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Mathematical modeling of the dynamic yarn path depending on spindle speed in a ring spinning process

Mahmud Hossain¹, Christian Telke², Anwar Abdkader¹, Chokri Cherif¹ and Michael Beitelschmidt²

Abstract
This paper presents a mathematical model to predict the distribution of yarn tension and the balloon shape as a function of spindle speed in the ring spinning process. The dynamic yarn path from the delivery rollers to the winding point on the cop has been described with a non-linear differential equation system. These equations have been integrated with a Runge–Kutta method using MATLAB software. Since the numerical solution of the equations strongly depends on initial values, an algorithm of sensitivity analysis has been developed to predict the right choice of initial values in order to find a stable solution. For model validation purposes, the yarn tension has been measured between delivery rollers and yarn guide. Furthermore, a high-speed camera has been used to capture the balloon shape at different spindle angular velocities in order to compare the theoretically determined balloon shape with the one that actually occurs on the machine.

Keywords
modeling of yarn path, yarn tension prediction, balloon shape prediction, ring spinning, ring-traveler system, sensitivity analysis, validation

The yarn formation technology with the ring spinning method is well known from staple fiber production. According to the principle of the ring spinning process, which is shown in Figure 1, the fibrous material, called roving, is fed to the drafting system in order to stretch according to a required yarn count. Afterwards, the drafted roving is delivered to the ring-traveler system through the yarn guide to impart twist to the yarn. Finally, it is wound up on the cop at a constant speed. The traveler itself rotates on the ring and it is dragged with the spindle by the means of the yarn that is attached to it. Each cycle of rotation of the traveler along the ring inserts one turn of twist to the yarn.¹ A balloon shape occurs between the yarn guide and traveler, due to the centrifugal forces that arise by the rotation of the yarn about the spindle axis. In respect to that, it is important to analyze the non-linear behavior of the balloon shape and the corresponding yarn tension distribution in all regions (I–IV) of the yarn path at different spindle speeds in order to design efficient ring spinning machines.

Since the invention of the ring spinning machine in 1828, a large number of experimental and theoretical methods have been applied to predict the balloon shape and its associated yarn tension distribution during the spinning process.²⁻¹⁷ Mack²,³ introduced the balloon properties and solved the corresponding differential equation system analytically for a stationary yarn path. De Barr⁴⁻⁷ assumed a simplified approach of balloon profiles that are sinusoidal. Furthermore, he developed a concept wherein the balloon profile is interpreted as a standing wave, identical to circularly

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polarized transverse vibrations of a string. This solution describes the relations between the spinning tension, the balloon shape and the air drag in the ring spinning process. Lisini proposed a non-stationary model with partial differential equations considering the ring-rail movement. He solved the equations of motion with the Finite Element (FE) method. Batra et al. showed an integrated approach of the balloon shape and solved the quasi-stationary non-dimensional equations considering the dynamic equilibrium of the traveler. Fraser showed that the quasi-stationary non-linear equations of motion are relevant to ring spinning. The relation among the guide eye tension, traveler mass, balloon height and air drag were explained in detail. Furthermore, he showed that the yarn tension and the balloon shapes exhibit bifurcation characteristics of the equations of motion. He integrated the equations in non-dimensional formulation in order to represent them in a more general way and to considerably reduce the number of influencing parameters, for example spindle speed. Tang et al. recommends the air drag coefficients for the balloon in the case of cotton and wool.

