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Erstveröffentlichung in / First published in:

Proceedings of the Institution of Mechanical Engineers. Part Part K, Journal of multi-body dynamics. 2016, 230(4), S. 541 – 552 [Zugriff am: 19.08.2019]. SAGE journals. ISSN 2041-3068.

DOI: <https://doi.org/10.1177/1464419316633565>

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Vertical beam modal response in a broadband energy harvester

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Proc IMechE Part K:
J Multi-body Dynamics
2016, Vol. 230(4) 541–552
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DOI: 10.1177/1464419316633565
pik.sagepub.com

Abstract

We examined energy harvesting using a vertical composite laminate beam with an additional moving mass subjected to kinematic harmonic excitation. The gravity effect and the moving tip mass applied to the system cause considerable changes in effective spring-mass characteristics of the bending beam. Simultaneously, we observed dynamical beam damping by an additional degree of freedom and non-linear effects including friction between the moving mass and the beam structure. The experiments were performed on the beam excited kinematically by a shaker, while beam velocity measurements were made by a scanning laser vibrometer. We applied modal analysis in the limit of a fairly low excitation level. The selected modal vibrations are illustrated by corresponding output time series.

Keywords

Non-linear vibrations, broadband energy harvesting, piezoceramic

Date received: 21 June 2015; accepted: 25 January 2016

Introduction

Broadband energy harvesting is based on the concept of non-linear dynamics.¹ This concept has been invented to power small portable devices and/or charge batteries in changeable ambient conditions. Such harvesters can consist of a mechanical resonator and a piezoelectric transducer connected to an electrical circuit.² The non-linear effects introduced to the system modify effective potential of the mechanical resonator and, consequently, the resonance frequency region including its broadening by resonance backbone inclination and via the appearance of additional non-linear solutions. Some of the proposed broadband energy harvesters considered introducing additional fields coupling (electric and magnetic fields),^{3–6} while others were based on impact phenomena.⁷ The recent configuration invented by Friswell et al.⁸ studied the effects of gravitational field acting on a vertical beam on a horizontally moving frame with a tip mass excited by harmonic frame displacement. These studies were continued with the stochastic force excitations.⁹ Using a modal analysis, we identified the leading modes in the system because this information could be important for optimising energy output.

The influence of an additional degree of freedom usually provokes an additional frequency response. This effect has already been used for energy harvesting to broaden the region of frequency response. Nana and Wofo¹⁰ suggested the use of an array of

two or more harvesters to increase the power delivered into load. Shahruz¹¹ analyzed a set of parallel single degree-of-freedom harvesters tuned at slightly different resonant frequencies, whereas Erturk et al.¹² considered a harvester as a serial set of two L-shaped beams. Recently, Litak et al.,¹³ Kucab et al.,¹⁴ and Litak¹⁵ studied two coupled magnetopiezoelectric oscillators with mistuning. They reported the effects of non-linearities on the synchronization of oscillators and, consequently, on power output. Finally, the concept of impact dampers using a moving mass and piezoelectric beams was discussed by Afsharfard and Farshidianfar.¹⁶

In our case, the moving mass made of steel coupled to a vertical cantilever laminate beam (Figure 1) would create impact and friction phenomena. In general, the non-linear effects improve the effectiveness of energy harvesting systems by broadening frequency intervals in the resonance region.^{13,17,18} Such energy

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Figure 1. Photo of the experimental stand with marked laser scanned points on the vertical beam (a). Schematic plot of the beam with moving element (b). Moving mass on the corrugated surface (c); $\mathbf{G}$, $\mathbf{R}$ $\mathbf{F}$ are the gravity, normal-reaction, and tangent-friction forces, while $\mathbf{v}$ and $\mathbf{a}$ are the velocity of the moving mass and acceleration of the beam, respectively. The vertical beam is supported at its lower end by the horizontally moving shaker. The resonance frequency (50 kHz) of the shaker was far beyond the working frequency of the piezoelectric beam model.

