ON THE USE OF SINGULAR PERTURBATION BASED MODEL HIERARCHIES OF AN ELECTROHYDRAULIC DRIVE FOR VIRTUALIZATION PURPOSES

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ABSTRACT

Virtualization of products means the representation of some of their properties by models. In a stronger digitalized world, these models will gain a much broader use than models had in engineering so far. Even for one modelling aspect different models of the same product will be used, depending on the specific need of the model user. That need may change in the course of product life, between first product concepts till over the different phases of development, to product use, maintenance, or even recycling. Since a digitalized world use of these diverse models will not be limited to experts model consistency will play a much stronger role. Model hierarchies will play a stronger role and can serve also as means for teaching product users a deeper understanding of product properties. A consistent model hierarchy leading from a simple to a more advanced property representation can support this learning process. In this paper perturbation methods are analyzed as a means for setting up model hierarchies in a consistent manner. This is studied by models for the behavior of a electrohydraulic drive, which consists of a variable speed motor, a pump, a double stroke cylinder and a counterbalance valve. Model hierarchy is achieved by model reduction in the sense of perturbation theory. The use of these different models for different questions in a system design context and their interrelations are exemplified.

Keywords: model hierarchies, virtualization, singular perturbation, electrohydraulic drive, digital twin, model order reduction, stability.

1. GENERAL INSTRUCTIONS

Since decades, formal models represented by data structures and processed by simulation programs in computers are indispensable means in modern engineering. So far, those models have been set up for a well-defined use in certain phases, like, for instance, for product planning, development, manufacturing, for control development, and for maintenance. Mainly experts created and used those models or gave assistance to users with lower modelling and simulation expertise. The trend towards a stronger digitalization of the technical world - currently heavily emphasized in the public and in industry and promoted by various political initiatives, like the German Industrie4.0 - gives models an additional role: to be virtual representatives of conventional products, e.g., of mechatronic systems. The word "digital twin", see e.g. [1], is the most popular term to express this model role. The

corresponding activity is virtualization, one of the strong consequences of digitalization in the engineering world. More persons than today will become stakeholders of the models. Centering digitalization to persons is one of the pillars of Industrie 4.0 (see [2]). Different models of a certain system or component will serve these stakeholders' different needs concerning the qualitative model purpose or the expected model accuracy. Model management will become much more challenging than today due to the diversity of use cases and to guarantee consistency between models and as well as a correct use which cannot longer be fully controlled by experts.

Models serve a certain purpose ([3]). In the envisaged product virtualization context, purposes are manifold, e.g.: to represent its building structure, its physical behavior, its functional decomposition structure, or its communication capabilities. These diverse

models of one system have something in common: a CAD model of the mechanical structure shares some information with a dynamical model of its vibration properties; the hydraulic schematic of the electrohydraulic drive in this paper is related to topological properties of a hydraulic system model. A coarse model of the basic hydraulic performance characteristics of this hydraulic drive, like its maximum force, pressure, speed, size and motor speed relations is in some way also included in a much more refined model of the nonlinear system dynamics. The refined hydraulic drive model may be used to derive the steady state characteristics as well, but requires many more input data than the simple model. Some users might miss some of these data but may have sufficient information for the coarse model. They even might be too demanded understanding the results of the detailed model and drawing the right conclusions. Computing effort, numerical stability issues, model robustness properties are further aspects which, in general, are more challenging for more complex models. Therefore, it makes sense to provide different system models.

These models should give comparable answers to the same question, e.g., concerning the influence of system parameters on results. This vaguely characterized property might be called model consistency in a weak sense. Furthermore, it is helpful if the model differences can help understanding the system properties and which parameters influence certain properties strongly. Systematic approaches for establishing such models with strongly overlapping purpose which provide also a means to assure or validate consistency are desirable. Model hierarchies have been discussed to some extent in the mechatronic system development context often in a more general sense than in this paper, e.g., by [4], [5], [6], [7], since hierarchies provide a simple ordering and ease the understanding of relationships. In this paper a perturbation approach is discussed for this purpose. It works for model classes which have a common base in form of a most complex model from which simpler models can be derived by setting certain and typically small parameters zero. This is a quite natural approach, since most used models disregard certain effects which play a negligible role for the specific model purpose. Further on, perturbation theory is a highly developed discipline which can help to systematically

compare models, hence quantify model consistency, and it defines a model hierarchy in a natural and consistent way. That is done in an exemplary fashion for a nowadays very popular electrohydraulic drive.