Many of the researchers, like Fraser, have focused on determining the yarn tension and balloon shape considering the equations as dimensionless, so that the equations can be solved numerically without considering all of the influencing parameters, such as spindle speed. As the yarn tension increases according to spindle speed, it is important to model the dynamic yarn path with respect to spindle speed and predict the actual yarn tension. In this paper, the dynamic yarn path has been first presented in a dimensioned form considering the spindle speed for different regions (I–IV, Figure 1) of the yarn path in conventional ring spinning with a ring-traveler system. The differential equations with dimensioned variables have been solved with the numerical method. As the equations are extremely non-linear, the numerical solution of the equations strongly depends on the initial conditions. Therefore, an algorithm for a sensitivity analysis has been developed additionally for the numerical solution, which provides a solution space with valid stable initial values. Finally, the numerical solution has been conducted with these valid initial values using the Runge–Kutta method in a MATLAB Program. This model provides the following advantages compared to the above-mentioned models:

- the prediction of yarn tension in non-normalized (with physical units) values at all regions of the dynamic yarn path according to spindle speed, even at higher spindle speeds such as 50,000 rpm;
- the effect of Coriolis force caused by delivery speed is considered;
- the information about balloon shape and maximum balloon diameter, which is an important criteria for the machine construction;
– the model has been validated according to spindle speed by comparing the predicted yarn tension and balloon shape with the measured one, which results in a good correlation.

Theoretical model

As shown in Figure 1, the yarn path can be segmented into four regions: region I – from the clamping point of the delivery rollers to the yarn guide; region II – from the yarn guide to the traveler; region III – yarn passage through the traveler; and region IV – from the traveler to the winding point on the cop. Fundamentally, the yarn path in region II arises due to the dynamical forces that determine the yarn tension distribution and the balloon shape. The yarn tension distribution of this region is mainly influenced by the centrifugal force, the air resistance, the yarn count, the inertial force of the relative motion and, finally, by the Coriolis force. The yarn tension in region IV can be determined with the Euler formula, wherein the friction parameters between the ring and traveler, as well as between the yarn and traveler, are considered as constant. In comparison to the real spinning process, the major assumptions and limitations of the presented mathematical yarn tension and balloon shape prediction model are as follows:

– the friction between the ring and traveler, as well as between the yarn and traveler, are considered as constant;
– the effect of balloon control ring is not considered;
– the ring-rail position is considered as constant in the presented model and has validated the model with measured data accordingly.

Mathematical formulation

Due to the fact that the yarn path is nearly straight in region I, it has no fundamental influence on the balloon shape and its tension distribution. The dynamical forces within this region are not considered. In region II, a cylindrical co-ordinate system with unit vectors \( e_r, e_\theta, e_z \) and belonging co-ordinates \( r(s), \theta(s), z(s) \) has been defined for further formulations. This co-ordinate system rotates with a constant angular velocity \( \omega e_z \) about the \( z \)-axis of the inertial fixed reference frame coincident with the spindle. If the balloon shape is viewed from the rotating reference frame it can be considered to be stationary. In this rotating frame, the position vector of a material point \( M \) can be expressed as \( R(s, t) = re_r + ze_z \). It is assumed that the yarn is perfectly flexible, inextensible and uniform. The effect of yarn twist is also neglected.

If \( T(s, t) \) is the yarn tension at material point \( M \), as shown in Figure 2, then the equation of motion for the yarn element at \( M \) can be written

\[
m\left\{ \mathfrak{D}^2 R + 2\omega e_z \times \mathfrak{D} R + \omega^2 e_z \times (e_z \times R) \right\} = \frac{\partial}{\partial s}\left( T \frac{\partial R}{\partial s} \right) + F. \tag{1}\]

where \( m \) is the linear yarn density, \( \omega \) denotes the spindle angular velocity, \( \mathfrak{D}^2 R \) denotes the yarn acceleration, \( 2\omega e_z \times \mathfrak{D} R \) the Coriolis acceleration and \( \omega^2 e_z \times (e_z \times R) \) the yarn centripetal acceleration relating to the rotating reference frame.

