigma_z-1}{\sigma_z}}+(1-\alpha_z)\left(\frac{L_z}{L_z}\right)^{\frac{\sigma_z-1}{\sigma_z}}\right]^{\frac{1}{1-\sigma_z}}} \\ &= N^{\frac{\epsilon}{\epsilon-1}}\frac{Y_z}{K_z}\alpha_z. \qquad \text{(A.32)}\end{aligned}$$

Inserting (21), (22), and (23) into (A.32) yields (A.31) after some rearranging. The marginal product of capital, G_K, is thus continuous at the zth point of normalization.

Next, we show that the second partial derivative of aggregate production function (34) with respect to capital, G_{KK}, has jump discontinuities. At the zth point of normalization, the left-hand limit of G_{KK} is given by the following:

$$\lim_{K\to K_z^-}\frac{\partial^2 X}{\partial K^2} = \left(\frac{1}{\sigma_{z-1}}\right)\frac{QX}{K_z^2}\left[\frac{1}{1+\frac{1-\alpha_{z-1}}{\alpha_{z-1}}\left(\frac{K_z}{K_{z-1}}\right)^{\frac{1-\sigma_{z-1}}{\sigma_{z-1}}}\left(\frac{L_z}{L_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}}-1\right], \qquad \text{(A.33)}$$

where $Q = \left(N^{\frac{\epsilon}{\epsilon-1}}Y_{z-1}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\alpha_{z-1}\left(\frac{X}{K_z}\right)^{\frac{1-\sigma_{z-1}}{\sigma_{z-1}}}K_{z-1}^{\frac{1-\sigma_{z-1}}{\sigma_{z-1}}}$. Moreover, at the zth point of normalization, the right-hand limit of G_{KK} is given by the following:

$$\lim_{K\to K_z^+}\frac{\partial^2 X}{\partial K^2}=\left(\frac{1}{\sigma_z}\right)\alpha_z\left(\alpha_z-1\right)\frac{N^{\frac{\epsilon}{\epsilon-1}}Y_z}{K_z^2}. \tag{A.34}$$

Inserting of (21), (22), and (23) into (A.34) yields the following after rearranging:

$$\lim_{K\to K_z^-}\frac{\partial^2 X}{\partial K^2}=\left(\frac{1}{\sigma_z}\right)\frac{QX}{K_z^2}\left[\frac{1}{1+\frac{1-\alpha_{z-1}}{\alpha_{z-1}}\left(\frac{K_z}{K_{z-1}}\right)^{\frac{1-\sigma_{z-1}}{\sigma_{z-1}}}\left(\frac{L_z}{L_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}}-1\right] \tag{A.35}$$

which is identical to (A.33), except for the first term in curved brackets. Moreover, since the term in squared brackets is negative, it follows that at the zth point of normalization, G_{KK} has jump discontinuities according to the following:

$$\lim_{K\to K_z^-}G_{KK}<\lim_{K\to K_z^+}G_{KK}.$$

for all $\sigma_z>\sigma_{z-1}$.

Finally, we show that the limit of the marginal product of capital, G_K, in the final good production is zero as K approaches infinity. Plugging (34) into (35) gives, after some rearranging, the following expression of G_K:

$$G_K=N^{\frac{\epsilon-1}{\epsilon}}Y_z\alpha_z^{\frac{\sigma_z}{\sigma_z-1}}K_z^{-1}\left[1+\frac{1-\alpha_z}{\alpha_z}\left(\frac{L}{L_z}\right)^{\frac{\sigma_z-1}{\sigma_z}}\left(\frac{K}{K_z}\right)^{\frac{1-\sigma_z}{\sigma_z}}\right]^{\frac{1}{\sigma_z-1}}. \tag{A.36}$$

In the case of a Cobb-Douglas aggregate production function with $\sigma_z=1$, (A.36) reduces to the following:

$$G_K=N^{\frac{\epsilon-1}{\epsilon}}Y_zK^{\alpha_z-1}\left(\frac{1}{K_z}\right)^{\alpha_z}\left(\frac{L}{L_z}\right)^{1-\alpha_z}. \tag{A.37}$$