harvesting devices contain mechanical resonators and additional energy transducers for transforming ambient mechanical energy into an electric form. Following this concept, Gu and Livermore¹⁹ reported the experimental results of developing models of energy harvesting devices in which a lower frequency resonator increases the output frequency by impacts. Other experiments with impacts demonstrate that the efficiency of electrical power transfers can be significantly improved.^{7,20}

Finally, the present experimental model investigates the influence of a moving mass on the efficiency of piezoelectric transducers. This mass can cause an additional dissipation via friction phenomena. It is worth noting that interrupted contact of the moving mass with the cantilever (Figure 1(b)) does change effective spring characteristics of the vibrating beam, which increases the resonant frequency. While a thorough theoretical description of the corresponding model with a fixed mass can be found in literature,⁸ we provide a simplified frequency range analysis in the following section.

Frequencies in limiting cases

Here, for simplicity reasons, we analyse two limiting cases with respect to beam-mass system mutual interactions: one system with a fixed mass $M_t = m_2 + m_3$ and the other with a contact loss $M_t = m_2$. These two systems differ by the tip mass value $M = \text{const}$ which is higher for the fixed mass case (for the parameter values see Table 1). In a more realistic case, the tip mass is time dependent $M_t(t)$. The kinetic energy of the considered system (Figure 1) is as follows⁸

$$T = \frac{1}{2} \rho A \int_0^{l_1} [(\dot{v}_p(s, t) + \dot{y}(t))^2 + \dot{u}_p(s, t)^2] ds + \frac{1}{2} M_t(t) [(\dot{v}(t) + \dot{y}(t))^2 + \dot{u}(t)^2] + \frac{1}{2} I_0 \dot{\phi}^2 \quad (1)$$

Table 1. Parameters of the mechanical resonator and electrical circuit.

Symbol and value	Description
$l_1 = 198 \text{ mm}$	Beam length
$l_2 = 35 \text{ mm}$	Beam width
$h = 2.2 \text{ mm}$	Beam thickness
$l_3 = 10 \text{ mm}$	Position of the mass
$b_1 = 37 \text{ mm}$	The length of the piezoelectric layer
$b_2 = 18 \text{ mm}$	The width of the piezoelectric layer
$b_3 = 28 \text{ mm}$	Active layer length
$b_4 = 14 \text{ mm}$	Active layer width
$b_5 = 12 \text{ mm}$	Position of the piezoelectric layer
$C = 29.5 \text{ nF}$	Capacity of the piezoceramic layer
$m_1 = 16.30 \text{ g}$	Mass of the laminate with piezoelectric
$m_2 = 3.21 \text{ g}$	mass of steel handle
$m_3 = 2.28 \text{ g}$	Additional moving mass
$E = 10^4 \text{ MPa}$	Young modulus of the beam
$\rho = 1070 \text{ kg/m}^3$	Mass density of the beam

Note: The MFC piezoelectric (27F12-049D) and piezoelectric parameters were provided by the Smart Materials Company.

while the corresponding potential energy reads as

$$\Pi = \frac{1}{2} EI \int_0^{l_1} \kappa(s, t)^2 ds - g \rho A \int_0^{l_1} u_p(s, t) ds - M_t(t) g u(t) \quad (2)$$

where v and u denote the horizontal and vertical displacements of the mass (see Figure 14 in Appendix 2). P denotes an arbitrary point on the beam whose position is described by the coordinates s , v_p , and u_p . $g = 9.81 \text{ m/s}^2$ is the gravity acceleration, E is the Young modulus and is $I = l_2 h^3 / 12$ the geometrical moment of inertia, ρ is the density of the beam mass. I_0 is the inertial moment of the beam.

Figure 2. Natural frequencies of the beam system (moving mass with contact and no-contact cases) against the Young modulus of the beam, E .

Additionally, parameter and symbol descriptions are included in Table 1 and Appendix 1.