The drag force per yarn length \( F \) can be expressed as follows:

\[
F = -D_n v_n r_n, \quad \text{with} \quad D_n = \frac{1}{2} \rho c_d d \text{ and } d = \sqrt{\frac{\rho u}{\rho_m}}, \tag{2}\]

\( \rho \) describes the air density in kg/m\(^3\), \( d \) is the yarn diameter in m, \( c_d \) denotes the air drag coefficient, \( D_n \) the air drag constant, \( \rho_m \) is the density of yarn in kg/m\(^3\) and \( v_n \) describes the normal component of the yarn velocity at point \( M \). The operator \( \mathfrak{D} \) is defined as

\[
\mathfrak{D} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial s}. \tag{3}\]
Since the rotational speed of the traveler is much higher than the delivery speed of the yarn, the problem can be considered as quasi-stationary\(^1\) and the differential operator \(\mathfrak{D}\) simplifies to
\[
\mathfrak{D} = v \frac{d}{ds}.
\]
\(^{(4)}\)

where \(v\) is the delivery velocity of the yarn at the delivery rollers.

As the yarn has been considered inextensible, the required equation for the inextensibility condition is
\[
\frac{dR}{ds} \cdot \frac{dR}{ds} = \left( \frac{dr}{ds} \right)^2 + \left( r \frac{d\theta}{ds} \right)^2 + \left( \frac{dz}{ds} \right)^2 = \left( r' \right)^2 + (r\theta')^2 + (z')^2 = 1
\]
\(^{(5)}\)

\((')\) describes the spatial derivative with respect to yarn paths.

Equation (1) can be rewritten as
\[
m v \left\{ \frac{d^2R}{ds^2} + 2 \omega e_z \times \frac{dR}{ds} + \omega^2 e_z \times (e_z \times R) \right\} = T \frac{dR}{ds} + T \omega^2 R + F.
\]
\(^{(6)}\)

The cylindrical polar components of Equation (6) with \(F\) from the Equation (2) are given below:
\[
mv^2 (r^2 - \theta^2) - 2m\omega vr\theta - m\omega^2 r = T(r' + T(r^2 - \theta^2)) + D_\omega \omega^2 r^2 r\theta' \sqrt{r^2 + z^2},
\]
\(^{(7)}\)
\[
mv^2 (r\theta' + 2r\theta') + m_\omega \omega^2 r' = T(r\theta' + T(r\theta' + 2r\theta')) - D_\omega \omega^2 r^2 (r^2 + z^2)^{3/2},
\]
\(^{(8)}\)
\[
m^2 z' = T z' + T z' + D_\omega \omega^2 r^3 \theta' \sqrt{r^2 + z^2}
\]
\(^{(9)}\)

If Equations (7)–(9) are multiplied by \(r'\), \(r\theta'\) and \(z'\), respectively, and added with considering equation (5), the following results:
\[
\frac{dT}{ds} = -m\omega^2 rr'.
\]
\(^{(10)}\)

The integration of (10) yields
\[
T(s) = T_0 - \frac{1}{2} m\omega^2 r^2.
\]
\(^{(11)}\)

\(T_0\) is the yarn tension at the guide eye and \(r\) is the balloon radius at material point \(M\).

**Boundary conditions**

The balloon equations need to be solved considering the boundary conditions at the guide eye and at the traveler.

For the guide eye (\(O\) (Figure 2)), the boundary conditions are
\[
r(0) = 10^{-3}m, \theta(0) = 0, z(0) = 0.
\]
\(^{(12)}\)

The first boundary condition at the traveler, \(N\) (Figure 2) is
\[
r(s_t) = a, z(s_t) = h.
\]
\(^{(13)}\)

\(s_t\) describes the in length of the yarn between guide eye and traveler, \(a\) is the ring radius and \(h\) is the balloon height (Figure 2).