From (A.36) and (A.37), it follows that

$$\lim_{K\to\infty}G_K=\begin{cases}0 & ,\text{if}\quad \sigma_z<1\\ 0 & ,\text{if}\quad \sigma_z=1\\ N^{\frac{\epsilon-1}{\epsilon}}Y_z\alpha_z^{\frac{\sigma_z}{\sigma_z-1}}K_z^{-1} & ,\text{if}\quad \sigma_z>1\end{cases}. \tag{A.38}$$

This completes the proof of Proposition 3.

A.7 Derivation of ΔY_t

In this appendix, we derive ΔY_t, the additional output an intermediate firm is able to produce when applying an intermediate production technology characterized by $\sigma_z > \sigma_{z-1}$. Utilizing equation (20) for two different technologies, indexed by z and $z-1$, ΔY_t can be written as follows:

$$\begin{aligned}\Delta Y_t = Y_z \left[\alpha_z \left(\frac{K_t}{K_z}\right)^{\frac{\sigma_z-1}{\sigma_z}} + (1-\alpha_z)\left(\frac{L_t}{L_z}\right)^{\frac{\sigma_z-1}{\sigma_z}}\right]^{\frac{\sigma_z}{\sigma_z-1}} \\ - Y_{z-1}\left[\alpha_{z-1}\left(\frac{K_t}{K_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}} + (1-\alpha_{z-1})\left(\frac{L_t}{L_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\right]^{\frac{\sigma_{z-1}}{\sigma_{z-1}-1}}.\end{aligned} \tag{A.39}$$

Moreover, to provide a better comparison between both technologies, we use (21), (22), and (23) to show that the second term on the right-hand side of (A.39) can also be written as follows:

$$\begin{aligned}&Y_{z-1}\left[\alpha_{z-1}\left(\frac{K_t}{K_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}} + (1-\alpha_{z-1})\left(\frac{L_t}{L_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\right]^{\frac{\sigma_{z-1}}{\sigma_{z-1}-1}} \\ &= \frac{Y_{z-1}\left[\alpha_z\left(\frac{K_t}{K_z}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}} + (1-\alpha_z)\left(\frac{L_t}{L_z}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\right]^{\frac{\sigma_{z-1}}{\sigma_{z-1}-1}}}{\left[\alpha_{z-1}\left(\frac{K_z}{K_{z-1}}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}} + (1-\alpha_{z-1})\left(\frac{L_z}{L_{z-1}}\right)^{\frac{1-\sigma_{z-1}}{\sigma_{z-1}}}\right]^{\frac{\sigma_{z-1}}{\sigma_{z-1}-1}}} \\ &= Y_z\left[\alpha_z\left(\frac{K_t}{K_z}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}} + (1-\alpha_z)\left(\frac{L_t}{L_z}\right)^{\frac{\sigma_{z-1}-1}{\sigma_{z-1}}}\right]^{\frac{\sigma_{z-1}}{\sigma_{z-1}-1}}.\end{aligned} \tag{A.40}$$

Plugging (A.40) into (A.39) obtains equation (38).

A.8 Proof of Proposition 5

A steady state in the R&D model requires $sG(K,L,z^*) = \delta K$. Substituting $G(K,L,z^*)$ by

$$G(K,L;z^*) = N^{\frac{\epsilon}{\epsilon-1}} Y_{z^*}\left[\alpha_{z^*}\left(\frac{K}{K_{z^*}}\right)^{\frac{\sigma_{z^*}-1}{\sigma_{z^*}}} + (1-\alpha_{z^*})\left(\frac{L}{L_{z^*}}\right)^{\frac{\sigma_{z^*}-1}{\sigma_z^*}}\right]^{\frac{\sigma_{z^*}}{\sigma_{z^*}-1}}, \tag{A.41}$$

and solving for K obtains equation (39). To establish the existence and uniqueness of K^*, note that $G(K, L; z)$ is continuously differentiable. Moreover, we have $G(0, L; z) = 0$, $\delta < \lim_{K \to 0} s\frac{\partial G(K,L;z)}{\partial K} = \infty$, $\delta > \lim_{K \to \infty} s\frac{\partial G(K,L;z)}{\partial K} = 0$, and $\frac{\partial^2 G(K,L;z)}{\partial K^2} < 0$ for all $\sigma_z \in (0, 1)$, such that there exists a K^* that satisfies (39).

