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Udo Broll
Peter Welzel
Kit Pong Wong

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Hedging and the Regret Theory of the Competitive Firm

Udo Broll†
TU Dresden

Peter Welzel
University of Augsburg

Kit Pong Wong
University of Hong Kong

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Abstract

This paper examines the production and hedging decisions of the competitive firm under price uncertainty when the firm is not only risk averse but also regret averse. Regret-averse preferences are characterized by a modified utility function that includes disutility from having chosen ex-post suboptimal alternatives. The extent of regret depends on the difference between the actual profit and the maximum profit attained by making the optimal production and hedging decisions had the firm observed the true realization of the random output price. While the separation theorem holds under regret aversion, the prevalence of hedging opportunities may have perverse effect on the firm's optimal output level, particularly when the firm is sufficiently regret averse. The full-hedging theorem, however, does not hold. We derive sufficient conditions under which the regret-averse firm's optimal futures position is an under-hedge (over-hedge). We further show that the firm optimally increases (decreases) its futures position when the price risk possesses more positive (negative) skewness.

JEL classification: D21, D24, D81

Keywords: Futures, Production, Regret theory

†Corresponding author: Udo Broll, Department of Business and Economics; School of International Studies (ZIS), Technische Universität Dresden, 01062 Dresden, Germany. E-mail: udo.broll@tu-dresden.de (U. Broll).

1. Introduction

The seminal work of Sandmo (1971) has inspired a large body of research on the theory of the competitive firm under price uncertainty. One important strand of this literature examines the behavior of the firm when a futures market exists for hedging purposes, from which two celebrated theorems emanate (see, e.g., Broll and Zilcha, 1992; Danthine, 1978; Feder et al., 1980; Holthausen, 1979; to name just a few). First, the separation theorem states that the firm's optimal output level depends neither on the risk attitude of the firm, nor on the incidence of the price uncertainty. Second, the full-hedging theorem asserts that the firm should fully hedge its exposure to the price risk if the futures market is unbiased.[1]

Most of the extant models in the literature assume that the firm's preferences admit the standard von Neumann-Morgenstern expected utility representation. Such a modeling approach rules out the possibility that the firm may have desires to avoid consequences wherein ex-post suboptimal decisions appear to have been made even though these decisions are ex-ante optimal based on the information available at that time. To account for this consideration, Bell (1982, 1983) and Loomes and Sugden (1982) propose regret theory that defines regret as the disutility arising from not having chosen the ex-post optimal alternative, which is later axiomatized by Quiggin (1994) and Sugden (1993). Regret theory is supported by a large body of experimental literature that documents regret-averse preferences among individuals (see, e.g., Loomes, 1988; Loomes et al., 1992; Loomes and Sugden, 1987; Starmer and Sugden, 1993).

In this paper, we incorporate regret theory into the competitive firm that is allowed to hedge the price risk by trading futures. Specifically, we characterize the firm's regret-averse preferences by a modified utility function that includes disutility from having chosen ex-post suboptimal alternatives.[2] The extent of regret depends on the difference between

[1] The full-hedging theorem is analogous to a well-known result in the insurance literature that a risk-averse individual fully insures at an actuarially fair price (Mossin, 1968).

[2] For other applications of regret aversion using a similar modified utility function, see Braun and Muermann (2004) and Wong (2012) in the case of demand for insurance, Muermann et al. (2006) in a portfolio choice problem, Tsai (2012) and Wong (2011) in the banking context, and Broll et al. (2015, 2016) in the

the actual profit and the maximum profit attained by making the optimal production and hedging decisions had the firm observed the true realization of the random output price. We are particularly interested in examining the robustness of the separation and full-hedging theorems when regret aversion prevails.

We show that the separation theorem holds in that the firm can sell its output forward by trading futures so as to lock in the marginal revenue at the predetermined futures price. However, the prevalence of hedging opportunities may not have the usual output-enhancing effect under regret aversion, contrary to what the conventional wisdom suggests. We show that the regret-averse firm optimally produces more when it is banned from trading futures for hedging purposes if regret aversion plays a sufficiently more important role than risk aversion in determining the firm's production decision. The full-hedging theorem does not hold. We derive sufficient conditions under which the regret-averse firm optimally opts for an under-hedge (over-hedge). These are in stark contrast to the findings of Guo et al. (2015) and Michenaud and Solnik (2008), both of which show that the optimal futures position under regret aversion and an unbiased futures market is an under-hedge. Guo et al. (2015) assume that the firm cannot trade futures had the firm observed the true output price, whereas Michenaud and Solnik (2008) assume that the firm can opt for neither a long futures position nor an over-hedge. These assumptions are unduly strong. We adopt a more realistic assumption that there are position limits for long and short futures positions due to margin requirements and other liquidity reasons, which accounts for the more general findings in our setting. Finally, we show that the firm optimally increases (decreases) its futures position when the price risk possesses more positive (negative) skewness.

The rest of this paper is organized as follows. Section 2 delineates the model of the competitive firm under price uncertainty when the firm's preferences exhibit not only risk aversion but also regret aversion. Section 3 examines the firm's optimal production decision. Section 4 examines the firm's optimal hedging decision. The final section concludes.

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decision to export.

2. The model

Consider the competitive firm under price uncertainty *à la* Sandmo (1971). There is one period with two dates, 0 and 1. To begin, the firm produces a single commodity according to a deterministic cost function, $C(Q)$, where $Q \geq 0$ is the output level, and $C(Q)$ is compounded to date 1 with the properties that $C(0) = C'(0) = 0$, and $C'(Q) > 0$ and $C''(Q) > 0$ for all $Q > 0$.[3] The firm sells its entire output, Q, at the per-unit price, $\tilde{P}$, at date 1.[4] The firm regards $\tilde{P}$ as a positive random variable that is distributed according to a known cumulative distribution function, $F(P)$, over support $[\underline{P}, \overline{P}]$, where $0 < \underline{P} < \overline{P}$.