The second boundary condition at the traveler is defined as
\[
T_1[g \sin \phi - a\theta(s_t)] = \mu \sqrt{[T_1(r'(s_t) + g \cos \phi) - m_r r^2 a^2] + [T_1 z'(s_t)]^2}.
\]
\(^{(14)}\)

This second condition relates the yarn tension at the bottom of the balloon \(T_1\) and the traveler mass \(m_r\) to the geometrical quantities \(r'(s_t), \theta(s_t), z'(s_t)\) and \(\phi\) and the friction parameters \(\mu\) and \(g = \omega_\mu a^2 = 1.7\) as constant.\(^13\) The winding tension \(T_2\) (Figure 2) is estimated by Euler equation as follows:
\[
T_2 = T_1 \cdot \omega_\mu a^2
\]
\(^{(15)}\)

where \(\mu\) denotes the friction between the ring and traveler, \(\omega_\mu\) is the coefficient of friction between the yarn and traveler and \(a\) is the wrap angle (Figure 2).

**Numerical solution of the theoretical model**

In this section, the solution process to determine the balloon shape and its yarn tension distribution are described. The numeric solution according to above-mentioned second-order non-linear differential equations (7)–(9) and their boundary conditions (12)–(14) is done in MATLAB. A two-point boundary value problem is solved by using the shooting method.\(^18\) First of all, the equations are rewritten to a couple of first-order differential equations. A Runge–Kutta method with variable spatial step width has been used to integrate the equations. In order to find the correct set of initial
values \((r', T_0)\), which fulfills the above-mentioned boundary conditions, a reformulation of these boundary conditions as a squared error residual sum is necessary. Afterwards, an optimization with the balloon shape as the target function in order to minimize the residual error is performed. The solution to Equations (7)–(9) is, due to their highly non-linearity, strongly dependent on the initial values. That is why a sensitivity analysis in terms of the initial values is necessary to find an optimal solution efficiently and automatically to the problem with the above-mentioned iterative optimization algorithm. The fundamental steps of sensitivity analysis are given in Figure 3.

The initial values of the non-linear equations have been calculated from the sensitivity analysis. The numerical process has been conducted for a set of \(r(0)\) and \(T_0\) for different spindle speeds. Exemplarily, the solution space of a sensitivity analysis of \(T_0\) at the spindle speed of 10,000 rpm is given in Figure 4. According to Figure 4, the sensitivity analysis of \(T_0\) has been shown for \(r(0)\) of 0.15 at a spindle speed of 10,000 rpm. The algorithm of the sensitivity analysis can automatically identify the information about the balloon shape as follows:

- valid initial conditions, which fulfill the boundary conditions;
- invalid solutions, where the boundary conditions are not fulfilled;
- unexpected solution for unwanted balloon shape although the boundary conditions are fulfilled;
- effects of non-linearity and numerical stability.

The residual errors \(\Gamma_1\) and \(\Gamma_2\), \(\Gamma_k \in \mathbb{R}\) with their weighting factors \(\xi_k \in \mathbb{R}\) can be written according to Equations (13) and (14) in terms of the initial shooting values \(T_0\) and \(r'(0)\) as follows:

\[
\Gamma_1 = \xi_1 (r(s_i, T_0, r') - a)^2,
\]

\[
\Gamma_2 = \xi_2 \left[ T_1 [g \sin \phi - a\theta'(s_i, T_0, r')] - \mu \left\{ [T_1 r'(s_i, T_0, r') + g \cos \phi - m r \theta^2 a]^2 + [T_1 z'(s_i, T_0, r')]^2 \right\} \right]^2.
\]
$T_0$ is the yarn tension at the guide, $T_1$ is the yarn tension at the traveler, $a$ describes the ring radius, $\phi$ the angle between the yarn path in the winding zone and the line connecting the traveler with the center of the cop (Figure 2), $m_T$ denotes the traveler mass, $\omega$ is the angular velocity of the spindle, $\mu$ describes the coefficient of friction between the ring and traveler and $g$ is the friction parameter between the yarn and traveler.

