Identification of Parameters for the Middle Ear Model

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Key Words

Finite-element model · Middle ear · Parameter estimation · Tympanic membrane · Modal analysis · Laser scanning vibrometer

Abstract

This paper presents a method of parameter identification for a finite-element model of the human middle ear. The parameter values are estimated using a characterization of the difference in natural frequencies and mode shapes of the tympanic membrane between the model and the specimens. Experimental results were obtained from temporal bone specimens under sound excitation (300–3,000 Hz). The first 3 modes of the tympanic membrane could be observed with a laser scanning vibrometer and were used to estimate the stiffness parameters for the orthotropic finite-element model of the eardrum. A further point of discussion is the parameter sensitivity and its implication for the identification process.

Introduction

Increasingly, the functionality of the human hearing organ is investigated by the help of models. Two groups of models are currently utilized. The first group consists of

electroacoustic circuit models which have a long history based on the close link between acoustic and electrical engineering [Hudde and Weistenhöfer, 1997; Goode et al., 1994]. The second group is made up of structural mechanical models, mainly finite-element models which gain in importance more and more [Eiber and Kauf, 1994; Wada et al., 1992; Beer et al., 1997; Williams et al., 1997]. The latter have the advantage that mechanical functionality is directly related to mechanical parameters, thus avoiding complicated analogies.

The quality of the model essentially depends on two factors, the model's structure which determines its basic capabilities and the proper choice of parameters. Parameter identification is therefore as important as the structural design. Several mechanical parameters for use in finite-element models have already been presented by various authors. Previous investigations had been made in very different ways. Von Békésy [1949] and Kirikae [1960] for instance determined Young's modulus on specially prepared pieces of tympanic membrane. Other authors used frequency response functions (FRFs) of the entire middle ear for parameter estimation, where parts of the specimen were successively removed [Wada et al., 1990]. Funnell and Laszlo [1982] published a review on mechanical properties of the eardrum.

Looking at the currently available data, the task of parameter identification is still present. The advanced models contain many parameters to describe the middle

Fig. 1. Finite-element model of the human tympanic membrane with malleus and ligaments (the musculus tensor tympani was removed for the calculations. CG = Center of gravity).

ear in more detail, and only a few parameters have been identified with the required precision. The main problem, however, arises from the inverse formulation of the parameter estimation problem. It is in principle an improperly posed problem and there is no unique solution. That means parameter values can only be validated by comparing measurements and model predictions. For this reason, we need several comparable investigations and quantities of comparison with a high content of information.

In this paper, we want to introduce a method of parameter identification based on dynamic investigations. The objective of the work is to estimate material parameters for a mechanical model of the human middle ear. The investigations were performed on a submodel consisting of the tympanic membrane with malleus and ligaments. The corresponding boundary value problem in the form of finite elements is presented in detail elsewhere [Beer et al., 1997] and is not the object of the present work. Quantities of comparison for parameter estimation are natural frequencies and mode shapes.

In the first section we introduce the parameters of the model and the numerical results from Beer et al. [1997], which were used for the identification. The second part focuses on the experimental work, the measurement of vibration patterns of the tympanic membrane via laser

scanning vibrometry and the modal analysis to obtain natural frequencies and mode shapes. The following section then presents the parameter estimation method and deals with problems of identification.

Parameters of the Finite-Element Model

The identification is based on a model consisting of tympanic membrane, malleus and ligaments. Figure 1 shows the finite-element model where the eardrum is modeled as a thin curved shell, the malleus as a rigid body with inertial properties and the ligaments as pipe elements (with longitudinal, bending and torsional stiffness). A detailed description of the model is given by Beer et al. [1997].

The parameters of the model can be divided into 3 groups: geometrical, inertial and stiffness parameters. Items of the first 2 groups were determined on the basis of geometric measurements [Drescher, 1995]. The inertial parameters were calculated with density values from the literature [Kirikae, 1960] as described elsewhere [Beer et al., 1997]. The present investigations deal with the stiffness parameters for the eardrum. The eardrum is formed by shell elements with orthotropic material properties.

Fig. 2. Modes of the finite-element model with initial parameters (**a–c**) and final parameters (**g–i**) and modes from experimental modal analysis (**d–f**); view from the tympanic cavity with the malleus positioned downwards; similar colors mean similar displacements, for mode 3 all areas vibrate in phase, for mode 4 and 5 red and blue areas vibrate out of phase: **d** The points of the FRFs of figure 3. 1 = Red line; 2 = green line; 3 = blue line; p.s. = posterior-superior region; p.i. = posterior-inferior; a.i. = anterior-inferior; a.s. = anterior-superior.

The two parts pars tensa and pars flaccida are modeled with different materials, each part having the following parameters: radial and tangential Young's modulus (according to the fiber arrangement), shear modulus and Poisson's ratio. An additional factor *RMI* enables the bending stiffness *K* to be reduced according to the following formula:

$$K = RMI \frac{E h^3}{12 (1 - \nu^2)},$$

where *E* is Young's modulus, *h* is the thickness and *ν* is Poisson's ratio [ANSYS®]. Values of the *RMI* factor are in the range from 0 to 1.