To hedge the price risk, $\tilde{P}$, the firm can trade infinitely divisible futures contracts at date 0. Each futures contract calls for delivery of one unit of the commodity at date 1 at the predetermined futures price, $P^f \in (\underline{P}, \overline{P})$. Let H be the number of the futures contracts sold (purchased if negative) by the firm at date 0. The firm is subject to position limits in that its short futures position cannot exceed S and its long futures position cannot exceed L due to margin requirements and other liquidity reasons, where $S > 0$ and $L > 0$ are exogenously given short and long position limits, respectively.[5] The futures position, H, is said to be an under-hedge, a full-hedge, or an over-hedge, depending on whether H is smaller than, equal to, or greater than the output level, Q, respectively. We assume that the position limits, S and L, are large enough that the firm can opt for an under-hedge, a full-hedge, or an over-hedge at the optimum.[6] The firm's profit at date 1 is, therefore, given by

$$\Pi(P) = PQ - C(Q) + (P^f - P)H, \tag{1}$$

for all $P \in [\underline{P}, \overline{P}]$, subject to the constraint that the short and long position limits do not

[3]The strict convexity of the cost function reflects the fact that the firm's production technology exhibits decreasing returns to scale.

[4]Throughout the paper, random variables have a tilde (~) while their realizations do not.

[5]The short and long position limits are not only realistic features of a futures market, but also necessary for prohibiting the firm from taking an infinite short (long) futures position should the firm be perfectly informed that the true realization of the random output price is below (above) the predetermined futures price.

[6]This assumption requires that $S > Q^*$, where Q^* solves $C'(Q^*) = P^f$. See Proposition 1.

bind, i.e., $-L < H < S$.

Following Braun and Muermann (2004) and Wong (2014), we assume that the firm's preferences are represented by the following bivariate utility function (Paroush and Venezia, 1979):

$$V(\Pi, R) = U(\Pi) - \beta G(R), \tag{2}$$

where $\beta > 0$ is a constant regret coefficient, $U(\Pi)$ is a von Neumann-Morgenstern utility function with $U'(\Pi) > 0$ and $U''(\Pi) < 0$ for all $\Pi > 0$, and $G(R)$ is a regret function defined over the magnitude of regret, R, such that $G(0) = 0$, and $G'(R) > 0$ and $G''(R) > 0$ for all $R > 0$.[7] The magnitude of regret, $R = \Pi^{\max} - \Pi$, is gauged by the difference between the actual profit, Π, and the maximum profit, $\Pi^{\max}$, that the firm could have earned at date 1 should the firm have made the optimal production and hedging decisions based on knowing the true per-unit price, P. Since Π cannot exceed $\Pi^{\max}$, the firm experiences disutility from forgoing the possibility of undertaking the ex-post optimal production and hedging decisions. As the constant regret coefficient, β, increases, regret aversion becomes increasingly more important in representing the firm's preferences as compared to risk aversion.

To characterize the regret-averse firm's optimal production and hedging decisions, we have to first determine the maximum profit, $\Pi^{\max}$. If the firm could have observed the realized per-unit price, P, the maximum profit would be achieved by choosing $Q(P)$ that solves $C'\Big(Q(P)\Big) = P$ and $H = S$ if $P < P^f$ and $H = -L$ if $P > P^f$. Note that $Q'(P) = 1/C''\Big(Q(P)\Big) > 0$. The maximum profit as a function of P is given by

$$\Pi^{\max}(P) = PQ(P) - C\Big(Q(P)\Big) + \max[(P^f - P)S, (P - P^f)L], \tag{3}$$

for all $P \in [\underline{P}, \overline{P}]$. The magnitude of regret, $R(P)$, is given by

$$R(P) = \Pi^{\max}(P) - \Pi(P), \tag{4}$$

[7] Bleichrodt et al. (2010) provide empirical evidence that regret functions are indeed convex.

for all $P \in [\underline{P}, \overline{P}]$, where $\Pi(P)$ and $\Pi^{\max}(P)$ are given by Eqs. (1) and (3), respectively.

We can now state the regret-averse firm's ex-ante decision problem. At date 0, the firm chooses an output level, Q, and a futures position, H, so as to maximize the expected value of its regret-theoretical utility function:

$$\max_{Q\geq 0,\ H} \ \mathrm{E}\Big[U\Big(\Pi(\tilde{P})\Big) - \beta G\Big(R(\tilde{P})\Big)\Big], \tag{5}$$

where $R(P)$ is given by Eq. (4), and $\mathrm{E}[\cdot]$ is the expectation operator with respect to $F(P)$. The first-order conditions for program (5) are given by

$$\mathrm{E}\bigg[\Big[U'\Big(\Pi^*(\tilde{P})\Big) + \beta G'\Big(R^*(\tilde{P})\Big)\Big][\tilde{P} - C'(Q^*)]\bigg] = 0, \tag{6}$$

and

$$\mathrm{E}\bigg[\Big[U'\Big(\Pi^*(\tilde{P})\Big) + \beta G'\Big(R^*(\tilde{P})\Big)\Big](P^f - \tilde{P})\bigg] = 0, \tag{7}$$

where an asterisk (*) indicates an optimal level. The second-order conditions for program (5) are assumed to be satisfied.