The summation of the weighted residual errors of (16) and (17) yields

$$\Gamma(s_i, T_0, r') = \sum_{i=1}^{N} \xi_i \Gamma_i(s_i, T_0, r').$$  \hspace{2cm} (18)

The minimization of (18) with respect to the abovementioned initial trial values $T_0$ and $r'(0)$ is calculated from the sensitivity analysis:

$$\min_{T_0, r'} \Gamma(s_i, T_0, r') = \min_{T_0, r'} \sum_{i=1}^{N} \xi_i \Gamma_i(s_i, T_0, r')$$  \hspace{2cm} (19)

delivers an optimal set of trial values that fulfill the boundary conditions (13) and (14).

The variables, which are used for the mathematical modeling and for the experimental investigation, are given in Table 1.

The whole numerical method is represented as a block diagram in Figure 5. In order to perform an automated analysis and solution, the process is based on a parameter database that contains parameters such as spindle speed, yarn count, balloon height, etc., to define the input parameter space. To determine the area of stable initial values within the solution space with respect to the given boundary conditions at the guide eye and the traveler, a sensitivity analysis has been performed as a preprocessing step.

With this a valid preset of initial values can be determined that guarantees a stable optimization determination within the final solution step afterwards. With these selected initial values the final numerical solution step is performed using the Runge–Kutta method in order to calculate the balloon shape and its yarn tension distribution at different regions.

**Results and discussion**

**Calculated results with the numerical solution**

The yarn tension in different regions of the yarn path (I–IV) is calculated with the developed numerical model and is shown in Table 2. The yarn tension in region II has been calculated at three different locations (namely the balloon heights of 175, 100 and 5 mm) so that the yarn tension $F(\text{II})$ close to the yarn guide, balloon control ring and traveler can be predicted. For the calculation, the balloon height is considered as 180 mm. The maximal balloon diameter $r_{\text{max}}$ and its position $h_{\text{max}}$ are...
also calculated with this model, which is important for the machine construction. The yarn tension in all regions (I–IV) increases with the higher spindle speed. The maximum yarn tension occurs at region IV between the traveler and cop. This tension is higher than that of other regions, due to the friction between the ring and traveler and between the yarn and traveler. Moreover, the yarn tension $F(I)$ at the yarn guide (region I) is greater than that at the balloon (region II) and at the traveler (region III).

The yarn tension in region II decreases with the balloon diameter and the minimal tension occurs at the maximum balloon diameter $r_{\text{max}}$ (Equation (10)). The calculated balloon shapes corresponding to yarn tension (Table 2), for example at the spindle speeds of 5000, 10,000 and 15,000 rpm, are shown in Figure 6(a). The friction between the ring and traveler, $\mu$, and the friction between the yarn and traveler, $\mu_y$, are assumed for the theoretical model as listed in Table 1. The other conditions of Table 1 are taken into account both for the theoretical model and experimental investigation.

### Table 2. Calculated values of yarn tension at different spindle speeds

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Spindle speed</th>
<th>$F(I)$</th>
<th>$F(II)$</th>
<th>$F(III)$</th>
<th>$F(IV)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[]</td>
<td>[rpm]</td>
<td>[cN]</td>
<td>[cN]</td>
<td>[cN]</td>
<td>[cN]</td>
</tr>
<tr>
<td>1</td>
<td>5000</td>
<td>7.73</td>
<td>7.73</td>
<td>7.55</td>
<td>7.49</td>
</tr>
<tr>
<td>2</td>
<td>7500</td>
<td>13.62</td>
<td>13.62</td>
<td>13.02</td>
<td>12.90</td>
</tr>
<tr>
<td>3</td>
<td>10,000</td>
<td>23.71</td>
<td>23.69</td>
<td>22.59</td>
<td>22.84</td>
</tr>
<tr>
<td>4</td>
<td>12,500</td>
<td>33.56</td>
<td>33.53</td>
<td>31.38</td>
<td>32.21</td>
</tr>
<tr>
<td>5</td>
<td>15,000</td>
<td>46.80</td>
<td>46.77</td>
<td>43.40</td>
<td>44.87</td>
</tr>
</tbody>
</table>

$F(I)$: yarn tension between the clamping point of the delivery rollers and yarn guide; $F(II)$: yarn tension between the yarn guide and traveler; $F(III)$: yarn tension at the traveler; $F(IV)$: yarn tension between the traveler and winding point in the cop; $h$: balloon height from the ring rail; $r_{\text{max}}$: maximum balloon diameter; $h_{\text{max}}$: balloon height from the ring rail at $r_{\text{max}}$.

