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Erstveröffentlichung in / First published in:

Journal of Elastomers & Plastics. 2016, 48(6), S. 523 - 534 [Zugriff am: 15.08.2019]. SAGE journals. ISSN 1530-8006.

DOI: <https://doi.org/10.1177/0095244315611168>

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Journal of Elastomers & Plastics

2016, Vol. 48(6) 523–534

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DOI: 10.1177/0095244315611168

jep.sagepub.com

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Abstract

A novel constitutive characterization method for uncured rubber behaviour has been developed in this article. A systematic measuring procedure was designed to fully investigate the uncured rubber complex stress–strain behaviour under different deformation patterns, which integrated three kinds of tests – the uniaxial tensile, the compression test and the shear test. It can be found from the observed behaviour that the uncured rubber has similar but much pronounced non-elastic stress–strain relationship, which is highly non-linear and highly rate dependent. A generalized Maxwell model with modified Yeoh model is developed to constitutively describe the observed phenomena in which parameters are identified by an evolution optimization scheme. Good agreement can be found between the model and the test data. Another finding is that, similar to vulcanized rubber, multi-test data are needed to obtain compatible constitutive models. The test results, findings and the developed model help rubber engineers deeply understand the uncured rubber's mechanical behaviour and provide a base for rubber manufacturing simulation.

Keywords

Uncured rubber, constitutive experiment, constitutive characterization, viscoelastic constitutive model, manufacturing simulation

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Introduction

The objective of this article is to develop a systematic procedure to constitutively characterize the mechanical properties of the uncured rubber. Constitutive modelling plays a very important role in the design and analysis of rubber components. Vulcanized rubber, for example, bulk rubber in automobile tyres, is successfully modelled as hyperelastic^{1–6} or hyperviscoelastic material^{7–9} using finite element simulation. However, during the manufacturing process, rubber remains uncured, which mechanical behaviour is not well understood and constitutively modelled. With the numerical techniques developed in recent years, simulation tools like finite element method have been increasingly used in optimizing the structure of rubber product.^{10,11} These techniques give much importance to the constitutive characterization and modelling of uncured rubber. Unfortunately, in contrast to vulcanized rubber, very few reports are available in the published literature for characterizing the mechanical behaviour of uncured rubber.^{12,13}

The basic hypothesis is that the uncured rubber can be treated as solid rheological body with rate- and time-dependent mechanical behaviour. Firstly, in the 'Experimental' section, a systematic experimental procedure integrating tensile, compressive and shear tests is developed to fully characterize uncured rubber under different deformation patterns. Secondly, in the 'Discussion of test results' section, comparison of rheological behaviour between uncured rubber and cured rubber are discussed. Finally, in 'Constitutive modelling' section a 3D-generalized Maxwell model-coupled Yeoh elastic part is used to model the observed phenomena in which the parameters are identified by an evolution algorithm. The model is validated in comparison with the predicted and tested data of stress–strain relationship, followed by a short summary at the end.

Experimental

This section introduces the preparation of the test specimen and then the test procedures, which are different from the traditional cured rubber.

Materials

A tread compound for a truck radial tyre was selected as the testing material, which was provided by industry. Uncured rubber is sticky and soft at room temperature, thus, preparing test specimens is difficult, especially for compression and shear specimens because the thicknesses of these two types of specimens is too large for samples to be cut. Three moulds were developed to solve this problem (Figure 1).

Rubber without sulphur was taken from the mixing chamber and put into a milling machine. It was then calendared into strips of two different thicknesses: 4 mm and 10 mm. Strips of 4 mm thickness were inserted into a square mould, shown in Figure 1(a), heated for 15 min at 120°C. The strip thus becomes a plate of 2 mm thickness so that it can be cut into dumb-bell specimens. The strips of 10 mm thickness were inserted into the distributed square and circular moulds as shown in Figure 1(b) and (c) and heated under the same conditions described above so that square and cylindrical specimens with a thickness of 8 mm can be formed.

Figure 1. Mould for: (a) dumb-bell specimens, (b) square specimens and (c) cylindrical specimens.

Testing

To fully characterize the mechanical properties of uncured rubber, an experimental procedure system integrating three tests was developed: the uniaxial tensile test, the simple shear test and the compression test, all with loading and unloading cycles. During testing, the environmental temperature was fixed at 25°C.