2. EXAMPLE ELECTROHYDRAULIC DRIVE

2.1. Governing Equations

This drive concept (see Fig. 1) uses a speed variable electric motor to rotate a constant displacement pump (displacement volume V_p) and to move the cylinder piston. The different oil flows of both cylinder ports are compensated by a counterbalance (CBV) and a check valve (CHK). Closed loop control is achieved by measuring the velocity of the piston, comparing it to the command variable and feeding back the difference to a PI-controller. The drive is loaded by a process force F which is assumed being constant enabling rigorous stability statements. Those would be also feasible if F is slowly time dependent with a sufficient frequency gap between $F(t)$ and the fast dynamics of the self excited oscillations of the Hopf-bifurcation instability. The assumptions for the mathematical model of the overall system are that the compressibility of the hydraulic oil can be represented in linearized form, there is no stiction between piston and cylinder, all mechanical parts can be described sufficiently accurate as rigid bodies, nonlinear effects of the electric motor (as e.g. stray flux) can be neglected, there are no spatial pressure gradients in the two volumes of the chambers A and B, the valve's flow rates can be described by a quadratic relation between pressure and flow rate and that the inertia of the hydraulic oil is neglected. With these assumptions effects like cavitation, wave propagation, elasticity of the piston and the poppet in the CBV are ignored and the mathematical model reduces to a system of eight coupled nonlinear ordinary differential equations.

$$\dot{p}_A = \frac{E_{Fl}(-A_A w - q_A + q_P)}{A_A x + V_{A0}} \quad (1a)$$

$$\dot{p}_B = \frac{E_{Fl}(A_B w + q_B - q_P)}{-A_B x + V_{B0}} \quad (2b)$$

$$\dot{x} = w \quad (3c)$$

$$\dot{w} = \frac{-F(t) + p_A A_A - p_B A_B - d_W w}{m_F} \quad (4d)$$

$$\dot{z} = v \quad (5e)$$

$$\dot{v} = \frac{-c(z + z_0) + p_A A_{P1} + p_B A_{P3} - d_V v}{m_P} \quad (6f)$$

$$\dot{\omega}_M = \frac{M_N - (p_A - p_B)V_P/(2\pi) - d_P \omega_M}{\theta_{tot}} \quad (7g)$$

$$\dot{e}_I = \frac{w_d - w}{\varepsilon_I} \quad (8h)$$

Equations (9a) and (10b) describe the pressure build-up in chambers *A* and *B* respectively which is affected by inflowing oil and by the movement of the piston using a linearized compressibility relation. Equations (11c) and (12d) account for the motion of the piston which results from the external force *F* and the weighted pressure difference between chamber *A* and *B*. The equations of motion for the poppet of the CBV are illustrated in Equations (13e) and (14f). The model of the CBV consists of a simple mass-spring system with a damping coefficient d_V proportional to the velocity of the poppet. The equation of motion for the electric motor (15g) incorporates the torque M_N delivered by the electric motor, the resulting torque on the displacement pump due to the pressure difference and a damping factor d_P which models effects as viscous friction of the bearings and other dissipative actions. The internal state of the PI-controller is described in Equation (16h) where ε_I stands for the inverse I-coefficient of the PI-controller. The torque M_N (which is the PI-controller output) and the flow rate q_P of the displacement pump as well as the flow rate q_A through the CBV are algebraic functions of the state variables (Eq. (2)). A square root relation between flow rate and pressure drop which is scaled by the opening *z* of the valve is used to model the CBV. This is motivated by the assumption that the electrohydraulic drive is used for long (and fast) stroke actuation which involves large flow rates through the CBV.