Besides, the electrical field energy, E_{elf} , inside the piezoelectric which includes the work, W , of the piezoelectric patch in moving or extracting electrical charge, and electrostatic energy on the electrodes of a piezoelectric patch, E_{els} are the following²¹

$$E_{elf} = \int_{V_{ol}} \mathbf{E} \mathbf{D} dV_{ol}, \quad W = \theta v U, \quad E_{els} = QU \quad (3)$$

where $\mathbf{E}$ is an external electrical field between the electrodes, $\mathbf{D}$ is an electrical displacement vector in the piezoceramic layer, V_{ol} is a volume of the piezo patch, Q is the charge on the electrodes, and θ is an effective transduction constant dependent on the piezoelectric material properties and the beam and piezoelectric patch geometry.

Consequently, the natural frequency of the mechanical system of a pre-buckled case is given by (Appendix 2)

$$f_n(E) = \frac{1}{2\pi} \sqrt{\frac{EIP_6 - g(M_t P_4 + \rho A P_9)}{M_t + \rho A P_1 + I_0 P_5^2}} \quad (4)$$

The natural frequencies of the beam system for both cases: fixed and moving mass with no-contact, against the Young modulus of the beam E , are plotted in Figure 2.

In our case, $E = 10^4$ MPa and frequencies $f_1 = 9.08$ Hz, and $f_2 = 10.22$ Hz for the fixed and freely moving masses, respectively. Constants P_i are defined in Appendix 2. This result has been obtained by assuming a constant tip mass, $M_t = \text{const}$. For variable mass, these two frequencies would roughly define a frequency shift. The moving mass frequency spectrum would also be influenced by a friction effect

Figure 3. Electrical circuit set with an electro-motive force on the piezoceramic element; electric resistors with the Data AcQuisition DAQ system and voltage divider. R_{eq} and R_{DAQ} are the equivalent and internal resistances of the DAQ computer card, respectively.

caused by a relative motion with the beam-mass contact contributing to energy dissipation.

Experimental model

In the experiment, we investigated the system (Figure 1) with a piezoelectric patch using a piezoelectric module d_{31} , which is a measure of voltage transduction due to deformation in a transverse direction. During the beam's vibration, electric power generated by the piezoelectric loaded the corresponding resistors. The electrical output was measured by means of a data acquisition set (DAQ) (see Figure 3) as

$$U_{DAQ} = U \frac{R_1}{R} \quad (5)$$

where the ratio R_1/R is the voltage divider presented in Figure 3, applied to avoid the exceeding of a measurement voltage range. U is the output voltage. In our experiments, we used a piezoceramic patch layer attached to the beam. The diagram of the beam used in the experiments together with the labels of dimensions and corresponding sizes is shown in Figure 4.

To measure the corresponding velocities of the beam and the frame, we used a scanning laser vibrometer. For a low level of excitations, we performed a standard modal analysis to find characteristic mode shapes and its eigen frequencies (see Figure 5).

Discussion of the results

Following the scanning measurements (see the grid points depicted on the excited beam and the reference frame in Figure 1(a)), we performed modal analysis. This analysis focuses on movement behavior of a mechanical structure; specifically, its configuration in space and time. The natural base of indicators of this movement is given by eigenvalues, consisting of eigen frequencies, modal dampings, and eigenvectors.

Modal analysis

The eigenvectors (deflection shapes of the structure) are closely related to the eigen frequencies. The shapes

Figure 4. Diagram of the beam with MFC (Macro Fiber Composite). The marked dimensions are given in Table I.

Figure 5. Modal analysis of the vibrating beam in the fixed mass case. The corresponding bending mode shapes are plotted below and in Roman numerals as I–VI, while IIa represents the twisting mode.

Figure 6. Modal analysis of the vibrating beam in the moving mass case. The corresponding mode shapes are plotted below and marked by Roman numerals.

are only determined by the inner composition of the structure and its parameters. Additionally, there exist operational deflection shapes induced by a specific excitation due to action of the structure. In this case, the resulting vibrational behavior is determined by both the composition of structure response and excitation. Usually, excitation defines both time dependence and its location. In our case, the excitation is realized by the inertial force of the moving frame.