3. Optimal production decision

In this section, we examine the firm's optimal production decision. To this end, we add Eq. (6) to Eq. (7) to yield

$$\mathrm{E}\Big[U'\Big(\Pi^*(\tilde{P})\Big) + \beta G'\Big(R^*(\tilde{P})\Big)\Big][P^f - C'(Q^*)] = 0. \tag{8}$$

Given that $U'(\Pi) > 0$, Eq. (8) reduces to $C'(Q^*) = P^f$, thereby invoking our first proposition.

Proposition 1. *Given that the regret-averse competitive firm can trade the futures contracts for hedging purposes, the firm's optimal output level, Q^*, is the one at which the marginal cost of production, $C'(Q^*)$, is equated to the predetermined futures price, P^f.*

The intuition for Proposition 1 is as follows. By producing one more unit of the commodity, the firm receives the marginal revenue, $\tilde{P}$, which is stochastic. The firm can sell this additional unit forward via trading one futures contract to lock in the marginal revenue at P^f. At the optimum, the firm equates the marginal revenue, P^f, to the marginal cost, $C'(Q^*)$. This then gives rise to the usual optimality condition, $C'(Q^*) = P^f$, that determines the optimal output level, Q^*.

An immediate implication of Proposition 1 is that the firm's optimal production decision depends neither on its attitude towards risk and regret, nor on the underlying uncertainty. Proposition 1 as such extends the separation theorem to the case in which the firm is regret averse.

While Proposition 1 shows that the separation theorem holds under regret aversion, it is completely silent about how the prevalence of hedging opportunities affects the regret-averse firm's optimal output level. To address this issue, we consider a benchmark case wherein the firm is banned from trading the futures contracts for hedging purposes. In this benchmark case without hedging, the firm's optimal output level, Q°, is the solution to the following first-order condition:

$$\mathrm{E}\Big[\Big[U'\Big(\Pi_0^\circ(\tilde{P})\Big) + \beta G'\Big(R_0^\circ(\tilde{P})\Big)\Big][\tilde{P} - C'(Q^\circ)]\Big] = 0, \tag{9}$$

where $\Pi_0^\circ(P) = PQ^\circ - C(Q^\circ)$ and $R_0^\circ(P) = PQ(P) - C\Big(Q(P)\Big) - PQ^\circ + C(Q^\circ)$ for all $P \in [\underline{P}, \overline{P}]$.

To have a fair comparison, we assume that the predetermined futures price, P^f, is set equal to the expected value of $\tilde{P}$, i.e., $P^f = \mathrm{E}[\tilde{P}]$. To compare Q^* with Q°, we evaluate the left-hand side of Eq. (9) at $Q = Q^*$ to yield

$$\mathrm{E}\Big[\Big[U'\Big(\Pi_0^*(\tilde{P})\Big) + \beta G'\Big(R_0^*(\tilde{P})\Big)\Big][\tilde{P} - C'(Q^*)]\Big]$$

$$= \mathrm{Cov}\Big[U'\Big(\Pi_0^*(\tilde{P})\Big) + \beta G'\Big(R_0^*(\tilde{P})\Big), \tilde{P}\Big], \tag{10}$$

where $\Pi_0^*(P) = PQ^* - C(Q^*)$ and $R_0^*(P) = PQ(P) - C\Big(Q(P)\Big) - PQ^* + C(Q^*)$ for all $P \in [\underline{P}, \overline{P}]$, $\mathrm{Cov}[\cdot,\cdot]$ is the covariance operator with respect to $F(P)$, and the equality follows from the property of the covariance operator and Proposition 1.[8] It then follows from Eqs. (9) and (10) that $Q^* > (<)\ Q^\circ$ if, and only if, the right-hand side of Eq. (10) is negative (positive).

When $\beta = 0$, the right-hand side of Eq. (10) reduces to $\mathrm{Cov}\Big[U'\Big(\Pi_0^*(\tilde{P})\Big), \tilde{P}\Big] < 0$, where the inequality follows from $U''(\Pi) < 0$ and $\Pi_0^{*\prime}(P) = Q^* > 0$. Hence, we have $Q^\circ < Q^*$ when $\beta = 0$, which is the well-known result that the firm, being risk-averse, produces less than Q^* so as to limit its exposure to the price uncertainty (Sandmo, 1971). In the following proposition, we show that for β sufficiently small, introducing regret aversion to the firm would not substantially alter such a risk-reducing incentive so that $Q^\circ < Q^*$.

Proposition 2. *Given that $U'''(\Pi) \geq 0$ and $G'''(R) \geq 0$, the regret-averse competitive firm optimally produces less when it is banned from trading the futures contracts for hedging purposes, i.e., $Q^\circ < Q^*$, if the constant regret coefficient, β, is sufficiently small such that*

$$\beta \leq \frac{U'\Big(\Pi_0^*(P^f)\Big) - U'\Big(\Pi_0^*(\overline{P})\Big)}{G'\Big(R_0^*(\overline{P})\Big) - G'(0)}. \tag{11}$$

Proof. Since $\Pi_0^*(P^f) < \Pi_0^*(\overline{P})$ and $U''(\Pi) < 0$, we have $U'\Big(\Pi_0^*(P^f)\Big) > U'\Big(\Pi_0^*(\overline{P})\Big)$. Likewise, $R_0^*(\overline{P}) > 0$ and $G''(R) > 0$ imply that $G'\Big(R_0^*(\overline{P})\Big) > G'(0)$. Hence, the threshold of β in condition (11) is strictly positive. Let $\Phi(P) = U'\Big(\Pi_0^*(P)\Big) + \beta G'\Big(R_0^*(P)\Big)$. Then, we have

$$\mathrm{Cov}\Big[U'\Big(\Pi_0^*(\tilde{P})\Big) + \beta G'\Big(R_0^*(\tilde{P})\Big), \tilde{P}\Big] = \mathrm{E}\Big[[\Phi(\tilde{P}) - \Phi(P^f)](\tilde{P} - P^f)\Big]. \tag{12}$$