Tensile test

Figure 2(a) shows the uniaxial tensile test used in this study, in which a standard dumb-bell-shaped sample was employed. Five test rates, 0.1, 1.0, 2.0, 5.0 and 10 mm/s, were adopted to investigate the rate-dependent behaviour of the specimens. Three different

Figure 2. Test set-up of: (a) tensile test, (b) compression test and (c) simple shear test.

types of tensile tests were designed to understand the material behaviour in different deformation patterns.

1. Monotonic tensile test:

The sample was stretched at a constant rate until break.

2. Force control cyclic tensile test:

Unloading ended when the force equals zero. In each loading step, the strain deformation increased by 20% of the gage length l_0 of specimen. A 1 s dwelling time was set before next loading step.

3. Displacement control cyclic tensile test:

Unloading ended when the displacement of the two clamps equals zero. In contrast, the force control cyclic test, a much higher dwelling time of 180 s, was set before next loading step to investigate the multistep relaxation behaviour of materials.

Compression test

The compression test set-up is shown in Figure 2(b). Three cylindrical specimens were assembled as a total cylindrical coupon of 24 mm thickness. The compression rate was 0.5 mm/s. To establish a frictionless condition between the specimen and the pressure head, two thin films were placed at the bottom and top of the specimen.

1. The monotonic compression test:

Figure 3. The results of the first uniaxial cyclic tensile test (speed: 0.1 mm/s).

The compression displacement was -75% of the original thickness.

2. The force control cyclic compression test:

The unloading terminated when the force of the transducer reaches zero. A 5-s dwelling time was given before next loading step.

Shear test

The shear test set-up is shown in Figure 2(c). Three coarse aluminium plates and two rectangular specimens formed a sandwich construction, which was fixed to the base of the testing machine, and the movable clamp of the machine stretched the middle aluminium plate to apply the shear deformation to the specimen. Each shear test involved five loading and unloading cycles, and the shear rate was 0.2 mm/s. In each loading process, the displacement of the upper clamp was increased by 1.0 mm. The unloading process terminated when the force of the transducer was zero.

Discussion of test results

This section will describe the main test results, especially those remarkable phenomena observed compared to a normal vulcanized compound. In contrast to those vulcanized compounds, the uncured rubber shows much pronounced non-elastic deformation and complex rate-dependent behaviour. For example, as shown in Figure 3, the results of the force control tensile test reveal that the non-zero strain increases approximately 5% in each loading–unloading cycle, and this non-elastic strain increase is much more obvious than corresponding vulcanized compound.¹⁴

On the other hand, it is clear from Figure 4 that the results of the monotonic tensile test and the force control cyclic tensile test are shown together, and the curve for the

Figure 4. The results of the monotonic tensile test and the first cyclic tensile test.

Figure 5. The results of the first cyclic tensile test at different speeds.

monotonic tensile test is the envelope of those of the cyclic test. One may conclude this is a solid rheological behaviour.^{12,15,16}

Figure 5 represents the results of the force cyclic tensile test at different tensile speeds, which clearly show that when the tensile speed increases, the slope during the loading process likewise increases remarkably. This rate-dependent behaviour is also much stronger than a normal corresponding vulcanized compound.¹⁷

In Figure 6, the blue line represents the results of the monotonic compression test, and the red line represents the cyclic compression test. Similar to Figure 4, the curve of the monotonic compression test is the envelope of the curves of the cyclic test.

Figure 6. The results of the monotonic compression test and cyclic compression test.

Figure 7. The results of the cyclic shear test.

The results of the shear test are shown in Figure 7, which clearly show that there is 0.15 rad non-zero non-elastic strain increase at each loading and unloading cycle.

Constitutive modelling

In this section, a 3D non-linear viscoelastic constitutive model is developed to fully understand the above observed behaviour of uncured rubber under different conditions. As observed from Figures 3 to 7, uncured mechanical behaviour is highly non-linear,

Figure 8. Generalized Maxwell model.

complex and rate dependent; hence, the generalized Maxwell model is used as shown in Figure 8. The modified Yeoh model is selected to model the elastic part since it is highly non-linear.