The acceleration-current-transmission factor and force-current-transmission factors characterizing the laser vibrometer system were $K_{ac} = 25 \text{ m/(s}^2\text{A)}$ and $K_{fc} = 15 \text{ N/A}$, respectively. The output voltage of the *Polytec* frequency generator was 1 V, and the gain of the amplifier was 5. The sampling rate was 10 kHz, with the sampling rate of the vibrometer set to 2.56 kHz. We used a measuring time of 3.2 s for a bandwidth ranging from 0 to 2 kHz and 3200 FFT lines (a resolution of 312.5 mHz). [Is the unit 'mHz' correct as given? Kindly check.] Finally, the shaker amplitude was fixed at 12 mm.

The results of the modal analysis for the beam with the moving mass in the frequency domain and a selection of modal shape pictures are presented in Figure 5. The modal analysis is designed for linear systems. In our studies, we employed this analysis to a fairly low excitation level.

However, it is possible to see that the mode shapes do not represent smooth surfaces as was expected of the system affected by friction and impacts. Similarly, the line broadening observed for higher frequencies is caused by these non-linearities. In Figure 6, we compare the modal analysis results obtained for the case of the moving mass (violet line) and the fixed mass (black line) for the first three modes which are vital for energy harvesting. One can observe that the spectrum is deformed by an extra degree of freedom of the moving mass, and the non-linear effects of friction and impacts, including additional damping. However, the increased bandwidth caused by adding one degree of freedom to the system is not precisely quantified.

The second-order set of linearized equations of motion for a discretized mechanical system of N DOF (corresponding to the number of measured points)

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\dot{\mathbf{y}} + \mathbf{C}\mathbf{y} = \mathbf{f}(t) \quad (6)$$

where $\mathbf{y}(t)$ and $\mathbf{f}(t)$ are the vectors of generalized coordinates and generalized force, while $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ are the mass, damping and stiffness matrices, respectively. Note that the modal parameters ($\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$) are unknown, and they are to be found experimentally, together with corresponding mode shapes, as eigenvectors.

Figure 7. Magnification of the first three vibration modes for the system with moving (violet) and fixed (black) masses.

Figure 8. Output voltage U time series for the first bending mode of the beam with fixed tip mass at the frequency $f = 9.96$ Hz (a) and with moving mass at $f = 9.96$ Hz (b). Note that in each sub-figure there is the voltage output U above, while below we plot the measured reference shaker velocity U_{Ref} (measured in proportional units (V)).

Note that the original non-linear system is linearized for modal investigations. In this case, linearization is justified by a small amplitude vibration limit. This limit could be confusing because a clearance degree of freedom occurs in the moving mass case. Consequently, the modal analysis provides averaging of the contact and contact loss inertial mass. The studies conducted by Friswell et al.⁸ were dedicated to fairly larger amplitudes of vibration and single-mode approximation at the same time. On the other hand, the relative motion of the moving mass and the beam would also influence the damping parameter.

The most visible effect to be observed is the shift of the mode III frequency to a higher frequency region. This could be a non-linear effect of stiffness changes with respect to the contact between the moving mass and the beam. Simultaneously, this mode is

characterized by a smaller velocity compared to the fixed mass case. Comparing the peak values in the resonance, we observe a decrease in the characteristic range velocities (Figure 5) as the modes are not so pronounced in the moving mass case (see Figure 3), where additional damping and non-linear effects are present. On the other hand, the first mode response is smeared in the system with the moving mass. This additional damping in the beam could be related to frictional effects caused by the movement of additional sliding inertia. This movement induces friction oscillations and, consequently, energy absorption by an additional degree of freedom.

Furthermore, it is easy to notice a difference between the fixed and the moving mass time series. The moving mass introduces irregular perturbation to both the piezoelectric output voltage

Figure 9. Wavelet spectra of the piezoelectric voltage U and the reference velocity U_{ref} of the first bending mode in the case of fixed (a) and moving (b) tip mass at the frequency $f = 9.96$ Hz. The colours are related to the magnitudes of $P_W(s, n)$ (equation (8)) according to the logarithmic scale $\log_2$ (see right panels marked with the corresponding scale exponents). The dark contour lines enclose regions of greater than 95 % confidence and the area below the thin U-shaped curve denotes the cone of influence.