Differentiating $\Phi(P)$ twice with respect to P yields

$$\Phi'(P) = U''\Big(\Pi_0^*(P)\Big)Q^* + \beta G''\Big(R_0^*(P)\Big)[Q(P) - Q^*], \tag{13}$$

[8] For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{Cov}[\tilde{X}, \tilde{Y}] = \mathrm{E}[\tilde{X}\tilde{Y}] - \mathrm{E}[\tilde{X}]\mathrm{E}[\tilde{Y}]$. Proposition 1 implies that $C'(Q^*) = P^f = \mathrm{E}[\tilde{P}]$.

and

$$\Phi''(P) = U'''\Big(\Pi_0^*(P)\Big)Q^{*2} + \beta G'''\Big(R_0^*(P)\Big)[Q(P) - Q^*]^2 + \beta G''\Big(R_0^*(P)\Big)Q'(P). \quad (14)$$

Since $U'''(\Pi) \geq 0$, $G''(R) > 0$, $G'''(R) \geq 0$, and $Q'(P) > 0$, Eq. (14) implies that $\Phi''(P) > 0$ for all $P \in [\underline{P}, \overline{P}]$. Since $Q^* = Q(P^f)$ and $Q'(P) > 0$, we have $Q(P) < (>)\ Q^*$ for all $P < (>)\ P^f$. It follows from Eq. (13) that $\Phi'(P) < 0$ for all $P \in [\underline{P}, P^f]$. Hence, $\Phi(P) > \Phi(P^f)$ for all $P \in [\underline{P}, P^f]$. Since $\Phi(P^f) = U'\Big(\Pi_0^*(P^f)\Big) + \beta G'(0)$, condition (11) ensures that $\Phi(P^f) \geq \Phi(\overline{P})$. Since $\Phi(P)$ is strictly convex in P and $\Phi'(P^f) < 0$, it follows from condition (11) that $\Phi(P) < \Phi(P^f)$ for all $P \in (P^f, \overline{P})$. The right-hand side of Eq. (12) as such is negative so that $Q^* > Q^\circ$. $\square$

Proposition 2 shows that the conventional wisdom under which the prevalence of hedging opportunities is output enhancing is valid if regret aversion is relatively unimportant as compared to risk aversion in determining the firm's optimal output level, i.e., β is sufficiently small. This suggests that the prevalence of hedging opportunities may give rise to perverse output effect, particularly when regret aversion becomes the dominant factor of the firm's production decision, i.e., β is sufficiently large. We show in the following proposition that it is indeed the case.

Proposition 3. *Given that $U'''(\Pi) \geq 0$ and $G'''(R) \geq 0$, the regret-averse competitive firm optimally produces more when it is banned from trading the futures contracts for hedging purposes, i.e., $Q^\circ > Q^*$, if $\mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big] > G'\Big(R_0^*(\underline{P})\Big)$ and the constant regret coefficient, β, is sufficiently large such that*

$$\beta \geq \frac{U'\Big(\Pi_0^*(\underline{P})\Big) - \mathrm{E}\Big[U'\Big(\Pi_0^*(\tilde{P})\Big)\Big]}{\mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big] - G'\Big(R_0^*(\underline{P})\Big)}. \quad (15)$$

Proof. Since $\Pi_0^*(\underline{P}) < \Pi_0^*(P)$ for all $P \in (\underline{P}, \overline{P}]$ and $U''(\Pi) < 0$, we have $U'\Big(\Pi_0^*(\underline{P})\Big) > \mathrm{E}\Big[U'\Big(\Pi_0^*(\tilde{P})\Big)\Big]$. If $\mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big] > G'\Big(R_0^*(\underline{P})\Big)$, the threshold of β in condition (15) is strictly positive. Since $\mathrm{E}[\Phi(\tilde{P})] = \mathrm{E}\Big[U'\Big(\Pi_0^*(\tilde{P})\Big)\Big] + \beta\mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big]$, $\mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big] > G'\Big(R_0^*(\underline{P})\Big)$ and condition (15) ensure that $\mathrm{E}[\Phi(\tilde{P})] \geq \Phi(\underline{P})$. Since $\Phi(P)$ is strictly convex in P and $\Phi'(P) < 0$ for all $P \in [\underline{P}, P^f]$, it must be true that $\mathrm{E}[\Phi(\tilde{P})] < \Phi(\overline{P})$. Hence, there must exist a unique point, $P^* \in (P^f, \overline{P})$, at which $\Phi(P^*) = \mathrm{E}[\Phi(\tilde{P})]$ such that $\Phi(P) < \mathrm{E}[\Phi(\tilde{P})]$ for all $P \in (\underline{P}, P^*)$ and $\Phi(P) > \mathrm{E}[\Phi(\tilde{P})]$ for all $P \in (P^*, \overline{P}]$. Then, we have

$$\mathrm{Cov}\Big[U'\Big(\Pi_0^*(\tilde{P})\Big) + \beta G'\Big(R_0^*(\tilde{P})\Big), \tilde{P}\Big] = \mathrm{E}\Big[\{\Phi(\tilde{P}) - \mathrm{E}[\Phi(\tilde{P})]\}(\tilde{P} - P^*)\Big] > 0, \tag{16}$$