Figure 1: Hydraulic schematic of the electrohydraulic drive and sketch of the counterbalance valve

$$M_N = k_P(w_d - w) + k_I e_I \quad (2a)$$

$$e_I = \int (w_d - w) dt \quad (2b)$$

$$q_P = \frac{\omega_M V_P}{2\pi} - \frac{p_A - p_B}{R_L} \quad (2c)$$

$$q_A = \frac{q_N \sqrt{p_A - p_T}}{\sqrt{p_N}} \left(\frac{z}{z_{max}} \right) \quad (2d)$$

2.2. Dimensionless Equations

To identify small parameters for a perturbation analysis the equations are transformed into dimensionless form. To this end, following transformations of the states and of the time *t* are performed: the pressures in chamber *A* and *B* $p_A = P_A p_S$ and $p_B = P_B p_S$ are scaled by the system pressure p_S , dimensionless position $x = X l_S$ is scaled by the piston's stroke length l_S , dimensionless piston velocity $w = W w_{ref}$ is scaled by a reference velocity $w_{ref} = l_S / T_{ref}$, dimensionless poppet position of CBV $z =$

Z_{max} is scaled by the maximum poppet position z_{max} , dimensionless velocity of CBV $v = Vv_{ref}$ is scaled by $v_{ref} = z_{max}/T_z$ with $T_z = 2\pi\sqrt{m_p/c}$, dimensionless angular velocity of motor (gear transmission ratio of clutch is $i = 1$) $\omega_M = \Omega\omega_N$ is scaled by the nominal rotation speed of the electric motor ω_N , dimensionless internal PI-controller state $e_I = E_I l_S$ is scaled by l_S and time $t = \tau T_{ref}$ is scaled by a reference time $T_{ref} = A_A l_S / q_N$ where q_N is the normal flow rate at nominal pressure p_N bar. These transformations and the substitution of equations (2a) – (2d) into (1) result in the dimensionless set of nonlinear differential equations.

$$\varepsilon_P \dot{P}_A = \frac{-\frac{\sqrt{P_A - P_T} Z}{\sqrt{P_N}} + \frac{(\Omega r_L + P_A - P_B)v_P}{r_L} - W}{X + v_{A0}} \quad (3a)$$

$$\varepsilon_P \dot{P}_B = \frac{-\gamma W + \frac{(\Omega r_L + P_A - P_B)v_P}{r_L}}{\gamma(X - v_{B0})} \quad (3b)$$

$$\dot{X} = W \quad (3c)$$

$$\varepsilon_F \dot{W} = -W\delta_w - P_B\gamma - f + P_A \quad (3d)$$

$$\varepsilon_Z \dot{Z} = V \quad (3e)$$

$$\varepsilon_V \dot{V} = -CV\delta_V + P_A\alpha_{P1} + P_B - C(Z + Z_0) \quad (3f)$$

$$\varepsilon_M \dot{\Omega}_M = v_P((E_I + K_P(W_d - W))m_N - \delta_P\Omega - P_A + P_B) \quad (3g)$$

$$\varepsilon_I \dot{E}_I = W_d - W \quad (3h)$$

Where following parameters were used:

$$\begin{aligned} v_{A0} &= \frac{V_{A0}}{A_A l_S} & v_{B0} &= \frac{V_{B0}}{A_B l_S} \\ \gamma &= \frac{A_B}{A_A} & D_P &= \frac{d_p p_S V_P}{2\pi\omega_N} \\ C &= \frac{CZ_{max}}{A_{P3} p_S} & \alpha_{P1} &= \frac{A_{P1}}{A_{P3}} \\ m_N &= \frac{M_N 2\pi}{V_P p_S} & q_N &= \frac{A_A l_S}{T_{ref}} \\ r_L &= \frac{R_L \omega_N V_P}{2\pi p_S} & v_P &= \frac{V_P T_{ref} \omega_N}{2\pi A_A l_S} \\ K_P &= k_P \omega_N / M_N & K_I &= k_I \omega_N T_{ref} / M_N \end{aligned}$$

where k_P and k_I are the parameter of the proportional and the integral term of the PI-controller respectively and

$$\varepsilon_P = \frac{p_S}{E_{Fl}} \quad \varepsilon_F = \frac{m l_S}{A_A T_{ref}^2 p_S} \quad \varepsilon_V = \frac{z_{max} m_P}{A_{P3} T_{ref}^2 p_S} \quad (4)$$

$$\varepsilon_I = \frac{1}{K_I} \quad \varepsilon_Z = \frac{2\pi\sqrt{m_P/c}}{T_{ref}} \quad \varepsilon_M = \frac{\theta_{tot} \omega_N^2}{A_A l_S p_S} \quad (5)$$

Eq. (4) and (5) are the perturbation parameters which – when their values are small – lead to a differential-algebraic system of equations. The process of reducing the system of ordinary differential equations is described in the next paragraph.