Figure 10. Output voltage U and reference velocity U_{ref} time series for the second bending mode of the beam with fixed tip mass at the frequency $f = 76.25$ Hz (a) and with moving mass at $f = 78.8$ Hz (b).

(Figures 8, 10, and 12) and the moving frame displacement excited by the shaker (Figures 5 to 7). Clearly, it disturbs the voltage output signal, which is noticeable in the upper sides of ‘b’ parts in Figures 8, 10, and 12 for the first, the second, and the third mode, respectively. Interestingly, one can observe the non-linear changes of the voltage output in the first mode even in the fixed mass case (Figure 8(a)). In spite of regularity visible in the reference signal, the shape of $U(t)$ is of a non-sinusoidal form. This may be due to the non-uniformity of beam mass distribution together with point mass attachment, and the geometrical non-linear effect of gravity (see Friswell et al.⁸).

Wavelets

For better clarity, we also performed wavelet analysis. Namely, we studied the time series of the voltage output and velocity $X(t)$ (here $X(t)$ represents a time series U or U_{ref}) given by a Continuous Wavelet

Transform (CWT).^{22,23} The corresponding CWT with respect to the wavelet function $\Psi()$ is defined as follows

$$W_{\xi,n}(X) = \sum_{i=1}^N \frac{1}{\xi} \Psi\left(\frac{i-n}{\xi}\right) \frac{(X(t_i) - \langle X \rangle)}{\sigma_X} \quad (7)$$

where $\langle X(t) \rangle$ and σ_X are the average and the standard deviation for the corresponding time series, and t_i is the sampling time. Finally, the wavelet $\Psi(t)$ is referred to as the mother wavelet, and the letters ξ and n denote the scale and the time index, respectively. The wavelet power spectrum (WPS) of the X time series is defined as the square modulus of the CWT

$$P_W = |W_{\xi,n}|^2 \quad (8)$$

In our calculations, we have used a complex Morlet wavelet as the mother wavelet. The Morlet wavelet consists of a plane wave modulated by a Gaussian

Figure 11. Corresponding voltage Wavelet spectra of the piezoelectric voltage U and reference velocity U_{ref} for the second bending mode of the beam with fixed tip mass at the frequency $f = 76.25$ Hz (a) and with moving mass at $f = 78.8$ Hz (b).

Figure 12. Time series for the third bending mode of the beam with fixed tip mass at the frequency $f = 197.19$ Hz (a) and with moving mass at $f = 215.6$ Hz (b).

function and is described by

$$\Psi(\eta) = \pi^{-1/4} e^{i\theta_0\eta} e^{-\eta^2/2} \quad (9)$$

where θ_0 is the center frequency, also referred to as the order of the wavelet, and η is the renormalized time variable.

The value of θ_0 defines the number of oscillations in the wavelet, and thus controls the time/frequency resolutions. In our analysis, we used $\theta_0 = 6$; this choice provides a good balance between the time and frequency resolutions. Also, for this choice, the scale is approximately equal to the period, and therefore the terms, scale and period, can be interchanged for interpreting results. For a general discussion see literature.^{22–24}

The particular time series of the examined system will be analysed by wavelets in the next subsection.

Time series analysis

In this section, we analyse the non-linear properties indicated in the model description. To this end, we show the time series of measured values (U – the output voltage

and U_p – the corresponding beam velocity) together with the corresponding wavelet spectra in Figures 8 to 13. In this analysis, the focus is put on the third three resonant values representing different mode shapes (see Figure 7 and Table 2). Note that the non-linearities visible in Figure 8 were confirmed by Wavelet transform (Figure 9) by detecting of the higher harmonics (lower periods) in the upper panel (output values) with respect to the mostly sinusoidal signal of a period about 10^{-2} s (about 10 Hz). Modulations of a higher frequency (lower period) form vertical structures. This structure is more clearly visible in the moving mass case (Figure 9(b)). Note that the irregularities in the $U(t)$ have been also found in the higher modes (Figures 10 to 13). A clearly modulated vertical structure can also be observed in Figures 11(b) and 13(b) (upper panel).