which implies that $Q^* < Q^\circ$. $\square$

The intuition for Proposition 3 is as follows. Since $G'\Big(R_0^*(P)\Big)$ is U-shaped and reaches a unique minimum at $P = P^f$, there must exist a unique point, $P^\circ \in (P^f, \overline{P})$, such that $G'\Big(R_0^*(P^\circ)\Big) = \mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big]$ if $\mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big] > G'\Big(R_0^*(\underline{P})\Big)$. Hence, we have $G'\Big(R_0^*(P)\Big) < \mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big]$ for all $P \in [\underline{P}, P^\circ)$ and $G'\Big(R_0^*(P)\Big) > \mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big]$ for all $P \in (P^\circ, \overline{P}]$, which imply that

$$\mathrm{Cov}\Big[G'\Big(R_0^*(\tilde{P})\Big), \tilde{P}\Big] = \mathrm{E}\Big[\Big\{G'\Big(R_0^*(\tilde{P})\Big) - \mathrm{E}\Big[G'\Big(R_0^*(\tilde{P})\Big)\Big]\Big\}(\tilde{P} - P^\circ)\Big] > 0. \tag{17}$$

When the firm is risk neutral, i.e., $U(\Pi) = \Pi$, the right-hand side of Eq. (10) reduces to $\mathrm{Cov}\Big[G'\Big(R_0^*(\tilde{P})\Big), \tilde{P}\Big]$, which is positive from Eq. (17). As such, the firm, being purely regret averse, has more concerns about the disutility from the discrepancy of its output level, $Q(P) - Q^*$, when high realizations of $\tilde{P}$ are revealed. To minimize regret, the firm optimally adjusts its output level upward from Q^* so that $Q^\circ > Q^*$. Since the firm is in fact risk averse, there is a countervailing incentive that induces the firm to reduce its output level. Condition (15) ensures that the firm is sufficiently regret averse, i.e., β is sufficiently large, in that the incentive driven by regret aversion dominates the opposing incentive driven by risk aversion, thereby inducing the firm to optimally produce more if it is banned from trading the futures contracts for hedging purposes.

4. Optimal hedging decision

In this section, we examine the firm's optimal hedging decision. To focus on the firm's hedging motive, we assume that the futures contracts are unbiased in that the predetermined futures price, P^f, is set equal to the expected value of $\tilde{P}$, i.e., $P^f = \mathrm{E}[\tilde{P}]$. In this case, Eq. (7) becomes

$$\mathrm{Cov}\Big[U'\Big(\Pi^*(\tilde{P})\Big) + \beta G'\Big(R^*(\tilde{P})\Big), \tilde{P}\Big] = 0. \tag{18}$$

Since covariances can be interpreted as marginal variances, Eq. (18) simply says that the optimal futures position, H^*, is the one at which the variability of the firm's marginal utility under regret aversion is minimized.

Evaluating the left-hand side of Eq. (18) at $H = Q^*$ yields

$$\mathrm{Cov}\Big[G'\Big(\Pi^{\max}(\tilde{P}) - \Pi^*(P^f)\Big), \tilde{P}\Big]. \tag{19}$$

It then follows from Eq. (7) and the second-order conditions for program (5) that $H^* <$ $(>)$ Q^* if, and only if, the covariance term is positive (negative). We state and prove the following proposition.

Proposition 4. *The competitive firm, being regret averse, optimally opts for an under-hedge, i.e., $H^* < Q^*$, or an over-hedge, i.e., $H^* > Q^*$, depending on whether*

$$\mathrm{E}\Big[G'\Big(\Pi^{\max}(\tilde{P}) - \Pi^*(P^f)\Big)\Big] \geq G'\Big(\Pi^{\max}(\underline{P}) - \Pi^*(P^f)\Big), \tag{20}$$

or

$$\mathrm{E}\Big[G'\Big(\Pi^{\max}(\tilde{P}) - \Pi^*(P^f)\Big)\Big] \geq G'\Big(\Pi^{\max}(\overline{P}) - \Pi^*(P^f)\Big), \tag{21}$$

respectively.

Proof. Differentiating $G'\Big(\Pi^{\max}(P) - \Pi^*(P^f)\Big)$ with respect to P yields

$$\begin{aligned}&\frac{\partial}{\partial P} G'\Big(\Pi^{\max}(P) - \Pi^*(P^f)\Big)\\&= G''\Big(\Pi^{\max}(P) - \Pi^*(P^f)\Big)\Big[Q(P) - SI_{\{P<P^f\}} + LI_{\{P>P^f\}}\Big],\end{aligned} \tag{22}$$

where $I_{\{A\}}$ is an indicator function that takes on a value equal to unity if event A occurs and zero otherwise. Since $S \geq Q^* > Q(P)$ for all $P \in [\underline{P}, P^f)$ and $G''(\Pi^{\max} - \Pi) > 0$, it follows from Eq. (22) that $G'\Big(\Pi^{\max}(P) - \Pi^*(P^f)\Big)$ is decreasing (increasing) in P for all $P < (>)\ P^f$. Hence, $G'\Big(\Pi^{\max}(P) - \Pi^*(P^f)\Big)$ is U-shaped and reaches a unique minimum at $P = P^f$. If condition (20) holds, there must exist a unique per-unit price, $P^\circ \in (P^f, \overline{P})$, such that

$$G'\Big(\Pi^{\max}(P^\circ) - \Pi^*(P^f)\Big) = \mathrm{E}\Big[G'\Big(\Pi^{\max}(\tilde{P}) - \Pi^*(P^f)\Big)\Big]. \tag{23}$$