References

Acemoglu, D. and Cao, D. (2015). Innovation by entrants and incumbents. *Journal of Economic Theory*, 157:255–294.

Acemoglu, D. and Guerrieri, V. (2008). Capital Deepening and Nonbalanced Economic Growth. *Journal of Political Economy*, 116(3):467–498.

Aghion, P., Akcigit, U., and Howitt, P. (2014). What Do We Learn From Schumpeterian Growth Theory? In Aghion, P. and Durlauf, S., editors, *Handbook of Economic Growth*, volume 2B, pages 515–563. North Holland.

Aghion, P. and Howitt, P. (1992). A Model of Growth Through Creative Destruction. *Econometrica*, 60(2):323–351.

Alvarez-Cuadrado, F., Van Long, N., and Poschke, M. (2017). Capital-labor substitution, structural change, and growth. *Theoretical Economics*, 12(3):1229–1266.

Antony, J. (2009a). Capital/Labor substitution, capital deepening, and FDI. *Journal of Macroeconomics*, 31(4):699–707.

Antony, J. (2009b). A dual elasticity of substitution production function with an application to cross-country inequality. *Economics Letters*, 102(1):10–12.

Antony, J. (2010). A class of changing elasticity of substitution production functions. *Journal of Economics*, 100(2):165–183.

Arrow, K. J., Chenery, H. B., Minhas, B. S., and Solow, R. M. (1961). Capital-Labor Substitution and Economic Efficiency. *The Review of Economics and Statistics*, 43(3):225–250.

Bekar, C., Carlaw, K. I., and Lipsey, R. G. (2018). General purpose technologies in theory, application and controversy: A review. *Journal of Evolutionary Economics*, 28(5):1005–1033.

Benassy, J.-P. (1996). Taste for variety and optimum production patterns in monopolistic competition. *Economics Letters*, 52(1):41–47.

Benassy, J.-P. (1998). Is there always too little research in endogenous growth with expanding product variety? *European Economic Review*, 42(1):61–69.

Bergholt, D. (2011). A note on CES functions. *BI Norwegian Business School*, pages 1–6.

Bernard, A. B., Eaton, J., Jensen, J. B., and Kortum, S. (2003). Plants and Productivity in International Trade. *The American Economic Review*, 93(4):1268–1290.

Bloom, N., Jones, C. I., Van Reenen, J., and Webb, M. (2020). Are Ideas Getting Harder to Find? *American Economic Review*, 110(4):1104–1144.

Bond-Smith, S. (2019). The Decades-long Dispute over Scale Effects in the Theory of Economic Growth. *Journal of Economic Surveys*, 33(5):1359–1388.

Bresnahan, T. F. and Trajtenberg, M. (1995). General purpose technologies 'Engines of Growth'? *Journal of Econometrics*, 65(1):83–108.

Brown, M. (1966). *On the Theory and Measurement of Technological Change*. Cambridge University Press.

Brown, M. and De Cani, J. S. (1963). Technological Change and the Distribution of Income. *International Economic Review*, 4(3):289–309.

Cantore, C., Ferroni, F., and León-Ledesma, M. A. (2017). The dynamics of hours worked and technology. *Journal of Economic Dynamics and Control*, 82:67–82.

Carlaw, K. I. and Lipsey, R. G. (2006). GPT-Driven, Endogenous Growth. *The Economic Journal*, 116(508):155–174.

Carlaw, K. I. and Lipsey, R. G. (2011). Sustained endogenous growth driven by structured and evolving general purpose technologies. *Journal of Evolutionary Economics*, 21(4):563–593.