The moving mass causes changes in various modes in different ways. In the first mode (Figures 7 and 8), the course of $U(t)$ is simply dominated by the moving mass disturbances. In the second mode, the time series of the voltage output is deformed and modulated (Figures 9 and 10). Finally, in the third mode (Figures 10 and 11), the amplitude of voltage changes

Figure 13. Wavelet spectra of the piezoelectric voltage U and reference velocity U_{Ref} of the first mode in the case of fixed (a) and moving (b) tip mass at the frequency $f = 197.19$ Hz. and $f = 215.6$ Hz, respectively.

Table 2. Ratio of the standard deviation of the output voltage (measured on the piezoceramic part) to the standard deviation of the reference signal U/U_{Ref} (in the units V/(m/s)).

Vibration mode	Fixed mass (corresponding frequency)	Mass with clearance (corresponding frequency)
I	8.66 (9.96 Hz)	3.48 (9.96 Hz)
II	12.48 (76.25 Hz)	8.99 (78.80 Hz)
III	12.37 (197.19 Hz)	11.69 (215.60 Hz)

in the time domain to a fairly random degree. This disturbance effect of the moving mass generates the most characteristic frequencies detected in the modal analysis. In other words, the non-linear system with the fixed mass is susceptible to the synchronized motion, excluding multimode solutions. Furthermore, this synchronization is weakened by the moving mass, thus enabling multimode solutions to occur.

In Table 2, we present the standard deviation ratio of voltage output to reference signal. The ratio is smaller in all cases for the moving mass cases (the case with clearance). This is the effect of increasing damping in the system by enabling the mass to perform some relative motion.

Conclusions

We studied the effect of a moving mass in a simple beam system of energy harvesting. The modal analysis results and output voltage time series responses confirm the smearing of the resonance area, and consequently indicate the broadening of the output voltage around the mode I region. The non-linear effects of the moving mass (contact loss, impact, friction) are presumably due to a large frequency shift in the mode III area. Contrary to the fixed mass case, in the moving mass case we

observe a whole spectrum of the system characteristic frequencies which are also revealed by the modal analysis. In addition, it is worth noting that the improvement in the third vibration mode in the system with the moving mass is due to the change in mode shape. The third mode in the fixed mass model is torsional, while in the moving mass case it is flexural. Summing up, the discussed non-linear effects could improve device efficiency. It should be stressed that in contrast to other approaches with impacts,^{7,20} our proposal combines the effect of clearance, friction and impacts. Nonetheless, to provide more information about this effect, some more systematic studies are necessary for the fixed and moving mass configurations, including frequency sweeps response of voltage output.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was supported by the Polish National Science Center (A.R., M.B., and G.L.) under the grant agreement No. 2012/05/B/ST8/00080.