Since the model has been designed to investigate dynamic characteristics, we used results of dynamic analyses to identify the parameters. Natural frequencies and mode shapes were used as quantities of comparison. Figure 2a–c shows the modes 3–5 of the model which were calculated with initial parameter values taken from the literature and previous investigations. Modes 1 and 2 (not shown here) are mainly vibrations of the malleus in its suspensions. These modes were not used for parameter estimation because the three-dimensional malleus vibration could not be measured with the required accuracy.

Experimental Modal Analysis

Experiments were made on specially prepared temporal bone specimens. We used 3 specimens (with approximately the same size and shape) which gave very similar results. The results for one of these specimens are presented here.

The specimens were taken 48 h post mortem and stored in saline solution. In order to obtain the same boundary conditions as for the model, the inner ear as well as incus and stapes were removed. The specimens thus consisted of the tympanic membrane, malleus and ligaments and a part of the auditory canal. Due to the preparation the tensor tympani muscle was cut and therefore also removed from the model. Measurements were performed within 6 days after death. The specimen was excited by a sound source placed in the auditory canal. A microphone just in front of the tympanic membrane supplied the reference signal (sound pressure) for the measurement. The eardrum vibrations were measured with a laser scanning vibrometer from the side of the tympanic cavity (Polytec OFV 055 optical scanning head with close-up unit and OFV 3001 S controller). Measurements were performed in a frequency range of 300–3,000 Hz with

chirp excitation at about 90 dB sound pressure level. We obtained FRFs $H(j\omega)$ in the form of displacement versus sound pressure for about 100 evenly spaced points at the eardrum (fig. 3):

$$H(j\omega) = \frac{S_{yy}(j\omega)}{S_{xy}(j\omega)}.$$

This calculation of the transfer function compensates the input noise from the microphone since there was more noise on the input signal than on the output. The numerator $S_{yy}(j\omega) = Y(j\omega) Y(j\omega)^*$ is the auto-spectrum of the response, and the denominator $S_{xy}(j\omega) = Y(j\omega) X(j\omega)^*$ is the cross-spectrum between response and reference, with $Y(j\omega) = F\{y(t)\}$ the Fourier transform of the response (displacement) and $X(j\omega) = F\{x(t)\}$ the Fourier transform of the reference (sound pressure); * = complex conjugate form. The FRFs show 2 resonance regions and indicate a strongly damped structure. The strong damping causes higher modes not to appear as resonance peaks in the FRFs.

Considering these facts we performed a modal analysis to obtain the natural frequencies and mode shapes in the measured frequency range. Standard analyses of the commercially available program IDEAS were used for this purpose. Modal analysis yielded 3 modes with modal properties of (frequency *f*/modal damping *ζ*): mode 3 (908 Hz/10%), mode 4 (1,355 Hz/15%), mode 5 (2,012 Hz/15%), according to the definition of the eigenvalue $s = \delta + j\bar{\omega} = -\zeta\omega + j\omega\sqrt{1 - \zeta^2}$ with $\omega = 2\pi f$. These modes were assumed to correspond to the calculated modes 3–5 of the model and were numbered accordingly. The measured mode shapes were very complex, i.e. the phase angles between different points took on values between 0 and 180°, but the modes were forced to real mode shapes, i.e. phase angles of either 0 or 180°. This simplification was necessary in order to compare measured and calculated mode shapes. There is no damping included in the model so far and even proportional damping as used in other models [Wada et al., 1992] will not solve this problem because the measured complex mode shapes indicate nonproportional damping. At this point some further investigations are necessary.

The modes obtained from the modal analysis are shown in figure 2d–f. A small part of the pars tensa and nearly the whole pars flaccida were hidden by remaining parts of bone and could not be measured. The mode shapes are similar to the calculated ones of the model so that the model's structure and the initial parameters can be used as a good basis for parameter estimation.

Fig. 3. FRFs (amplitude in m/Pa), measured at different points of the tympanic membrane (see fig. 2d).

Parameter Identification

An important step in the process of parameter identification is the investigation of the parameter's sensitivity, i.e. how do modifications of parameter values influence certain results as for instance natural frequencies and eigenvectors (mode shapes). The aim of these investigations is to extract those parameters with most influence on the results because these parameters can be most precisely estimated. It is seldom possible to estimate all parameters at once because of numerical problems. Therefore the parameters with the highest sensitivity are estimated first and then the others successively. In figure 4 sensitivities of natural frequencies due to parameter modifications are shown. Dark squares indicate high sensitivity and white squares lower sensitivity. The figure indicates that for instance the ligaments' Young's moduli hardly influence the natural frequencies in contrast to the radial Young's modulus of the pars tensa. According to the sensitivity matrix the parameters for the ligaments can hardly be obtained. For this reason they were not modified. The other parameters were successively estimated, first the radial Young's moduli and the RMI factors (dark squares for these parameters at natural frequencies 3, 4 and 5). The results for the parameters of the pars flaccida are less reliable because the sensitivity value are low. Furthermore there is only little measurement information since most of the pars flaccida was hidden.