Validation of calculated yarn tension between the delivery rollers and yarn guide (region I)

For further validation of calculated yarn tension in region I, a three-point tensile force measurement sensor M1330 from Tensometric Messtechnik GmbH was used to measure the yarn tension between the delivery rollers and yarn guide. The sensor was positioned 100 mm away from the clamping point of the delivery rollers. The balloon height, that is, the distance between the yarn guide and ring rail, is set to be constant at 180 mm. The measurement setup is shown in Figure 7(a). The yarn tension was measured several times in the normal ring spinning process to receive statically verified measurement results. Figure 7(b) shows the results for the yarn tension measured with the sensor compared with the calculated one at different spindle speeds.

It is shown that the simulated values display good correlation with the measured values and confirm the prediction for different spindle speeds. The yarn tension increases with higher spindle speed both in the case of calculated results and experimental ones, as shown in Table 3. However, the calculated and measured values differ at higher spindle speed. The calculated yarn tensions vary only 2–3% from the measured ones at the spindle speeds of 5000 and 7500 rpm. This variation is remarkable as the spindle speed increases. Therefore, the
measurement method of yarn tension needs to be developed for further experiments, especially at higher spindle speed. Moreover, the frictional parameters, such as friction between the ring and traveler and the friction between the yarn and traveler have been considered constant in the case of theoretical calculation. The friction between the yarn and traveler decreases at higher spindle speed, which also influences the

Figure 6. Comparison of balloon shapes: (a) the calculated balloon shapes; (b) the measured balloon shapes.
measured values of yarn tension. Moreover, the measured yarn tension is slightly affected by the position of the sensor as well. The process parameters of the machine, such as the variation of spindle speed, variation of balloon form, etc., affect the measured values too. Furthermore, the balloon collides with machine parts at higher spindle speed as the measurement is set up without the balloon control ring and thus decreases the balloon form and the yarn tension slightly.

Finally, the twisting of yarn due to the rotation of the traveler propagates up to the spin triangle, where the yarn is almost untwisted. The sensor part positioned between the yarn guide and delivery rollers prevents the twist to propagate further and causes end breakages. That is why the end breakages occur during the yarn tension measurement with the three-point sensor, particularly at higher spindle speed.

**Prediction of yarn tension at higher spindle speed**

The tension was predicted up to 50,000 rpm with the help of the model shown in Table 4.

As the frictional heat between the ring and traveler increases, the yarn can be spun with a maximum spindle speed of 30,000 rpm. However, this theoretical model with some modification can be applied in the case of the spinning yarn with a friction-free new superconducting magnetic bearing, where the spindle speed is expected to increase significantly.

---

**Table 3.** Comparison of calculated and experimental values of yarn tension at different spindle speeds

<table>
<thead>
<tr>
<th>Spindle speed [rpm]</th>
<th>Calculated yarn tension, F(I) [cN]</th>
<th>Measured yarn tension, F(I) [cN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>7.73</td>
<td>7.50</td>
</tr>
<tr>
<td>7500</td>
<td>13.62</td>
<td>13.36</td>
</tr>
<tr>
<td>10,000</td>
<td>23.71</td>
<td>20.83</td>
</tr>
<tr>
<td>12,500</td>
<td>33.56</td>
<td>33.00</td>
</tr>
<tr>
<td>15,000</td>
<td>46.80</td>
<td>42.00</td>
</tr>
</tbody>
</table>