3. MODEL REDUCTION USING SINGULAR PERTURBATION THEORY

The mathematical model of a physical system always comes along with simplifications by, e.g., dimensionality reduction. A consistent theory which constitutes the process of simplification of a certain class of dynamical systems is the singular perturbation theory for ordinary differential equations in a form introduced by [8]. It basically states that a dynamical system which consists of two coupled subsystems with slow and fast dynamics has an invariant manifold close to the solution of the reduced problem. The latter is obtained if the fast dynamics is neglected.

The dimensionless state space model (3) exhibits the structure of a singular perturbation model. Due to proper scaling the right hand side of these equations are of order 1 so that the coefficients (ε_i with $i = \{P, F, Z, V, M, I\}$) in front of the state's time derivatives determine how fast the states are changing in time. As $\varepsilon_i \rightarrow 0$ the change of the corresponding state happens instantaneously and the differential equation degenerates to an algebraic equation. If the algebraic equation is fulfilled at isolated points for the fast state variables the singular perturbation model is said to be in standard form.

3.1. System Reduction: $\varepsilon_P, \varepsilon_F, \varepsilon_Z, \varepsilon_V \rightarrow 0$

The variable ε_P is the ratio between system pressure p_S (typical 300 bar) and bulk modulus of the hydraulic oil E_{Fl} (typical 14000 bar) and therefore, its value is in the order of 10^{-2} . The value of ε_F is in the order of 10^{-4} . As the ratio

between $\varepsilon_F: \varepsilon_V$ is $\varepsilon_F/\varepsilon_V = (m/m_p)(l/z_{max})/(A/A_{p3})$ where each factor in brackets scales approximately with size of the actuated piston compared to the size of the CBV and the overall effect ($m \sim scale^3$, $A \sim scale^2$ and $l \sim scale$) ends up being $\varepsilon_F/\varepsilon_V \sim scale^2$. The scaling factor $scale$ is in the range of $scale \approx 10^4$, therefore, ε_V is negligible as $\varepsilon_F \rightarrow 0$. The value of ε_Z is defined by $\varepsilon_Z = T_Z/T_{ref}$. T_Z is the inverse resonance frequency of the mass-spring-system of the poppet CBV which is in the range of several milliseconds and T_{ref} is the time that is needed to drive the piston from one end to the other when the CBV delivers nominal flow rate. Therefore, ε_Z is in the range of 10^{-3} . Taking these small values into account it is a valid simplification to reduce system (3) to the slow model (6) with the singular perturbation parameters $\varepsilon_{\{P,F,Z,V\}} = 0$ and to algebraic equations (7) which further simplify to the following form if leakage is neglected, i.e. leakage resistance $r_L \rightarrow \infty$.

$$\dot{X} = W \quad (6a)$$

$$\varepsilon_M \dot{\Omega}_M = v_P((E_I + K_P(W_d - W))m_N - \delta_P \Omega - P_A + P_B) \quad (6b)$$

$$\varepsilon_I \dot{E}_I = W_d - W \quad (6c)$$

$$V = 0 \quad (7a)$$

$$W = \frac{\Omega v_P}{\gamma} \quad (7b)$$

$$Z = \frac{-C\gamma^2 Z_0 + \gamma^2 P_A \alpha_{p1} + \gamma P_A - \Omega \delta_W v_P - f\gamma}{C\gamma^2} \quad (7c)$$

$$P_B = \frac{-\Omega \delta_W v_P - f\gamma + \gamma P_A}{\gamma^2} \quad (7d)$$

$$\sqrt{P_A - P_T} = \gamma \Omega v_P \sqrt{P_N} C(\gamma - 1) / ((CZ_0 - P_A \alpha_{p1})\gamma^2 + (f - P_A)\gamma + \Omega \delta_W v_P) \quad (7e)$$

Equation (7b) describes a relation between velocity W and rotational velocity Ω of the pump representing an idealistic view on system behavior, i.e. a rotational movement of the pump immediately results in a linear actuation of the piston. P_A is expressed in an implicit formula (7e) which basically equates the flow rate through the valve and the flow rate of the pump q_P out of chamber A.