Then, we have

$$\begin{aligned}&\mathrm{Cov}\Big[G'\Big(\Pi^{\max}(\tilde{P}) - \Pi^*(P^f)\Big), \tilde{P}\Big]\\&= \mathrm{E}\Big[\Big[G'\Big(\Pi^{\max}(\tilde{P}) - \Pi^*(P^f)\Big) - G'\Big(\Pi^{\max}(P^\circ) - \Pi^*(P^f)\Big)\Big](\tilde{P} - P^\circ)\Big].\end{aligned} \tag{24}$$

Given that $G'\Big(\Pi^{\max}(P) - \Pi^*(P^f)\Big)$ is U-shaped, condition (20) and Eq. (23) imply that the right-hand side of Eq. (24) is positive, thereby rendering that $H^* < Q^*$. The proof that $H^* > Q^*$ given condition (21) can be done analogously and thus is omitted. □

The intuition for Proposition 4 is as follows. If condition (20) holds, it follows from Eq. (24) that $G'\Big(\Pi^{\max}(\tilde{P}) - \Pi^*(P^f)\Big)$ is positively correlated with $\tilde{P}$, thereby inducing the regret-averse firm to raise more concerns about the disutility from the discrepancy of its futures position, Q^*, from the long position limit, L, when high realizations of $\tilde{P}$ are revealed. To minimize regret, the firm optimally adjusts its futures position downward from Q^* so that $H^* < Q^*$. On the other hand, if condition (21) holds, then $G'\Big(\Pi^{\max}(\tilde{P}) - \Pi^*(P^f)\Big)$

is negatively correlated with $\tilde{P}$. The regret-averse firm as such raises more concerns about the disutility from the discrepancy of its futures position, Q^*, from the short position limit, S, when low realizations of $\tilde{P}$ are revealed. To minimize regret, the firm optimally adjusts its futures position upward from Q^* so that $H^* > Q^*$.

Finally, we examine the marginal effect of price risk on the firm's optimal hedging decision. To this end, we let $\hat{F}(P)$ be a new CDF of $\tilde{P}$. The following definition is adopted from the definition of downside risk *à la* Menezes et al. (1980).[9]

Definition 1. *The CDF, $\hat{F}(P)$, is said to have more simple positive (negative) skewness than the CDF, $F(P)$, if, and only if,*

$$\int_{\underline{P}}^{\overline{P}} [\hat{F}(P) - F(P)]\mathrm{d}P = 0, \tag{25}$$

$$\int_{\underline{P}}^{\overline{P}} \left\{ \int_{\underline{P}}^{P} [\hat{F}(x) - F(x)]\mathrm{d}x \right\}\mathrm{d}P = 0, \tag{26}$$

$$\int_{\underline{P}}^{P} [\hat{F}(x) - F(x)]\mathrm{d}x \leq (\geq)\ 0 \ \ \textit{for all}\ P \leq P^f, \tag{27}$$

$$\int_{\underline{P}}^{P} [\hat{F}(x) - F(x)]\mathrm{d}x \geq (\leq)\ 0 \ \ \textit{for all}\ P \geq P^f, \tag{28}$$

and

$$\int_{\underline{P}}^{P} \left\{ \int_{\underline{P}}^{x} [\hat{F}(y) - F(y)]\mathrm{d}y \right\}\mathrm{d}x \leq (\geq)\ 0 \ \ \textit{for all}\ P \in [\underline{P}, \overline{P}]. \tag{29}$$

Eq. (25) ensures that $\tilde{P}$ has the same mean under $F(P)$ and $\hat{F}(P)$. Eq. (26) ensures that $\tilde{P}$ has the same variance, denoted by σ^2, under $F(P)$ and $\hat{F}(P)$. Eq. (29) ensures that $\tilde{P}$ has more positive (negative) skewness under $\hat{F}(P)$ than under $F(P)$, while Eqs. (27) and

[9] An increase in downside risk in the sense of Menezes et al. (1980) is simply a third-degree increase in risk in the sense of Ekern (1980).

(28) ensure a single-crossing property. To see this, we compare the central third moment under $\hat{F}(P)$ and that under $F(P)$:

$$\int_{\underline{P}}^{\overline{P}} \left(\frac{P-P^f}{\sigma}\right)^3 \mathrm{d}[\hat{F}(P)-F(P)]$$

$$= \frac{6}{\sigma^3}\int_{\underline{P}}^{\overline{P}} (P-P^f)\bigg\{\int_{\underline{P}}^{P}[\hat{F}(x)-F(x)]\mathrm{d}x\bigg\}\mathrm{d}P$$

$$= \frac{6}{\sigma^3}\int_{\underline{P}}^{\overline{P}} (P-P^f)\bigg\{\int_{\underline{P}}^{P}[\hat{F}(x)-F(x)]\mathrm{d}x\bigg\}\mathrm{d}P$$

$$= -\frac{6}{\sigma^3}\int_{\underline{P}}^{\overline{P}} \bigg\{\int_{\underline{P}}^{P}\bigg\{\int_{\underline{P}}^{x}[\hat{F}(y)-F(y)]\mathrm{d}y\bigg\}\mathrm{d}x\bigg\}\mathrm{d}P, \tag{30}$$

where the first equality follows from integration by parts and Eq. (25), the second equality follows from Eq. (26), and the last equality follows from integration by parts and Eq. (26). If $\hat{F}(P)$ has more simple positive (negative) skewness than $F(P)$, the right-hand side of Eq. (30) is positive (negative) so that the third central moment under $\hat{F}(P)$ is indeed larger (smaller) than that under $F(P)$.