Crafts, N. (2004). Steam as a general purpose technology: A growth accounting perspective. *The Economic Journal*, 114(495):338–351.

David, P. A. (1990). The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox. *The American Economic Review*, 80(2), Papers and Proceedings of the Hundred and Second Annual Meeting of the American Economic Association:355–361.

de La Grandville, O. (1989). In Quest of the Slutsky Diamond. *American Economic Review*, 79(3):468–481.

de La Grandville, O. (2007). The 1956 contribution to economic growth theory by Robert Solow: a major landmark and some of its undiscovered riches. *Oxford Review of Economic Policy*, 23(1):15–24.

de La Grandville, O. (2017). *Economic Growth: A Unified Approach*. Cambridge University Press, second edition.

de La Grandville, O. and Solow, R. M. (2017). Capital-Labour Substitution and Economic Growth. In *Economic Growth - A Unified Approach*, pages 114–154. Cambridge University Press, Cambridge.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *The American Economic Review*, 67(3):297–308.

Feenstra, R. C., Inklaar, R., and Timmer, M. P. (2015). The Next Generation of the Penn World Table. *The American Economic Review*, 105(10):3150–3182.

Fenichel, E. P. and Zhao, J. (2015). Sustainability and Substitutability. *Bulletin of Mathematical Biology*, 77(2):348–367.

Gómez, M. A. (2020). Factor substitution, long-run growth, and the speed of convergence in the one-sector convex endogenous-growth model. *Metroeconomica*, 71(1):2–21.

Griliches, Z. (1990). Patent Statistics as Economic Indicators: A Survey. *Journal of Economic Literature*, 28(4):1661–1707.

Griliches, Z. (1994). Productivity, R&D, and the Data Constraint. *The American Economic Review*, 84(1):1–23.

Grossman, G. M. and Helpman, E. (1991a). *Innovation and Growth in the Global Economy*. MIT Press, Cambridge, MA.

Grossman, G. M. and Helpman, E. (1991b). Quality Ladders in the Theory of Growth. *The Review of Economic Studies*, 58(1):43–61.

Growiec, J. (2008a). A new class of production functions and an argument against purely labor-augmenting technical change. *International Journal of Economic Theory*, 4(4):483–502.

Growiec, J. (2008b). Production functions and distributions of unit factor productivities: Uncovering the link. *Economics Letters*, 101(1):87–90.

Growiec, J. (2013). A microfoundation for normalized CES production functions with factor-augmenting technical change. *Journal of Economic Dynamics and Control*, 37(11):2336–2350.

Growiec, J. (2018). Factor-specific technology choice. *Journal of Mathematical Economics*, 77:1–14.

Hardy, G. H., Littlewood, J. E., and Pólya, G. (1952). *Inequalities*. Cambridge University Press, Cambridge.

Herrendorf, B., Herrington, C., and Valentinyi, A. (2015). Sectoral Technology and Structural Transformation. *American Economic Journal: Macroeconomics*, 7(4):104–133.

Irmen, A. (2011). Steady-state growth and the elasticity of substitution. *Journal of Economic Dynamics and Control*, 35(8):1215–1228.

Jiang, W. and León-Ledesma, M. (2018). Variable markups and capital-labor substitution. *Economics Letters*, 171:34–36.

Jones, C. I. (1995). R&D-Based Models of Economic Growth. *Journal of Political Economy*, 103(4):759–784.

Jones, C. I. (2003). Growth, Capital Shares, and a New Perspective on Production Functions. *University of California, Berkeley*.

Jones, C. I. (2005). The Shape of Production Functions and the Direction of Technical Change. *The Quarterly Journal of Economics*, 120(2):517–549.

Jovanovic, B. and Rousseau, P. L. (2005). General Purpose Technologies. In Aghion, P. and Durlauf, S. N., editors, *Handbook of Economic Growth*, volume 1 B, chapter 18, pages 1181–1224. North Holland.