3.2. Additional system reduction:

$$\varepsilon_M, \varepsilon_I \rightarrow 0$$

Many users of the presented drive appreciate its simple handling, including those control tasks which they have to provide in the course of system integration. What they expect is a kind of perfect gearbox which transmits the desired speed signal w_d (or W_d) with an acceptable accuracy into the actual piston speed w (or W). In other words, their model concerning system response is $w \equiv w_d$. Of course, that requires acceptable signals w_d and process forces $F(t)$ as well as a sufficient drive performance. This system behavior corresponds to a further reduced system, by setting also ε_M and ε_I zero. The dimensionless state equations degenerate to a system of algebraic equations, given by (8), if $\delta_W = 0$.

$$V = 0 \quad (8a)$$

$$W = W_d \quad (8b)$$

$$\Omega = \frac{\gamma W_d}{v_P} \quad (8c)$$

$$Z = \frac{-C\gamma Z_0 + \gamma P_A \alpha_{p1} + P_A - f}{C\gamma} \quad (8d)$$

$$P_B = \frac{-f + P_A}{\gamma} \quad (8e)$$

$$P_A = \frac{W_d(1 - \gamma) + \frac{\sqrt{P_A - P_T}}{\sqrt{P_N}} Z}{X + v_{A0}} \quad (8f)$$

The simple algebraic expressions in (8) as well as the corresponding slow model shall help the stakeholder to get a comprehensive understanding of the system behavior. Of course, the simple models are restricted to certain constraints like system stability which for the system under study is decided by the fast dynamic subsystem. The following section shall lead the reader through a rough concept phase and highlights the limits of the reduced models.

4. USING THE REDUCED MODEL FOR DESIGN

4.1. Exploitation of reduced models in a virtualization context

Equation (8f) is basically a third order polynomial in P_A if Z is inserted from (8d). A compact approximate solution is

$$P_A \approx \frac{\gamma f_{PL} + f}{1 + \gamma \alpha_{P1}} + \frac{(\gamma - 1) \gamma \sqrt{\frac{\gamma f_{PL} + f}{\gamma \alpha_{P1} + 1}} W_d \sqrt{P_N}}{\gamma f_{PL} + f} C \quad (9)$$

In this relation the tank pressure P_T is set to zero and the spring preload distance Z_0 of the CBV is replaced by its equivalent dimensionless preload force f_{PL} : $Z_0 = f_{PL}/C$ which is an essential adjustment parameter.

This relation provides adjustment rules for the CBV: For the considered retraction ($W_d < 0$) scenario, a positive load ($f > 0$), and the requirement $P_B > 0$ the following approximate design rule for f_{PL} results

$$f_{PL} > \alpha_{P1} \max f \quad (10)$$

Spring stiffness C leads to an increase of P_A , since $\gamma < 1$. A small C reduces this effect but this is traded-off by a higher Z_0 , i.e. a longer counterbalance spring, and a slower CBV dynamics. The relation seems to require a preload $f_{PL} > -\min f/\gamma$. But this is technically an erroneous conclusion. For $f < 0$ and a small f_{PL} Z goes to its limit, $Z = 1$. The equilibrium is handled by an end stop contact force. P_A computes to

$$P_A = P_N W_d^2 (\gamma - 1) \quad (11)$$

Similar design considerations can be made for the CBV area ratio α_{P1} . Also approximate expressions for the efficiency η of stationary retraction motion can be derived from the reduced problem, if the terms of basic formula

$$\eta = \left(\frac{FW}{\omega_M M_M} \right)^{-\text{sign}(f)} \quad (12)$$

are successively replaced by their dimensionless counterparts and solutions of the reduced problem. For pulling forces ($f < 0$) the following series approximation of η in W_d results:

$$\eta \approx \frac{f v_p (\gamma \alpha_{P1} + 1)}{\gamma (v_p (\alpha_{P1} + 1) f + f_{PL} v_p (\gamma - 1))} \quad (13)$$

$$- \frac{f a (C \sqrt{P_N} (\gamma - 1)^2 a b + \gamma \delta_p a (\gamma f_{PL} + f)) / v_p}{\gamma (f (\alpha_{P1} + 1) + f_{PL} (\gamma - 1))^2 (f_{PL} \gamma + f)} W_d$$

with

$$a = \sqrt{\frac{\gamma f_{PL} + f}{\gamma \alpha_{P1} + 1}} \quad b = \gamma \alpha_{P1} + 1 \quad (14)$$

For the CBV opening the following approximate solution can be derived

$$Z \approx \frac{(\gamma - 1) \sqrt{\gamma \alpha_{P1} + 1} W_d \sqrt{P_N}}{\sqrt{\gamma f_{PL} + f}} \quad (15)$$

Obviously, $f > \gamma f_{PL}$ must hold true, otherwise the valve goes to its fully open state $Z = 1$. That gives another formula, which is not presented here because of the page limits.