Since $\tilde{P}$ has the same mean under $F(P)$ and $\hat{F}(P)$ from Eq. (25), the firm's optimal output level remains Q^* when the original CDF, $F(P)$, is replaced by the new CDF, $\hat{F}(P)$. The firm's optimal futures position, $H^\dagger$, is the solution to the following first-order condition:

$$\int_{\underline{P}}^{\overline{P}} \Big[U'\Big(\Pi^\dagger(P)\Big)+\beta G'\Big(\Pi^{\max}(P)-\Pi^\dagger(P)\Big)\Big](P^f-P)\mathrm{d}\hat{F}(P)=0, \tag{31}$$

where a dagger (†) signifies an optimal level. To compare $H^\dagger$ with H^*, we evaluate the left-hand side of Eq. (31) at H^* to yield

$$\int_{\underline{P}}^{\overline{P}} \Big[U'\Big(\Pi^*(P)\Big)+\beta G'\Big(\Pi^{\max}(P)-\Pi^*(P)\Big)\Big](P^f-P)\mathrm{d}\hat{F}(P)$$

$$= \int_{\underline{P}}^{\overline{P}} \Big[U'\Big(\Pi^*(P)\Big)+\beta G'\Big(\Pi^{\max}(P)-\Pi^*(P)\Big)\Big](P^f-P)\mathrm{d}[\hat{F}(P)-F(P)], \tag{32}$$

where the equality follows from Eq. (7). It then follows from Eq. (31) that $H^\dagger > (<)\ H^*$ if, and only if, the right-hand side of Eq. (32) is positive (negative). We state and prove the following proposition.

Proposition 5. *Given that $U'''(\Pi) \geq 0$ and $G'''(\Pi^{\max} - \Pi) \geq 0$, the regret-averse competitive firm increases (decreases) its optimal futures position, i.e., $H^\dagger > (<)\ H^*$, when the CDF of the uncertain per-unit price, $\tilde{P}$, shifts from $F(P)$ to $\hat{F}(P)$, where $\hat{F}(P)$ has more simple positive (negative) skewness than $F(P)$.*

Proof. Let $\Phi(P) = \Big[U'\Big(\Pi^*(P)\Big) + \beta G'\Big(\Pi^{\max}(P) - \Pi^*(P)\Big)\Big](P^f - P)$. Then, we have $\Phi''(P) = \Psi(P) - 2U''\Big(\Pi^*(P)\Big)(Q^* - H^*)$, where

$$\Psi(P) = U'''\Big(\Pi^*(P)\Big)(Q^* - H^*)^2(P^f - P)$$
$$+\beta G'''\Big(\Pi^{\max}(P) - \Pi^*(P)\Big)\Big]\Big[Q(P) - SI_{\{P<P^f\}} + LI_{\{P>P^f\}}\Big]^2(P^f - P)$$
$$+\beta G''\Big(\Pi^{\max}(P) - \Pi^*(P)\Big)\Big](P^f - P)Q'(P)$$
$$-2\beta G''\Big(\Pi^{\max}(P) - \Pi^*(P)\Big)\Big]\Big[Q(P) - SI_{\{P<P^f\}} + LI_{\{P>P^f\}}\Big]. \tag{33}$$

Since $Q(P) < (>)\ Q^*$ whenever $P < (>)\ P^f$ and $Q'(P) > 0$, Eq. (33) implies that $\Psi(P) > (<)\ 0$ whenever $P < (>)\ P^*$. Using $\Phi(P)$, we can write the right-hand side of Eq. (32) as

$$\int_{\underline{P}}^{\overline{P}} \Phi(P)\mathrm{d}[\hat{F}(P) - F(P)]$$
$$= \int_{\underline{P}}^{\overline{P}} \Big[\Psi(P) - 2U''\Big(\Pi^*(P)\Big)(Q^* - H^*)\Big]\Big\{\int_{\underline{P}}^{P}[\hat{F}(x) - F(x)]\mathrm{d}x\Big\}\mathrm{d}P$$
$$= \int_{\underline{P}}^{\overline{P}} \Psi(P)\Big\{\int_{\underline{P}}^{P}[\hat{F}(x) - F(x)]\mathrm{d}x\Big\}\mathrm{d}P$$

$$+2(Q^* - H^*)^2 \int_{\underline{P}}^{\overline{P}} U'''\Big(\Pi^*(P)\Big)\Big\{\int_{\underline{P}}^{P}\Big\{\int_{\underline{P}}^{x}[\hat{F}(y) - F(y)]\mathrm{d}y\Big\}\mathrm{d}x\Big\}\mathrm{d}P, \tag{34}$$

where the first equality follows from integration by parts and Eq. (25), and the second equality follows from integration by parts and Eq. (26). Since $\Psi(P) > (<)\ 0$ whenever $P < (>)\ P^f$, Eqs. (27) and (28) imply that the first term on the right-hand side of Eq. (34) is negative (positive). Eq. (29) implies that the second term on the right-hand side of Eq. (34) is also negative (positive). Hence, we conclude that $H^\dagger < (>)\ H^*$ if $\hat{F}(P)$ has more simple positive (negative) skewness than $F(P)$. □

The intuition for Proposition 5 is as follows. When $\hat{F}(P)$ has more simple positive skewness than $F(P)$, realizations of $\tilde{P}$ close to $\underline{P}$ are much less likely to be seen than those close to $\overline{P}$. The regret-averse firm as such raises more concerns about the disutility from the discrepancy of its futures position, H^*, from the long position limit, L, when high realizations of $\tilde{P}$ are revealed. To minimize regret, the regret-averse firm optimally adjusts its futures position downward from H^*. Prudence, i.e., $U'''(\Pi) \geq 0$, further reinforces the firm's preferences for positive skewness and thus $H^\dagger < H^*$. On the other hand, when $\hat{F}(P)$ has more simple negative skewness than $F(P)$, realizations of $\tilde{P}$ close to $\underline{P}$ are much more likely to be seen than those close to $\overline{P}$. The regret-averse firm as such optimally adjusts its futures positive upward from H^* to reduce the discrepancy of its futures position, H^*, from the short position limit, S, when low per-unit prices are revealed. Prudence implies that the firm would like to minimize its exposure to negative skewness and thus $H^\dagger > H^*$.