Karagiannis, G., Palivos, T., and Papageorgiou, C. (2005). Variable Elasticity of Substitution and Economic Growth: Theory and Evidence. In Diebolt, C. and Kyrtsou, C., editors, *New Trends in Macroeconomics*, pages 21–37. Springer, Berlin, Heidelberg.

Karbarbounis, L. and Neiman, B. (2014). The Global Decline of the Labor Share. *The Quarterly Journal of Economics*, 129(1):61–103.

Klump, R. and de La Grandville, O. (2000). Economic Growth and the Elasticity of Substitution: Two Theorems and Some Suggestions. *American Economic Review*, 90(1):282–291.

Klump, R., McAdam, P., and Willman, A. (2007a). Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach. *The Review of Economics and Statistics*, 89(1):183–192.

Klump, R., McAdam, P., and Willman, A. (2007b). The Long-Term SucCESs of the Neoclassical Growth Model. *Oxford Review of Economic Policy*, 23(1):94–114.

Klump, R., McAdam, P., and Willman, A. (2012). The normalized CES production function: Theory and empirics. *Journal of Economic Surveys*, 26(5):769–799.

Klump, R. and Preissler, H. (2000). CES Production Functions and Economic Growth. *The Scandinavian Journal of Economics*, 102(1):41–56.

Knoblach, M., Roessler, M., and Zwerschke, P. (2020). The Elasticity of Substitution Between Capital and Labour in the US Economy: A Meta-Regression Analysis. *Oxford Bulletin of Economics and Statistics*, 82(1):62–82.

Knoblach, M. and Stöckl, F. (forthcoming). What Determines the Elasticity of Substitution between Capital and Labor? A Literature Review. *Journal of Economic Surveys*.

Koeniger, W. and Licandro, O. (2006). On the Use of Substitutability as a Measure of Competition. *The B.E. Journal of Macroeconomics*, 6(1):1–9.

Kogan, L., Papanikolaou, D., Seru, A., and Stoffman, N. (2017). Technological Innovation, Resource Allocation, and Growth. *The Quarterly Journal of Economics*, 132(2):665–712.

Kortum, S. S. (1993). Equilibrium R&D and the Patent-R&D Ratio: U.S. Evidence. *The American Economic Review*, 83(2), Papers and Proceedings of the Hundred and Fifth Annual Meeting of the American Economic Association:450–457.

Kortum, S. S. (1997). Research, Patenting, and Technological Change. *Econometrica*, 65(6):1389–1419.

León-Ledesma, M. A., McAdam, P., and Willman, A. (2015). Production Technology Estimates and Balanced Growth. *Oxford Bulletin of Economics and Statistics*, 77(1):40–65.

León-Ledesma, M. A. and Satchi, M. (2019). Appropriate Technology and Balanced Growth. *The Review of Economic Studies*, 86(2):807–835.

Mallick, D. (2010). Capital-Labor Substitution and Balanced Growth. *Journal of Macroeconomics*, 32(4):1131–1142.

Mallick, D. (2012). The role of the elasticity of substitution in economic growth: A cross-country investigation. *Labour Economics*, 19(5):682–694.

Matveenko, A. (2011). Stochastic Models of the Technological Ideas Flow as a Foundation of the Production Function. In *International Days of Statistics and Economics, Prague*.

Matveenko, A. and Matveenko, V. (2015). Technological Choice Generating Normalized CES Production Functions. In *Conference on Dynamics, Economic Growth, and International Trade, Geneva*.

Matveenko, V. (2010). Anatomy of Production Functions: A Technological Menu and a Choice of the Best Technology. *Economics Bulletin*, 30(3):1906–1913.

Miyagiwa, K. and Papageorgiou, C. (2003). Elasticity of Substitution and Growth: Normalized CES in the Diamond Model. *Economic Theory*, 21(1):155–165.

Miyagiwa, K. and Papageorgiou, C. (2007). Endogenous Aggregate Elasticity of Substitution. *Journal of Economic Dynamics and Control*, 31(9):2899–2919.