Figure 2 shows the efficiency values for a retraction with vanishing speed ($W_d \rightarrow 0$) in those force regions where the formula above is valid. The reduced system (Eq. (8)) describe algebraic relations between all the system states. In the end, the curves describe how much power is dissipated in the CBV as this is the only loss which exists in the reduced model. The loss is characterized by the flow rate through the CBV (which corresponds to the desired speed of the piston) and the pressure combination in chamber A and B which is needed to, firstly, compensate

Figure 2: Efficiency η for different external force f .

the external force f and, secondly, open the CBV such that the right flow rate is reached. It is clear that for pulling forces (red curve) the pressure in A becomes low which results in a small pressure drop in the CBV and hence in a good efficiency.

4.2. Comparison between reduced and full model

In the previous section the use of the reduced problem was shown exemplary. In this section the reduced model, i.e. the dynamics of the slow model, shall be compared to the full model always having in mind a system design task. When applying a reduced model it is necessary to notice under which circumstances it is a valid approximation of the complete model. Validity of the reduced model cannot be checked directly by looking at the formulas of the reduced model.

One condition is that the fast system has to be stable on the so-called slow manifold, which basically is a subset in phase space where the solutions of the reduced system live. This condition can be shown by linearizing the fast subsystem and checking the Routh–Hurwitz stability criterion for all possible slow states and parameters. In most cases the inequalities that result from this procedure cannot be solved for certain parameters analytically because the expressions become very cumbersome when the number of fast states increases. In case of the electrohydraulic drive, the first two criteria are trivially fulfilled for physical meaningful quantities. The remaining inequalities are cumbersome and only two approximations are presented here.

$$\alpha_{P1} < \frac{Z}{2(P_A - P_T)(X + v_{A0})} \quad (16a)$$

$$\alpha_{P1} > \frac{k_1(-C(1 + k_1^2 k_2^3) - k_1 k_2^2 k_3 k_4 - \gamma k_4)}{k_3(\gamma k_1^2 k_2^2 k_4 + 1)} \quad (16b)$$

with

$$k_1 = 1/(\sqrt{P_A - P_T}\sqrt{P_N})$$

$$k_2 = 1/(X + v_{A0})$$

$$k_3 = \sqrt{P_A - P_T}/\sqrt{P_N}$$

$$k_4 = 1/(-X + v_{B0})$$

The two inequalities in (16) represent necessary conditions for stability, thus, for the reduced system to be valid. Such conditions restrict the derived relationship (10) for f_{PL} (including parameter C) and α_{P1} further and highlight limitations that come along when using only the reduced model for design purposes. Equation (16a) is derived by neglecting Figure 4. dissipative effects ($\delta_V = \delta_W = 0$). It shows the explicit connection between the pilot ratio α_{P1} and the position variable X as $1/(X + v_{A0})$ is decreasing rapidly when X grows to its maximum value. Similar sensitivity of the expression originate from pressure P_A in chamber A. This dependence is particularly interesting for fast transients rather than for quasi-static dynamics as P_A is kept low by large α_{P1} in steady state operation.

Another aspect is the validity of the model itself (e.g. the value of the valve opening Z can only take values between 0 and 1) if such boundary conditions are not explicitly integrated into the model. In the next paragraphs we take a closer look at the design parameter α_{P1} and analyze how the the eigenvalues behave on the slow manifold.

Figures 3 and 4 show the velocity w of the piston calculated from the full and the reduced model described by (6) and (7) and the desired velocity w_d for different values of the pilot ratio α_{P1} . The graphs can be interpreted as showing the pistons velocity with different level of detail which ranges from the velocity of the full model (blue) to the instantaneous step response $w = w_d$ (green) which results from the reduced system

Figure 3: Dynamic step response of velocity W with $\alpha_{P1} = 1/3$.

Figure 4: Dynamic step response of velocity W with $\alpha_{P1} = 1/1.5$

(8). It can be seen that the reduced model is not capable of predicting weak stability issues but rather approximates the qualitative behaviour of the fast states. In general increasing the pilot ratio α_{P1} in the electrohydraulic drive stabilizes the system which might result in longer life time due to fewer wear in the moving parts. A down-side is the increase on working pressure in both chambers which might negatively affect e.g. the pump's life-time.