5. Conclusion

In this paper, we incorporate regret theory into Sandmo's (1971) model of the competitive firm under price uncertainty. Regret-averse preferences are characterized by a modified utility function that includes disutility from having chosen ex-post suboptimal alternatives. The extent of regret depends on the difference between the actual profit and the maximum profit attained by making the optimal production and hedging decisions had the firm observed the true realization of the random output price. While the separation theorem holds under regret aversion, the prevalence of hedging opportunities may have perverse effect on the firm's optimal output level, contrary to what the conventional wisdom would suggest. We show that the regret-averse firm optimally produces more when it is banned from trading futures for hedging purposes if regret aversion plays a sufficiently more important role than risk aversion in determining the firm's production decision. We derive sufficient conditions under which the regret-averse firm optimally opts for an under-hedge (over-hedge). We further show that the firm optimally increases (decreases) its futures position when the price risk possesses more positive (negative) skewness. Regret aversion as such plays a distinctive role, vis-à-vis risk aversion, in shaping the production and hedging decisions of the competitive firm under price uncertainty.

References

Bell, D. E., 1982. Regret in decision making under uncertainty. Operations Research 30, 961–981.

Bell, D. E., 1983. Risk premiums for decision regret. Management Science 29, 1156–1166.

Bleichrodt, H., Cillo, A., Diecidue, E., 2010. A quantitative measurement of regret theory. Management Science 56, 161–175.

Braun, M., Muermann, A., 2004. The impact of regret on the demand for insurance. Journal of Risk and Insurance 71, 737–767.

Broll, U., Zilcha, I., 1992. Exchange rate uncertainty, futures markets and the multinational firm. European Economic Review 36, 815–826.

Broll, U., Welzel, P., Wong, K. P., 2015. Exchange rate risk and the impact of regret on trade. Open Economies Review 26, 109–119.

Broll, U., Welzel, P., Wong, K. P., 2016. The impact of regret on exports. German Economic Review 17, 192–205.

Danthine, J.-P., 1978. Information, futures prices, and stabilizing speculation. Journal of Economic Theory 17, 79–98.

Ekern, S., 1980. Increasing Nth degree risk. Economics Letters 6, 329–333.

Feder, G., Just, R. E., Schmitz, A., 1980. Futures markets and the theory of the firm under price uncertainty. Quarterly Journal of Economics 94, 317–328.

Guo, X., Wong, W.-K., Xu, Q., Zhu, X., 2015. Production and hedging decisions under regret aversion. Economic Modelling 51, 153–158.

Holthausen, D. M., 1979. Hedging and the competitive firm under price uncertainty. American Economic Review 69, 989–995.

Loomes, G., 1988. Further evidence of the impact of regret and disappointment in choice under uncertainty. Economica 55, 47–62.

Loomes, G., Starmer, C., Sugden, R., 1992. Are preferences monotonic—testing some predictions of regret theory. Economica 59, 17–33.

Loomes, G., Sugden, R., 1982. Regret theory: an alternative theory of rational choice under uncertainty. Economic Journal 92, 805–824.

Loomes, G., Sugden, R., 1987. Testing for regret and disappointment in choice under uncertainty. Economic Journal 97, 118–129.

Menezes, C.F., Geiss, C., Tressler, J., 1980. Increasing downside risk. American Economic Review 70, 921–932.

Mossin, J., 1968. Aspects of rational insurance purchasing. Journal of Political Economy 76, 553–568.

Muermann, A., Mitchell, O., Volkman, J., 2006. Regret, portfolio choice and guarantee in defined contribution schemes. Insurance: Mathematics and Economics 39, 219–229.

Paroush, J., Venezia, I., 1979. On the theory of the competitive firm with a utility function defined on profits and regret. European Economic Review 12, 193–202.

Quiggin, J., 1994. Regret theory with general choice sets. Journal of Risk and Uncertainty 8, 153–165.

Sandmo, A., 1971. On the theory of the competitive firm under price uncertainty. American Economic Review 61, 65–73.

Starmer, C., Sugden, R., 1993. Testing for juxtaposition and event-splitting effects. Journal of Risk and Uncertainty 6, 235–254.

Sugden, R., 1993. An axiomatic foundation of regret. Journal of Economic Theory 60, 159–180.

Tsai, J.-Y., 2012. Risk and regret aversion on optimal bank interest margin under capital regulation. Economic Modelling 29, 2190–2197.

Wong, K.P., 2011. Regret theory and the banking firm: the optimal bank interest margin. Economic Modelling 28, 2483–2487.

Wong, K. P., 2012. Production and insurance under regret aversion. Economic Modelling 29, 1154–1160.

Wong, K. P., 2014. Regret theory and the competitive firm. Economic Modelling 36, 172–175.

Wong, K. P., 2015. A regret theory of capital structure. Finance Research Letters 12, 48–57.