F(I): yarn tension between the delivery rollers and yarn guide in region I

**Table 4.** Prediction of the yarn tension at different spindle speeds

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Spindle speed [rpm]</th>
<th>F(I) at r_{\text{max}} [cN]</th>
<th>F(II) [cN]</th>
<th>F(III) [cN]</th>
<th>F(IV) [cN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17,500</td>
<td>59.30</td>
<td>53.42</td>
<td>56.75</td>
<td>100.80</td>
</tr>
<tr>
<td>2</td>
<td>20,000</td>
<td>78.08</td>
<td>70.59</td>
<td>74.75</td>
<td>132.73</td>
</tr>
<tr>
<td>3</td>
<td>22,500</td>
<td>99.13</td>
<td>89.74</td>
<td>94.91</td>
<td>168.51</td>
</tr>
<tr>
<td>4</td>
<td>25,000</td>
<td>105.64</td>
<td>88.26</td>
<td>100.44</td>
<td>179.60</td>
</tr>
<tr>
<td>5</td>
<td>50,000</td>
<td>142.00</td>
<td>120.11</td>
<td>121.18</td>
<td>241.39</td>
</tr>
</tbody>
</table>

F(I): yarn tension between the clamping point of the delivery rollers and yarn guide; F(II): yarn tension between the yarn guide and traveler; F(III): yarn tension at the traveler; F(IV): yarn tension between the traveler and winding point in the cop; r_{\text{max}}: maximum balloon diameter.
Conclusion

The dynamic yarn path of the ring spinning machine has been expressed in a differential equation system, including Coriolis force, considering the dimensioned variables such as spindle speed. Since the equations are non-linear and strongly dependent on initial values, a sensitivity analysis has been conducted to find the valid preset of initial values for a stable optimization and solution process. The yarn tension at all regions (I–IV) and the balloon shape have been calculated at different spindle speeds (5000–50,000 rpm) with the presented theoretical model. For the validation of the model, the yarn tension between the delivery rollers and yarn guide (region I) has been measured with a three-point sensor at spindle speeds from 5000 to 15,000 rpm. Moreover, the balloon shapes at different spindle speeds have been captured with a high-speed camera. All the experimental results have been compared with the calculated results. The experimental values, such as yarn tension in region I, are lower than those of the theoretical ones. This is because some assumptions of spinning parameters (such as the friction between the ring and traveler, the friction between the yarn and traveler, etc.) are taken as constant for the theoretical model, which vary in the case of the measured yarn tension. As the end breakages occur at higher speed during measurement, the model has been validated up to the spindle speed of 15,000 rpm. However, the calculated yarn tensions show good correlation with the measured values up to 15,000 rpm. Finally, this model is able to measure the yarn tension at any spindle speed, although it is important to find the limit of spinning due to end breakages caused by higher yarn tension.

Outlook

The presented developed numerical model is applied to predict the balloon shape and the tension distribution in a conventional ring spinning machine. However, in a research work of the DFG (German Research Foundation, Project Nr. CH-174-33-1), the existing ring-traveler system is replaced with a friction-free twisting element applying a superconducting magnetic bearing, which eliminates the main problem of increasing the productivity in the ring spinning machine. As shown in Figure 8, the superconducting magnet bearing consists of a circular superconductor and permanent magnet ring. After cooling the superconductor below its transition temperature, the permanent magnet ring levitates and is free to rotate above the superconductor ring according to the principles of superconducting levitation and pinning. Thus, it is expected to spin yarn with higher productivity compared to the existing system without deteriorating the yarn quality. In a future work, this above-mentioned mathematical model will be modified focusing on the dynamic yarn path with the superconducting magnet bearing replacing the existing ring-traveler system and, thus, it can determine the maximum limit of this system in terms of yarn tension. Moreover, both results can be compared to find the physical limitations of the new twisting system. The yarn tension during the unwinding process can also be predicted with slight modification of this model.

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