Another important system property which is indirectly influenced by α_{P1} is the maximum load the hydraulic actuator can hold because the CBV acts as a simple pressure relief valve when the piston is in steady state.

Besides the severe effect of α_{P1} on the stability of the overall system the position of the piston is a further crucial parameter in the sense of the fast system. A low stiffness of the hydraulic oil in chamber B which is decreased by increasing chamber B's dead volume is positively influencing the stability of the actuator. This can be seen in **Figure 5** where the largest eigenvalue at each point on the slow manifold for a given trajectory is plotted for different values of the piston position x and the parameter α_{P1} . Notice that for some pairs (x, α_{P1}) the fast system is becoming unstable, i.e. $\max \lambda_i > 0$. The area of the fast system's instability happens for piston position around the mechanical end stop $x \approx 1$ and for small $\alpha_{P1} < 0.5$.

Figure 5: Largest eigenvalue $\max \lambda_i$ on a trajectory.

5. CONCLUSION

Mathematical modelling of physical systems is a standard approach engineers are using for centuries to gain insights and predict the system's behaviour. Nowadays, in context of digitalization and political trends as Industrie4.0 there is the need for a variety of models which perhaps describe different aspects of the same physical system. At the same time it is crucial to keep consistency within such models. One attempt for preserving consistency and simultaneously obtaining model hierarchies which is discussed in this paper is to use the well-known method of singular perturbation. It was shown that simple, sometimes even algebraic equations lead to a useful model for the fast states of the system. Subsequently, this model was used to derive simple approximations, limitations for key parameters as well as a basic formula for the efficiency of the exemplary system of an electrohydraulic actuator. Besides showing the usefulness of such a model for design approach it was highlighted that due to the reduction procedure the static model might not even predict qualitative system behavior. This is for example the case when the fast subsystem loses its stability. Necessary bounds for preserving stability were derived by using Routh–Hurwitz stability criterion which implied further limitations on the parameter space.

NOMENCLATURE

θ_{tot}	Effective moment of inertia of pump and motor
$\omega_M(\Omega)$	Angular velocity of electric motor
$A_{A,B}$	Piston area A, B side
A_{p1}	Area of piloted poppet from side 1 of CBV
A_{p3}	Area of piloted poppet from side 3 of CBV
c	Spring constant of poppet preload spring
$d_v, (\delta_v)$	Damping factor of poppet (dimensionless)
$d_w, (\delta_w)$	Damping factor of piston (dimensionless)
$E_{\beta l}$	Bulk modulus oil
$e_i (E_i)$	Internal state of PI-controller (dimensionless)
$F (f)$	Process force (dimensionless)
$k_I (K_I)$	I-coefficient of PI-controller (dimensionless)
$k_P (K_P)$	P-coefficient of PI-controller (dimensionless)
m_F	Mass of piston
$M_N (m_N)$	Torque of electric motor (dimensionless)
m_P	Mass of poppet of CBV
$p_{A,B} (P_{A,B})$	Pressure chamber A, B (dimensionless)
p_N	Nominal pressure drop of CBV
$P_T, (P_T)$	Tank pressure (dimensionless)
$q_A (Q_A)$	Flow rate through CBV (dimensionless)
$q_B (Q_B)$	Flow rate through CHK (dimensionless)
q_N	Nominal flow rate through CBV
$q_P (Q_P)$	Pump flow rate (dimensionless)
$R_L (r_L)$	Leakage resistance (dimensionless)
$v (V)$	Velocity of poppet (dimensionless)
$V_{A0,B0}$	Dead volume chamber A, B
V_P	Displacement volume
$w (W)$	Piston velocity (dimensionless)
$w_d (W_d)$	Desired velocity of piston (dimensionless)
$x (X)$	Piston position (dimensionless)
$z (Z)$	Position of poppet (dimensionless)
$z_0 (Z_0)$	Preload deflection of poppet spring (dimensionless)
z_{max}	Maximum poppet displacement of CBV
γ	Area ration between piston side B and A
$\varepsilon_{P,F,V,Z,M}$	Perturbation parameter for pressure, piston, poppet displacement, poppet velocity, rotatory parts (pump and motor)
α_{p1}	Inverse pilot ratio of CBV
η	efficiency

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