Fig. 4. Sensitivities of natural frequencies due to modified parameters. E_r = Radial Young's modulus; E_t = tangential Young's modulus; G = shear modulus; ν = Poisson's ratio; RMI = factor to reduce bending stiffness; p.t. = pars tensa; p.fl. = pars flaccida. The figure shows sensitivities relative to each other, i.e. the darker the square, the higher the sensitivity.

Table 1. Final set of parameters

Pars tensa		
Radial Young's modulus (E_r)	85.7	Nmm ⁻²
Tangential Young's modulus (E_t)	48	Nmm ⁻²
Poisson's ratio (ν)	0.35	
Shear modulus (G)	6.2	Nmm ⁻²
RMI	0.78	
Pars flaccida		
Radial Young's modulus (E_r)	45.6	Nmm ⁻²
Tangential Young's modulus (E_t)	20	Nmm ⁻²
Poisson's ratio (ν)	0.43	
Shear modulus (G)	8.5	Nmm ⁻²
RMI	0.29	

Only modified parameters are shown. Other parameters of the model along with a detailed description of the model are given by Beer et al. [1997].

Similar investigations can be made in terms of the eigenvectors of the system. These sensitivity values yield insensitive elements of the eigenvector which are excluded from the parameter estimation to improve the condition of the problem. The investigations on parameter sensitivity were all performed with the initial model.

The parameter identification is based on results of dynamic analyses (modal analysis). The error between model and specimen is expressed by the vector of residues. This vector ($\underline{v}$) is formed from the differences between measured and calculated natural frequencies (ω_r) and eigenvectors ($\underline{u}_r$):

$$\underline{v}^T = (\dots, \omega_{rm} - \omega_r, \dots, \hat{\underline{u}}_{rm}^T - \hat{\underline{u}}_r^T, \dots).$$

The natural frequencies (ω_r) and eigenvectors ($\underline{u}_r$) of the model contain the unknown parameter values. The index m is used for measured quantities; r is the index for the mode. The present investigations were performed with modes 3–5.

We used a weighted least squares approach for the optimization problem:

$$Z(p) = \underline{v}^T \underline{G} \underline{v} \rightarrow \text{Min.}$$

$\underline{G}$ is a diagonal weighting matrix for the different accuracy and reliability of the elements of the residue vector. It is also used to normalize the elements of the residue vector since natural frequencies and eigenvectors have different units. According to the structure of the residue vector, the first part of $\underline{G}$ contains the squared reciprocal of the largest natural frequency. The second part contains for each eigenvector the squared reciprocal of the largest ele-

ment of this eigenvector. Zero values were placed in $\underline{G}$ for elements that were not utilized for the identification process. Since only a few elements (1–5) of each eigenvector were employed, the weighting matrix contains mainly zero elements in the second part. All used natural frequencies and elements of the eigenvectors were equally weighted for reasons of simplification.

Z is a function of the unknown parameter values and it will have a minimum for an optimal set of parameters. Since the problem is nonlinear in the parameters, the solution has to be obtained with iterative methods for nonlinear optimization like gradient methods or stochastic search methods. Initial parameters for the optimization problem were taken from the literature and previous investigations [Beer et al., 1997; Kirikae, 1960]. We used stochastic methods to scan the parameter space for possible minima of Z first. Following that gradient methods were applied to actually obtain the solution. A detailed description of the different optimization algorithms can be found in Natke [1983]. The result of the parameter estimation process, the final set of parameters, is shown in table 1.

Conclusions

We presented a method to identify parameters for a middle ear model on the basis of dynamic properties (i.e. natural frequencies and mode shapes). The investigations were performed on a submodel consisting of eardrum, malleus and ligaments.

The FRFs measured in a part of the middle ear contain more characteristics (i.e. clear resonance peaks) than transfer functions of the whole middle ear. Information about the system is therefore easier to detect in these functions. Another advantage is the reduced amount of parameters for submodels, thus improving numerical stability.

The sensitivity analysis showed the different influence of the parameters on the dynamic behavior of the model (fig. 4). It proved to be a good preliminary analysis to the identification process and helped to increase the accuracy of the results.

When comparing the parameter values with other models, the overall shape and structure have to be considered since they also have a considerable influence on the mechanical behavior. Different geometries with appropriate values for the material parameters lead to a similar dynamical behavior. The same applies to the boundary conditions.

In conclusion of the present investigations, we consider the following problems as fields for further activities: (1) including damping into the model as indicated by the experimental results; (2) the parameters must be evaluated with results of further measurements; these investigations should be aimed to get information about the range

for the parameters by measuring very different specimens; (3) parameters for the remaining parts of the model (especially for the ligaments and joints) must be identified in order to get a complete model. These investigations can be performed with other submodels and finally with the whole middle ear model.

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