The elastic part of the above-generalized Maxwell model can be described by a strain energy density function of modified Yeoh type:

$$W(I_{\bar{b}}) = C_{10} \cdot (I_{\bar{b}} - 3) + C_{20} \cdot (I_{\bar{b}} - 3)^2 + C_{30} \cdot (I_{\bar{b}} - 3)^3, \quad (1)$$

where C_{10} , C_{20} , C_{30} are material parameters, b is the left Cauchy-Green tensor expressed by the deformation gradient ($\underline{F}$):

$$\bar{b} = J^{-2/3} \cdot b = J^{-2/3} \cdot \underline{F} \underline{F}^T. \quad (2)$$

The Cauchy stress and the second Piola-Kirchoff stress can be described as:

$$\underline{\sigma} = 2J^{-1} \frac{\partial W(I_{\bar{b}})}{\partial I_{\bar{b}}} \bar{b}, \quad (3)$$

$$\underline{S}_0 = J \cdot \underline{F}^{-1} \underline{\sigma} \underline{F}^{-T}, \text{ and} \quad (4)$$

$$\text{DEV} \underline{S}_0 = \underline{S}_0 - \frac{1}{3} \cdot [C : \underline{S}_0] C^{-1}, \quad (5)$$

where C is the right Cauchy-Green tensor, DEV represents the deviator of a tensor.

Table 1. Material parameters for the generalized Maxwell model.

	C_{10} (N/mm ²)	C_{20} (N/mm ²)	C_{30} (N/mm ²)	g_j	τ_j
Hooke spring	0.0247	0.00075	0.000084	–	–
Maxwell element 1	–	–	–	0.1197	241.82
Maxwell element 2	–	–	–	0.4759	4.3704
Maxwell element 3	–	–	–	0.3475	0.7857

C_{10} , C_{20} and C_{30} : material parameters; g_j : material parameter; τ_j : relaxation time.

The total stress can then be calculated by the sum of the stress of the elastic part and the Maxwell element as:

$$\text{DEV}\underline{S}^{n+1} = \text{DEV}\underline{S}_0^{n+1} + \sum_{j=1}^N \underline{H}_j^{n+1}, \tag{6}$$

where the Maxwell stress can be derived from the elastic stress like Prony series approach:

$$\underline{H}_j(t) = \int_0^t \gamma_j \exp\left(-\frac{t-S}{\tau_j}\right) \frac{\partial \underline{S}_0}{\partial S} dS, \tag{7}$$

$$\gamma_j = \frac{g_j}{1 - \sum_{j=1}^N g_j}, \tag{8}$$

In the above equations, g_j and τ_j are material parameters. The latter τ_j represents the relaxation time of viscous components among Maxwell element.

An evolution strategy optimization is developed to fit the test data for identification of the parameter in Equations (1) to (8). The determined parameters are shown in Table 1, and the fitting effect is shown in Figures 9 and 10.

Figure 9 shows that the stress–strain curve predicted by the model is in good agreement with the tensile test results, both unloading curves basically identical. As for shear stress–strain, Figure 10 shows good agreement between the model and the test data as well, especially at large strain, experimental result can be accurately predicted basically.

One important finding for uncured rubber constitutive modelling is that, similar to vulcanized rubber, multi-test data are needed to obtain compatible constitutive models.^{18,19} If using only one test data, for example, shear data, to identify the constitutive model parameter, large errors may be found when one compare the model prediction behaviour for other deformation modes, for example, tensile test, as shown in Figure 11. This finding implies that the integration of different measurements, as proposed in this article, is necessary for obtaining reasonable constitutive models for both cured and uncured elastomers.

Summary and conclusion

In order to investigate the mechanical properties of uncured rubber, a systematic characterizing procedure was developed that integrated tensile, compressive and shear tests.

Figure 9. Comparison of tensile test results and modelling.

Figure 10. Comparison of shear test results and modelling.

It can be found from the observed behaviour that the uncured rubber has a similar but much pronounced non-elastic stress–strain relationship. The uncured rubber is highly non-linear and highly rate dependent. Nevertheless, these observed phenomena can still be treated according to a solid rheological body. Thus, a generalized Maxwell model with a modified Yeoh model is developed to constitutively describe the observed

Figure 11. Large error found when using only one test data for parameter identification.

phenomena in which the parameters are identified by an evolution optimization scheme. A good agreement can be found between the model and the test data.

Highlight

1. A systemic measuring procedure has been designed to fully investigate the uncured rubber complex stress–strain behaviour under different deformation patterns.
2. A generalized Maxwell model with modified Yeoh model has been developed to adequately model mechanical behaviour of uncured rubber.
3. A new finding is that, similar to vulcanized rubber, multi-test data are needed to obtain compatible constitutive models.
4. The test results, finding and the developed model help rubber engineers deeply understand the mechanical behaviour of uncured rubber and provide a base for manufacturing simulation.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: Project 51275265 and 5117528 was supported by the National Science Foundation of China. This study was partially supported by the Project Based Personnel Exchange Program with China Scholarship Council and German Academic Exchange Service (DAAD).

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