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Appendix I

Notation

D	electrical displacement vector in the piezoceramic layer
E	external electrical field between the electrodes
E_{elf}	electrical field energy
E_{els}	electrostatic energy on the electrodes of a piezoelectric patch
f_n and f	natural and excitation frequencies
$g = 9.81 \text{ m/s}^2$	gravity acceleration
G	gravity force
i	sampling index
$I = I_2 h^3 / 12$	geometrical moment of inertia
I_0	inertial moment of the beam
$K_{ac} = 25 \text{ m/(s}^2 \text{ A)}$	acceleration-current-transmission factor
$K_{fc} = 15 \text{ N/A}$	force-current-transmission factor
$\mathbb{L} = T - \Pi$	Lagrangian
M, K, and C	mass, damping, and stiffness matrices
M_i and M	tip mass
P	arbitrary point on the beam
P_W	wavelet power spectrum
P_i	constants defined in Appendix 1
Q	charge on the electrodes
R_{eq} and R_{DAQ}	equivalent and internal resistances of the DAQ computer card
R_1 and R	resistances in the voltage divider
s, v_p , and u_p	beam coordinates
t_i	sampling time
T	kinetic energy
U_{DAQ} and U	acquisition and output voltages
U_{Ref}	reference beam velocity
V_{ol}	volume of the piezo patch
W	work of the piezoelectric patch in moving electrical charge
$W_{\xi,n}(X)$	electrostatic energy on the electrodes of a piezoelectric patch
$X(t)$	wavelet transform of X
$\langle X(t) \rangle$ and σ_X	time series
y and f	average and standard deviation of X
$z = z_0 \cos \omega t$	vectors of generalized coordinates and generalized force
ξ and n	horizontal kinematic excitation of the beam
η	scale and time indices
θ	renormalized time variable
$\theta_0 = 6$	effective transduction constant
Π	center frequency
$\rho = 1070 \text{ kg/m}^3$	potential energy
$\Psi()$	mass density of the beam
$\psi(s)$	wavelet function
	mode shape of the beam

Appendix 2

The Lagrange equation of motion of the beam-mass system is derived as follows

$$\frac{d}{dt} \left(\frac{\partial \mathbb{L}}{\partial \dot{v}} \right) - \frac{\partial \mathbb{L}}{\partial v} = 0 \quad (10)$$

where $\mathbb{L} = T - \Pi$ is the Lagrangian. Corresponding parts of the above equations (with respect to the vertical bending beam geometry shown in Figure 14)

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{v}} \right) &= \rho A \left[\int_0^{l_1} \psi(s)^2 ds \ddot{v} + \int_0^{l_1} \psi(s) ds \ddot{z} \right. \\ &\quad \left. + (v\ddot{v} + 2\dot{v}^2) \int_0^{l_1} \left(\int_0^s \psi'(x)^2 dx \right)^2 ds v \right] \\ &\quad + M_t(t) \left[\ddot{v} + \ddot{z} + (v\ddot{v} + 2\dot{v}^2) \left(\int_0^{L_t} \psi'(s)^2 ds \right)^2 v \right] \\ &\quad + I_0 P_5^2 \left[\left(1 + \left(v^2 + \frac{1}{4} P_5^2 v^4 \right) (\psi'(l_1))^2 \right) \ddot{v} \right. \\ &\quad \left. + (2 + P_5^2 v^2) (\psi'(l_1))^2 v \dot{v}^2 \right], \\ \frac{\partial T}{\partial v} &= \left[\rho A \int_0^{l_1} \left(\int_0^s \psi'(x)^2 dx \right)^2 dx \right. \\ &\quad \left. + I_0 (\psi'(L_t))^4 \left(1 + \frac{1}{2} (\psi'(l_1))^2 v^2 \right) \right] v \dot{v}^2, \\ \frac{\partial \Pi}{\partial v} &= EI \left[v \int_0^{l_1} \psi''(s)^2 ds + 2v^3 \int_0^{l_1} \psi''(s)^2 \psi'(s)^2 ds \right. \\ &\quad \left. + \frac{3}{4} v^5 \int_0^{l_1} \psi''(s)^2 \psi'(s)^4 ds \right] \\ &\quad - g \left[\rho A \int_0^{l_1} \left(\int_0^x \psi'(s)^2 ds \right) ds \right. \\ &\quad \left. + M_t(t) \int_0^{L_t} \psi'(x)^2 ds \right] v \end{aligned} \quad (11)$$