Nakamura, H. (2009). Micro-Foundation for a Constant Elasticity of Substitution Production Function through Mechanization. *Journal of Macroeconomics*, 31(3):464–472.

Nakamura, H. (2010). Factor Substitution, Mechanization, and Economic Growth. *The Japanese Economic Review*, 61(2):266–281.

Nakamura, H. and Nakamura, M. (2008). Constant-Elasticity-of-Substitution Production Function. *Macroeconomic Dynamics*, 12(5):694–701.

Ngai, L. R. and Pissarides, C. A. (2007). Structural Change in a Multisector Model of Growth. *The American Economic Review*, 97(1):429–443.

Papageorgiou, C. and Saam, M. (2008). Two-Level CES Production Technology in Solow and Diamond Growth Models. *The Scandinavian Journal of Economics*, 110(1):119–143.

Ristuccia, C. A. and Solomou, S. (2014). Can general purpose technology theory explain economic growth? Electrical power as a case study. *European Review of Ecnomic History*, 18(3):227–247.

Rivera-Batiz, L. A. and Romer, P. M. (1991). Economic Integration and Economic Growth. *The Quarterly Journal of Economics*, 106(2):531–555.

Schaefer, A., Schiess, D., and Wehrli, R. (2014). Long-term growth driven by a sequence of general purpose technologies. *Economic Modelling*, 37:23–31.

Schumpeter, J. (1934). *The Theory of Economic Development*. Harvard University Press, Cambridge, MA.

Schumpeter, J. (1942). *Capitalism, Socialism and Democracy*. Harper and Brothers, New York.

Seegmuller, T. (2009). Capital-Labour Substitution and Endogenous Fluctuations: A Monopolistic Competition Approach with Variable Markup. *The Japanese Economic Review*, 60(3):301–319.

Segerstrom, P. S. (1991). Innovation, Imitation and Economic Growth. *Journal of Political Economy*, 99(4):807–827.

Segerstrom, P. S. (1998). Endogenous Growth without Scale Effects. *The American Economic Review*, 88(5):1290–1310.

Solow, R. M. (1956). A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1):65–94.

Solow, R. M. (1957). Technical Change and the Aggregate Production Function. *The Review of Economics and Statistics*, 39(3):312–320.

Spence, M. (1976). Product Selection, Fixed Costs, and Monopolistic Competition. *The Review of Economic Studies*, 43(2):217–235.

Stewart, K. G. (2018). Normalized CES supply systems: Replication of Klump, McAdam, and Willman (2007). *Journal of Applied Econometrics*, 33(2):290–296.

Stewart, K. G. and Li, J. (2018). Are factor biases and substitution identifiable? The Canadian evidence. *Canadian Journal of Economics*, 51(2):528–548.

Stöckl, F. (2020). Is Substitutability the New Efficiency? - Endogenous Investment in the Elasticity of Substitution between Clean and Dirty Energy. *DIW Discussion Paper No. 1886*.

Stokey, N. L. (1995). R&D and Economic Growth. *Review of Economic Studies*, 62(3):469–489.

Swan, T. W. (1956). Economic Growth and Capital Accumulation. *Economic Record*, 32(2):334–361.

Turnovsky, S. J. (2008). The Role of Factor Substitution in the Theory of Economic Growth and Income Distribution: Two Examples. *Journal of Macroeconomics*, 30(2):604–629.

Xue, J. and Yip, C. K. (2012). Factor Substitution and Economic Growth: A Unified Approach. *Macroeconomic Dynamics*, 16(4):625–656.

Xue, J. and Yip, C. K. (2013). Aggregate Elasticity of Substitution and Economic Growth: A Synthesis. *Journal of Macroeconomics*, 38(Part A):60–75.

Yang, X. and Heijdra, Ben, J. (1993). Monopolistic Competition and Optimum Product Diversity: Comment. *The American Economic Review*, 83(1):295–301.

Young, A. T. (2013). US Elasticities of Substitution and Factor-Augmentation at the Industry Level. *Macroeconomic Dynamics*, 17(4):861–897.