Consequently, the Lagrange equation

$$\begin{aligned} &\left[M_t(t) + \rho A \int_0^{l_1} \psi(s)^2 ds + I_0 (\psi'(L_t))^2 \right. \\ &\quad \left. + (\rho A \int_0^{l_1} \left(\int_0^s \psi'(x)^2 dx \right)^2 ds \right. \\ &\quad \left. + M_t(t) \left(\int_0^{L_t} \psi'(s)^2 ds \right)^2 + I_0 (\psi'(L_t))^4 \right] v \ddot{v} \\ &\quad + \left[\rho A \int_0^{L_t} \left(\int_0^s \psi'(x)^2 dx \right)^2 ds \right. \\ &\quad \left. + M_t(t) \left(\int_0^{L_t} \psi'(s)^2 ds \right)^2 + I_0 (\psi'(L_t))^4 \right] v \dot{v}^2 \\ &\quad - \left[g \left(M_t \int_0^{L_t} \psi'(s)^2 ds + \rho A \int_0^{l_1} \left(\int_0^x \psi'(x)^2 ds \right) ds \right) \right. \\ &\quad \left. - EI \left(\int_0^{l_1} \psi''(s)^2 ds + 2v^2 \int_0^{l_1} \psi''(s)^2 \psi'(s)^2 ds \right) \right] v \\ &= - \left[M_t + \rho A \int_0^{l_1} \psi(s) ds \right] \ddot{z} \end{aligned} \quad (12)$$

Figure 14. Schematic of the vertical beam harvester. M_t denotes the tip mass attached to the elastic beam, while v and u denote the horizontal and vertical displacements of the mass. Here, $L = l_1$, and $z = z_0 \cos \omega t$ is the kinematic excitation.

where ρ denotes the density of the beam mass, A is the transverse cross section of the beam, I_0 is the inertia moment of the beam, I denotes the geometrical moment of inertia, E is the Young modulus, l_1 (Table 1) is the length of the beam, $L_t = (l_1 - l_3)$, $\psi(s)$ is the function describes the shape mode of the beam, while prime means the spatial derivative $\psi'(s) = d\psi(s)/ds$. M_t denotes the sliding tip mass, which interacts on the beam by friction and impacts forces.

Assuming the displacement $\psi(s)$ as the first linear mode shape of the beam

$$\psi(s) = 1 - \cos\left(\frac{\pi s}{2l_1}\right) \quad (13)$$

In equation (12), one can define constants P_1 – P_9 as

$$P_1 = \int_0^{l_1} \psi(s)^2 ds = l_1 \left(\frac{3}{2} - \frac{4}{\pi} \right),$$

$$P_2 = \int_0^{l_1} \psi(s) ds = \frac{l_1}{\pi} (\pi - 2),$$

$$\begin{aligned}
P_3 &= \int_0^{l_1} \left(\int_0^s \psi'(x)^2 ds \right)^2 ds = \frac{\pi^2}{384l_1} (2\pi^2 - 9), \\
P_4 &= \int_0^{L_t} \psi'(s)^2 ds = \frac{\pi^2}{8L_t}, \\
P_5 &= \psi'(L_t) = \frac{\pi}{2L_t}, \\
P_6 &= \int_0^{l_1} \psi''(s)^2 ds = \frac{\pi^4}{32l_1^3}, \\
P_7 &= \int_0^{l_1} \psi''(s)^2 \psi'(s)^2 ds = \frac{\pi^6}{512l_1^5}, \\
P_8 &= \int_0^{l_1} \psi''(s)^2 \psi'(s)^4 ds = \frac{\pi^8}{4096l_1^7}, \\
P_9 &= \int_0^{l_1} \left(\int_0^s \psi'(x)^2 ds \right) ds = \frac{1}{16} (\pi^2 - 4) \quad (14)
\end{aligned}$$

Finally, equation (12) simplifies to

$$\begin{aligned}
& [M_t + \rho AP_1 + I_0 P_5^2 + (\rho AP_3 + M_t P_4^2 + I_0 P_5^4) v^2] \ddot{v} \\
& + [\rho AP_3 + M_t P_4^2 + I_0 P_5^4] v \dot{v}^2 \\
& - [g(M_t P_4 + \rho AP_9) - EI(P_6 + 2P_7 v^2)] v \\
& = -[M_t + \rho AP_2] \ddot{z} \quad (15)
\end{aligned}$$